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# THE EFFECT OF DYNAMIC HEDGING OF OPTIONS POSITIONS ON INTERMEDIATE-MATURITY INTEREST RATES

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**ABSTRACT**

When interest rates change, interest rate options dealers buy or sell securities to adjust the hedging positions that they have taken in order to offset their options exposures. The net result of this trading activity, which is unrelated to economic fundamentals, can be to push interest rates further in the direction they were moving. Such "feedback" effects interfere with the short-term dynamics of interest rate movements and can alter the shape of the yield curve, especially when changes in interest rates are large. Our empirical results confirm the existence of a positive feedback from the activity in the euro-denominated interest rate options market to the european yield curve. This finding can be useful for risk management purposes but also for analysts and policy makers when interpreting short-run movements in the yield curve as signals of future economic activity.

*Keywords*: Interest rate options; Dynamic hedging; European yield curve
*JEL classification*: E43; G130; G110; G140;

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## 1. Introduction

This paper focuses on a certain aspect of the interest rate options market in Europe. More precisely, it looks into the possible consequences from the hedging of positions by interest rate options dealers on medium term spot interest rates. The effect that the paper seeks to identify may be produced when the option dealers use dynamic hedging in order to maintain a so-called “delta neutral” position. Such adjustments in positions involve a pattern of buying (selling) the underlying security after its price has increased (fallen) which may finally result in a positive feedback to the price of the underlying security. In other words, the transactions realised on behalf of the options dealers in order to maintain a “delta neutral” position may result in further upward (downward) pressure on spot interest rates after an initial upward (downward) shock to rates has occurred, for example, following a monetary policy-induced change in intervention rates. Hull (1993) provides comprehensive information on the pricing and hedging of interest rate options. It should be noticed that the effect under investigation is implemented in a purely mechanistic way, i.e. it is unrelated to expectations of market participants about the future level of interest rates.

Kambhu and Mosser (2001) have studied the positive feedback effect from the dollar-denominated options market on the American yield curve. We, too, conduct an analysis, similar in spirit, that seeks to verify a positive feedback effect from the euro-denominated options market on the European yield curve. In fact, the data set we employ derives from the German markets. Furthermore, our findings can be interestingly compared to those of Kambhu and Mosser.

The article is organised as follows: Section 2 provides a description of the interest rate options market in terms of transaction volumes and the different tendencies observed. Section 3 looks into the details of dynamic hedging and the way it may be implemented. The model is described in Section 4. Section 5 presents the data sets used while Section 6 the estimation results. The study concludes with Section 7 where certain policy implications are also put forward. Additional details regarding the estimation procedure in terms of econometric tests and results are included in the Appendix.

## 2. The interest rate options market

The bulk of interest rate options are traded over-the-counter (OTC). They are mainly caps and floors but also swaptions. An interest rate option/warrant is an option contract that gives the right to pay or receive a specific interest rate on a predetermined principal for a set period of time. An interest rate cap (floor) is an option that pays the difference between a floating interest rate and the cap (floor) rate. A cap (or floor) is considered by both theorists and practitioners, and consequently treated for pricing and hedging purposes, as a portfolio of interest rate options. More specifically, a cap (floor) is treated as a string of call (put) options on the future values of the reference interest rate over the contract period. An interest rate swaption is an option to enter into an interest rate swap contract, purchasing the right to pay or receive a certain fixed rate. Swaptions, as instruments to relocate interest rate risk, are, to a significant extent, similar to caps (or floors).

Caps and floors are options on future short-term interest rates, usually six-month Libor - now Euribor - rates. In an interest rate cap (floor) contract, the buyer receives the difference between the Euribor rate prevailing at the time and the strike rate specified in the options contract when the reference rate is above (below) the strike rate, and nothing otherwise. Most cap and floor contracts are written for several years. The contract may cover the entire yield curve, up to ten years.

According to the most recent Bank for International Settlements (BIS) data (BIS, 2002), the size of the euro-denominated OTC interest rate derivatives market, in terms of daily turnover, is USD 231.5 billion. Almost two thirds of this is attributed to interest rate swap activity. Daily turnover in interest rate options amounts to USD 10.8 billion, which is only slightly below the corresponding figure (USD 12.2 billion) for the US dollar-denominated market for similar instruments. This fact reveals the comparable size of the two markets. Table 1 shows the evolution of positions in the global derivatives market by type of instrument.

The level of the market's net demand for interest rate options is of particular importance to our analysis. Dealers, by quoting two-way prices and being permanently present in the market, buy and sell options on a continuous basis. The total amount of options sold in excess of the total amount of options purchased is defined as the market's net demand. The BIS survey data reveal that, in the euro-denominated interest rate options market, dealers sell to customers some 25% more options than they purchase. They are therefore absorbing the market's net demand for interest rate options and, since they want to cover themselves for the interest rate risk inherent in their net options positions, they act appropriately in order to hedge those positions. Indeed, the ability of dealers to trade in a broad range of markets gives them the ability to execute hedging transactions faster and at a lower cost than other market participants, making them more willing than others to absorb the market's net demand for options.

Regarding the maturity of the contracts, from Table 2 it is apparent that there has been a robust growth in longer-term interest rate contracts (over five years). Between 1995 and 2001, the size of relevant positions increased almost fivefold. Although the figures shown concern positions globally, it is almost certainly the case that there has been a similar evolution in the euro-denominated interest rate derivatives positions, and, more precisely, in the interest rate options positions. The latter is of significant relevance to our analysis since the increase in the absolute size of positions and the lengthening of contract maturity may be factors that strengthen the effect that we are attempting to uncover.

## 3. Dynamic hedging

Hedging is central to the theory of options pricing. Arbitrage valuation models, such as that of Black and Scholes (1973), depend on the idea that an option, by using the underlying asset, can be perfectly replicated. It is thus possible to create a portfolio which hedges absolutely the position in the option. Delta-hedging strategies are recipes for replicating the payoff of a complex security by sophisticated dynamic trading of simpler securities. Participants in options markets commonly apply the kind of delta-hedging strategies implicit in Black and Scholes, at least approximately, in order to reduce their risk exposure.

The implementation of a delta-hedging strategy has two dimensions. First, there is the required size of the position in the underlying asset, which, in turn, depends upon the option-pricing model adopted. The option's delta is defined as the first partial derivative of the option price with respect to the price of the underlying asset. It shows the sensitivity of the option price to changes in the price of the underlying. The option's delta also indicates the size of the position in the asset required for hedging. The delta varies with the price of the asset and its value lies within the interval [0, 1]. Delta's rate of change is not constant either, being lower for delta values close to zero and higher for delta values close to one.

Since the seminar work of Black and Scholes (1973) and Black (1976), several more refined models for option pricing have emerged. Bühler *et al* (1999) offer a review of the various models developed as well as an empirical evaluation of the performance of the most popular interest rate options pricing models from the perspective of supervising the interest rate risk. In an earlier work, Stapleton and Subrahmanyam (1993) offer a transformation of the Black-Scholes formula as a simple and practical tool for valuing interest rate options by employing forward rates from the yield curve. We use this specification in the model below. Forward rates can be created synthetically by appropriately trading zero coupon bonds whose maturities straddle the maturity of the forward rate. Caps are priced as the sum of a series of call options for successively more distant reset dates. Since we are working on an aggregate (market) level, the "correct" hedging of particular positions is not of immediate interest to our analysis. The advent, however, of more refined pricing models may have had an effect on the functioning of the market in terms of the choice of instruments for delta-hedging available in different market segments with different liquidity characteristics.

The second dimension in implementing a delta-hedging strategy is the appropriate timing for rebalancing. Much of the theory of options assumes that markets are frictionless, a hypothesis that implies the absence of transactions costs. In fact, theoretically once an option (short) position is taken, the replicating portfolio is created containing delta units (long) of the underlying and, from that time on, it is continuously rebalanced in a self-financing manner. Continuous rebalancing means that at all times, from the creation of the position until the option's expiry and as delta varies over time, the replicating portfolio contains exactly delta units of the underlying asset.

In practice, rebalancing takes place only at discrete time intervals and the reason for that is the existence of transactions costs. The immediate consequence of discrete rebalancing is the emergence of the so-called *tracking error*. In discrete time and in between two consecutive rebalancings, the value of the replicating portfolio may not match exactly the size of the position in the derivative thus leaving this position in the interim less than fully hedged. Most importantly, rebalancing has an immediate effect on the dealer's P&L account: as the number of rebalancings increases, the cumulated loss due to transactions costs increases as well. Clearly, there is a tradeoff between the consequences on P&L and a less accurate hedging position or, in other words, between the size of the transactions cost and the degree of the dealer's risk aversion.

Several articles have been written on the issue of dynamic hedging under transactions costs. Clewlow and Hodges (1997) study the problem by maximising the trader's expected utility and then adopting an optimal control approach. One aspect of their solution indicates that even if an options book is currently exactly delta-hedged under Black-Scholes, taking into account transactions costs, it may nevertheless be optimal to move the hedge away from the Black-Scholes value. Bertsimas *et al* (2000) adopt a different approach in dealing with the issue by focusing their analysis uniquely on the aspect of the magnitude of the replication error. They derive a measure of the tracking error that can be used to infer an option-specific, optimal number of rebalancing periods.

Dynamic hedging in discrete time is directly incorporated into the model we adopt for our purposes. In fact, the model is explicitly written in terms of differences in the value of each explanatory variable -i.e. the forward rates. The difference is the change in the variable's value over a specific time interval. By investigating which difference should be considered as optimal within our model, we get an estimate of the general market tendency regarding the time period between rebalancings.

## 4. The model

Most cap and floor contracts are written for several years. The dealer's exposure to every single (e.g. six-month) forward interest rate may thus span the entire yield curve out to ten years. For maturities up to two or three years, dealers directly hedge every single forward interest rate exposure. In this time horizon, the underling market for forward interest rates has suitable characteristics in terms of contract availability and sufficient liquidity for this purpose. For maturities, beyond three years, however, dealers hedge longer sections of the yield curve in blocks, broken at those points where the markets for the underlying securities are most liquid. For example, an options dealer may hedge all of the (e.g. six-month) forward rates of between three and five years in terms of a single exposure to the three-to-five year forward interest rate. Similarly, all the six-month forward rates of between five and ten years are hedged in terms of a single exposure to the five-to-ten year forward interest rate.

A position in a forward interest rate is created synthetically by appropriate trading of bonds whose maturities straddle the maturity of the forward rate. Taking a long position in the five-year bond and a short position in the ten-year bond attains a long position in the five-to-ten year forward interest rate[1]. Similarly, taking a long position in the three-year bond and a short position in the five-year bond attains a long position in the three-to-five year forward interest rate. Therefore, hedging of exposures to either of these forward rates requires trading in the five-year bond. When positions are dynamically (delta-) hedged, they are adjusted at discrete time intervals as interest rates change over time.

We shall attempt to verify the existence of the feedback effect from dynamic hedging of options positions on the five-year spot rate by examining how the latter reacts to changes in the three-to-five year and five-to-ten year forward rates. Caps are hedged by maintaining long positions in the forward rates while floors by maintaining short positions in the forward rates.

An initial steepening of the yield curve[2] will cause forward rates to rise. Long positions in the three-to-five and five-to-ten forward interest rates will adjust in the following way: long positions in the three-to-five forward rate require further short selling of the five year bond, action that exerts downward pressure on the price of the bond, translating into an increase in its yield. Long positions in the five-to-ten forward rate require additional buying of the five-year bond, action that exerts upward pressure on the price of the bond and therefore a downward pressure on the its yield. Similar effects on the five-year rate will be produced when adjusting short

[1] This position gains money if the forward rate increases.
[2] Flattening, if the curve is inverted.

positions in the three-to-five and five-to-ten year forward rates following an initial steepening of the yield curve.

Table 3 summarises the effects of dynamic hedging adjustments on the five-year rate. Changes in the three-to-five forward rate will always induce changes in the five-year rate in the same direction. By contrast, changes in the five-to-ten forward rate will always induce changes in the five-year rate that are in the opposite direction.

We test for the presence of the described relationship by estimating the following equation:

$$\Delta r_5 = c + \alpha Z(-1) + \beta_1 \Delta F_{3,5}(-1,-t) + \beta_2 \Delta F_{5,10}(-1,-t) + \beta_3 \Delta r_5(-1,-t) + \varepsilon \qquad (1)$$

where

$\Delta r_5$ is the one-week change in the five-year interest rate

$\Delta F_{3,5}(-1,-t)$ is the $t$-1 week change in the three-to-five year forward rate, lagged one week

$\Delta F_{5,10}(-1,-t)$ is the $t$-1 week change in the five-to-ten year forward rate, lagged one week

$\Delta r_5(-1,-t)$ is the $t$-1 week change in the five-year spot rate, lagged one week

$Z$ (-1) is the error-correction term from the cointegration relationship in the levels of the five-year and forward rates, lagged one week.

If the positive feedback from dynamic hedging of options positions on the five-year rate is verified by our data, the estimated coefficients of the changes in forward rates must be statistically significant and have the appropriate signs, that is: $\beta_1 > 0$ and $\beta_2 < 0$. In addition, the coefficient $\alpha$ should have negative sign and be statistically significant. The latter guarantees the existence of a long-run equilibrium relationship within which short-term deviations due to the positive feedback effect take place.

## 5. The data set

The purpose of our analysis is to test for the existence of a positive feedback effect, due to dynamic hedging, from the euro-denominated options market to the European yield curve. We consider the German market as the benchmark market for the purposes of our analysis. The German OTC market for interest rate options started in 1989. Our data set is divided into two subsets, one from the German government paper market and the other from the German market for interest rate swaps (IRS) –the German IRS market. All data are weekly.

The sample period for the data from the German government paper market ranges from 1991 to 2002. The three, five and ten-year rate series are the respective yields from the German government bond yield curve. They were obtained from Bloomberg by retrieving the generic-yield series provided which go back as far as 1991. We have calculated the forward rates from those three, five and ten-year yields by employing the usual formula. Diagram 1 shows how the values of five-year yield, three-to-five and five-to-ten year forward rates have evolved over the sample period. There seems to be a change in behaviour between the pre-1992 period and the rest of the period under consideration. The beginning of the nineties was a period of particular turbulence for european markets which reached its peak with the ERM crisis in September 1992. Thereafter, markets were more convinced about the possibilities for the successful accomplishment of EMU and consequently they functioned more smoothly.

The sample period for the German IRS market is from 1990 to 2002. The three, five and ten-year swap rate series were also obtained from Bloomberg where the series provided go back as far as 1988. From those swap rates, we calculated the forward rates necessary for the estimation of our model. Diagram 2 depicts the evolution over time of the five-year swap rate and the three-to-five and five-to-ten year forward rates. The aforementioned change in behaviour between the pre-1992 period and the rest of the period under consideration is reconfirmed by the data from the swaps market. This change led us to restrict our sample to the period from 1993 to 2002[3].

[3] The need to restrict the sample was also confirmed by the fact that the cointegrating relationship was problematic when we employed full sample data. For the estimation of (1) using the data from the IRS

## 6. The results

Equation (1) was estimated in various ways by employing different time series. The coefficient $\beta_3$ was never statistically significant in any of the initial estimations. We, therefore, dropped the variable $\Delta r_5(-1,-t)$ from (1) and concentrated solely on $\beta_1$ and $\beta_2$. Technical details concerning our estimations can be found in the Appendix.

First, we deal with the issue of the appropriate value of $t$ in equation (1), i.e. the appropriate time span for calculating the difference in the forward rates. To this end, we estimate equation (1) repeatedly for $t = 2,3,4,...,50$ using data from the government paper market[4]. Diagram 3 presents the statistical significance of the relationship between spot and forward rates, as captured by the value of the F-statistic of the joint distribution of $\beta_1$ and $\beta_2$, corresponding to various periods of changes in forwards, ranging from one to forty-nine weeks. Taking the sixteen-week change in forward rates results in the highest statistical significance of the relationship between spot and forward rates. As observed in diagram 3, the statistical significance of the examined relationship increases progressively for differences from nine to sixteen weeks and then weakens, though relatively slowly, becoming insignificant for differences higher than forty-three weeks. In addition, diagram 4 shows the confidence bands for the estimated coefficients $\beta_1$ and $\beta_2$ corresponding to the various differences considered. The confidence bands depict the sign, size and statistical significance of the estimated coefficients. The estimated coefficients are both statistically significant and have the right sign when considering changes in forward rates ranging between nine and seventeen weeks.

The information contained in our data set, therefore, indicates that, in the european market, rebalancing of positions maintained for the purpose of hedging interest rate option exposures takes place every four months[5]. This finding is remarkably different from the relevant period that Kambhu and Mosser (2001) have

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market, the sample period was further reduced to the period 1994 – 2002 in order to obtain a satisfactory cointegrating relationship.

[4] Similar results are obtained when using data from the IRS market.

[5] This is a realised average period ensuing from the entire sample of observations. It must be interpreted with some caution since rebalancing, as shown below, depends upon the variability of forward rates.

estimated for the US dollar-denominated options market. They estimate a much shorter rebalancing period of five weeks, becoming only two weeks towards the end of the nineties. This difference can be explained by either the existence of much higher transactions costs in the european market and/or by differences in risk aversion. By the latter, it is not meant that european dealers are less risk averse than their american counterparts but their risk exposure may be, in effect, relatively limited. We have already pointed out that market activity figures show a net demand for euro-denominated interest rate options of 25%, while the respective figure presented by Kambhu and Mosser for the US dollar-denominated market is double that (50%). The two markets are of a comparable size.

The estimation results are presented in Tables 4 and 5. Equation (A) shows the relationship between the five-year spot rate (BOBL) and the three-to-five and five-to-ten year forward rates extracted from the government yield curve. The estimated coefficients $\beta_1$ and $\beta_2$ are statistically significant and have the appropriate signs. Overall, our results reveal that changes in forward rates do influence future spot rates. This influence, nevertheless, appears rather limited. The size of the estimated coefficients $\beta_1$ and $\beta_2$ shows that only some 7-8% of the change in forward rates is transmitted to the spot rate. Moreover, the model's explanatory power (as captured by the $R^2$) is low and reveals that the variation in forward rates explains only some 4% of the variation in the spot rate. This holds true on average for the entire sample of observations. Below, we explore the issue further by considering the size of the feedback effect during periods of large sustained changes in the five-year rate. The low explanatory power of the model also indicates that arbitrage opportunities between forward and spot rates are not systematically present. Consequently, the view of an efficient functioning of the market is reinforced. The results just described are similar to those of Kambhu and Mosser in the case of US dollar-denominated interest rate options market[6].

Equation (B) in Table 5 uses data from the German IRS market. The results obtained are statistically significant but not as strong as the results in equation (A). In

[6] According to Kambhu and Mosser , the variation in forward rates explains 4% of the variation in the spot rate while approximately 20% of the change in forward rates is transmitted to the spot rate.

general, the data extracted from the government yield curve appear more suitable for explaining the relationship under investigation.

Equation (C) shows the positive feedback effect on the BOBL yield from the forwards extracted from the IRS curve, while equation (D) shows the positive feedback effect on the five-year swap rate from the forwards extracted from the government yield curve. The statistical significance of the estimated coefficients in equation (D), compared to those in (C), is higher and the same holds true for the size of $R^2$. This finding indicates that, although the positive feedback is detected in both data sets, the direction of the spill over is from the market for government paper to the swaps market[7].

Given the heterogeneity of government bond markets, the euro swap curve has emerged recently as the new benchmark for european fixed income markets. The introduction of the euro led to the creation of a large and liquid market in euro-denominated swaps[8]. In addition, the swap market presents significant advantages over the government paper market for hedging purposes[9]. The detected direction of the spill over from the bond market to the swaps market is contradictory to this recent development (a development observed after the introduction of the euro) but can be possibly explained by the longer period covered by our sample (1993 – 2002).

In order to examine further the detected positive feedback effect, we partition our sample into two sub-samples, one including only large changes in forward rates and the second consisting of small changes in forward rates[10]. The relevant regression results appear in Table 6. The estimated coefficients $\beta_1$ and $\beta_2$ are statistically significant only in the case of the sample of large changes. Therefore, the feedback effect appears to be present only during periods of large changes in forward rates. In

[7]Contrary to what has prevailed during the earlier part of the 1990s, Kambhu and Mosser found that in the latter part of the decade the direction of "causality" for positive feedback effects is from the swaps market to the Treasury market.

[8] According to the BIS data, comparing activity in legacy currencies in 1998 with that in the euro in 2001, there was an increase of 104%.

[9] Those advantages are: a) IRS match exactly the maturity of the forward transactions; b) the swap yield curve serves as a basis for pricing forward transactions more generally; and c) transaction costs are low due to the very good liquidity in the IRS market (especially when compared to the cash market for non-benchmark bonds).

[10] This was achieved by increasing a cutoff value for the change in both forward rates in 1 bp increments until the two, so created, sub-samples have roughly the same size. The two sub-samples reached roughly the same size for a cutoff value of 38 bp.

the presence of transaction costs, dealers opt to adjust their hedge-related positions after forward rates have changed by more than certain amount[11].

Finally, we estimate the relationship between the spot five-year rate and lagged forward rates in periods leading up to episodes of large changes in the five-year rate. In other words, we investigate the size of the positive feedback effect in periods characterised by the building-up of strong expectations about an imminent significant change in the five-year yield[12]. This is achieved by compiling a sub-sample from our initial sample comprising observations (of the five-year spot rate and the related forward rates) for which the subsequent two-month change in the five-year rate lies within the largest 10% of rate changes. The relevant regression results appear in Table 7. They are remarkably different from the results obtained by using the full sample of observations. The size of the estimated coefficients $\beta_1$ and $\beta_2$ shows that 20% of the change in forward rates is transmitted to the spot rate. Moreover, the model's explanatory power increases considerably since the variation in forward rates explains now 30% of the variation in the spot rate[13].

## 7. Conclusions – Policy implications

We have verified the presence of a positive feedback effect due to dynamic hedging of interest rate options positions on the medium-term segment of the yield curve. This positive feedback induces a kind of overshooting during displacements of the midpoint of the curve. The effect is produced because positions are adjusted mechanistically, independently of economic fundamentals.

The effect is present during periods characterised by significant changes in interest rates (realised volatility). During those periods approximately 6% of the variation in the five-year yield is due to dynamic hedging of options positions (results

[11] This asymmetry in behaviour leads in practice to a rebalancing period the length of which varies with the size of change in forward rates.

[12] The underlying assumption here is that market participants are not usually caught by surprise regarding those large changes which is the result of significant shifts in economic fundamentals and/or policy stance. Market participants constantly monitor, for example, the evolution of different economic indicators and the comments made by policy makers.

[13] Kambhu and Mosser report similar results, though slightly different in size. The variation in forward rates explains 40% of the variation in the spot rate while more than 70% of the change in forward rates is transmitted to the spot rate.

in Table 6). This is not large and, in this respect, the detected effect appears to be only of second-order. Economic fundamentals and policy still overwhelmingly determine the short-term dynamics of the yield curve. The size of the effect, however, becomes important during periods of significant expected volatility in rates. During those periods some 30% of the variation in the five-year yield is attributed to the activity of dynamic hedging of options positions (results in Table 7).

The one-week change in the BOBL yield between 13/5 and 20/5/2003 was –13 basis points. On 20/5 there were strong expectations of an imminent significant reduction in interest rates[14]. Therefore, according to our estimates, some 4 bp out of the total reduction in the BOBL yield can be attributed to the positive feedback effect, i.e. the weekly drop in the yield would be 9 bp without the feedback. Assuming that on 20/5 there were no expectations of significant changes in interest rates, the positive feedback effect would be 0.8 bp.

Regarding risk management considerations, our results imply that there should be no problem for dealers to carry their hedge-related trades transacting at reasonable prices, when interest rate volatility is normal. In times, however, of large shifts in interest rates and furthermore during periods when expectations of significant changes are building up, dealers may find themselves exposed to far higher risks than they had anticipated. They may face prices at which it is too unfavourable to transact. In that case, the positive feedback effect due to dynamic hedging may be more disruptive since it amplifies market shocks.

Market participants and policy makers closely monitor and interpret yield curve movements because the curve is believed to reflect expectations of future economic fundamentals as well as one component of the monetary transmission mechanism, that from short-term to long-term interest rates[15]. According to our results, this information content of the yield curve is not significantly impaired under normal circumstances. Movements of the curve, however, may have to be interpreted more carefully when market shocks or significant policy changes occur (in terms of realised as well as expected volatility). Under these circumstances, changes in the

[14] In fact, both the European Central Bank (ECB) and the Federal Reserve Board (FED) eventually reduced interest rates. The ECB on 5/6/03 by 50 basis points and the FED on 25/6/03 by 25 basis points.

[15] See, for instance, the ECB Monthly Bulletin (May 2000).

curve may incorporate significant overshooting due to the positive feedback effect shown by our analysis.

# Appendix

## Estimation procedure and related econometric tests

## 1. Preliminary tests

The issue of non-stationarity has several important implications for econometric modelling. Depending on the degree of non-stationarity of the different variables involved, standard inference procedures may or may not apply.

In order to test for the presence of a unit root, we applied the method of Dickey and Fuller (1981). Applying the Augmented Dickey Fuller (ADF) test in the data series, we are unable to reject the null hypothesis that the data is a unit root series. On the other hand, when the unit root tests are applied to the first differences, the I(1) null hypothesis is easily rejected (Table A1). Thus, our data appear to contain a single unit root which cancels out at first differencing.

This suggests that the variables above are best modelled as I(1) processes. Consequently, our empirical model needs to be specified in terms of first differences.

## 2. Estimation procedure using German government bond yields

While the variables under consideration are non-stationary, it is still possible to find one or more linear combinations of the series that are stationary. In this case, the series themselves are said to be cointegrated and the stationary linear combination(s) is (are) referred to as the cointegrating equations and represent a long-run equilibrium relationship. In fact, cointegration exists if discrepancies from the presupposed relationship have bounded variability. Following a common shock, cointegrated series gradually converge to their long-run equilibrium relationships.

As noted by Engle and Granger (1987), time-series regressions involving relationships between the changes in cointegrated variables should include a lagged cointegration term in order to control for correlation between the contemporaneous levels of the regressions variables that would otherwise interfere with the consistent estimation of the equation coefficients. The error correction term is dynamic in the sense that it involves lags of the dependent and explanatory variables. It thus captures short–run adjustments to past disequilibria and to contemporaneous changes in the explanatory variables so allowing equilibrium to be achieved gradually. In particular, the size and the statistical significance of the error correction term measures the speed at which the dependent variable returns to its long–run equilibrium.

## *2.1 VAR specification testing and tests for cointegration among the integrated variables.*

We apply the Johansen (1991) maximum likelihood procedure that tests for the significance of all the cointegrating vectors between the variables.

The lag order of the VAR is often selected somewhat arbitrarily, with standard recommendations suggesting that we set it long enough to ensure that the residuals are white noise. However, if you chose it too large, the estimates become imprecise. Thus, four versions of the system are initially estimated involving four lags, six lags, eight lags and ten lags respectively. Two criteria (Akaike Information and Schwarz Baysian) and a likelihood ratio test are used to select the appropriate lag length. Both criteria and the LR test selects VAR(4).

In order to evaluate the number of cointegrating vectors between the variables, the trace test is conducted. Depending on the number of endogenous variables (n), the Johansen procedure produces the maximum number of the cointegrating vectors, which is n-1. Test statistics reject the null hypothesis of zero cointegrating vectors in favour of one cointegrating vector at the 5 per cent level of significance (Table A2). Thus, having determined that the variables under consideration are cointegrated, the vector error correction model can be applied.

Applying the VEC methodology the resulting cointegrating equation is specified as follows:

$$Z = BOBLi - 0.022 - \underset{(0.522)}{3.006}\, GovF_{3,5} + \underset{(0.520)}{1.942}\, GovF_{5,10} \qquad (1A)$$

The figures in parentheses are the standard errors. All the coefficients are significant at the 5 percent level (two-tailed) and with the anticipated sign.

### 2.2 Regression results

As already described, in order to determine the influence of the forward rates on the spot rate, we run several regressions in the form of (1) repeatedly for $t = 2,3,4,...,50$ so as to figure out the effect of changes in forward rates ranging from a one-week change up to 49-week changes. Z(-1) is the error correction term from the cointegration relationship in the level of rates as described earlier. Diagrams 3 and 4 show the F-statistics for the joint distribution of $\beta_1$ and $\beta_2$ as well as the confidence bands for $\beta_1$ and $\beta_2$ estimates. The test statistics in these diagrams were computed with White Heteroscedastisticy Consistent Covariances.

The regression results of the BOBL yield on forward rates from the German Government yield curve are shown in Table 4.

## 3. Estimation procedure using rates from the swap curve

We employ the same methodology regarding the VAR specification as in 2.1 above, using rates from the IRS curve. Test statistics reject the null hypothesis of zero cointegrating vectors in favour of one cointegrating vector at the 5 per cent level of significance (Table A3). Thus, having determined that the variables under consideration are cointegrated, the vector error correction model can be applied.

Applying the VEC methodology the resulting cointegrating equation is specified as follows:

$$Z = irs5Y \text{ -0.033 -3.1164}\, irsF_{3,5} \text{ +2.7607}\, irsF_{5,10}$$
$$\qquad\qquad (0.416) \qquad\quad (0.53)$$

The figures in parentheses are the standard errors. All the coefficients are significant at the 5 percent level (two-tailed) and have the anticipated sign.

The regression results of the spot rate on the forward rates from the IRS curve are shown in Table 5, equation (B).

## 4. Spill over effects between the German government bond and the German IRS markets.

### *4.1 Regression of BOBL yield on forward rates from IRS curve*

The regression results of the BOBL yield on the forward rates from the IRS curve are shown in Table 5, equation (C). The test statistics were computed with White Heteroskedasticity Consistent Covariances. Test statistics reject the null hypothesis of zero cointegrating vectors in favour of one cointegrating vector at the 5 per cent level of significance (Table A4). Thus, having determined that the variables under consideration are cointegrated, the vector error correction model can be applied.

Applying the VEC methodology the resulting cointegrating equation is specified as follows:

$$Z = BOBLi - \underset{(0.407)}{3.0487}\, irsF_{3,5} + \underset{(0.520)}{2.727}\, irsF_{5,10} - 0.0316$$

The figures in parentheses are the standard errors. All the coefficients are significant at the 5 percent level (two-tailed) and have the anticipated sign.

### *4.2 Regression of the spot 5Y rate from IRS on forward rates from the government yield curve.*

The regression results of the spot 5Y rate from IRS on forward rates from the government yield curve are shown in Table 5, equation (D). Test statistics reject the null hypothesis of zero cointegrating vectors in favour of one cointegrating vector at the 5 per cent level of significance (Table A5). Thus, having determined that the variables under consideration are cointegrated, the vector error correction model can be applied.

Applying the VEC methodology, the resulting cointegrating equation is specified as follows:

$$Z = irs5Y - \underset{(0.549)}{2.863}\, GovF_{3,5} + \underset{(0.530)}{1.771}\, GovF_{5,10} - 0.00313$$

The figures in parentheses are the standard errors. All the coefficients are significant at the 5 percent level (two-tailed) and with the anticipated sign.

## 5. Regressions for periods of large and small changes in the forward rates.

For the periods of large and small changes in the forward rates, the regression results are shown in Table 6. The error correction term Z has already been specified in (1A) above.

## 6. Regressions for periods of large changes in the five-year rate.

Regarding the sample of large changes in the spot 5Y rate, the regression results are shown in Table 7. The error correction term Z has been again specified in (1A) above.

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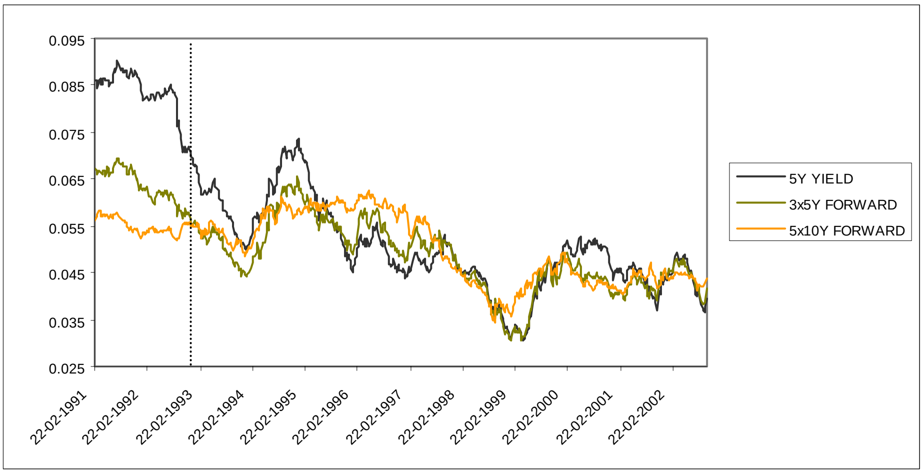


*Diagram 1. Spot and forward rates from the German government yield curve*

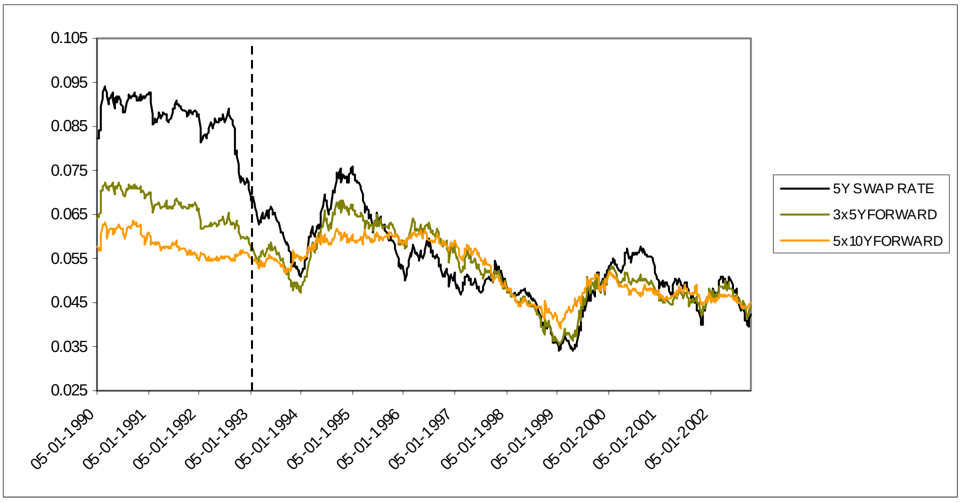


*Diagram 2. Spot and forward rates from the German IRS market*

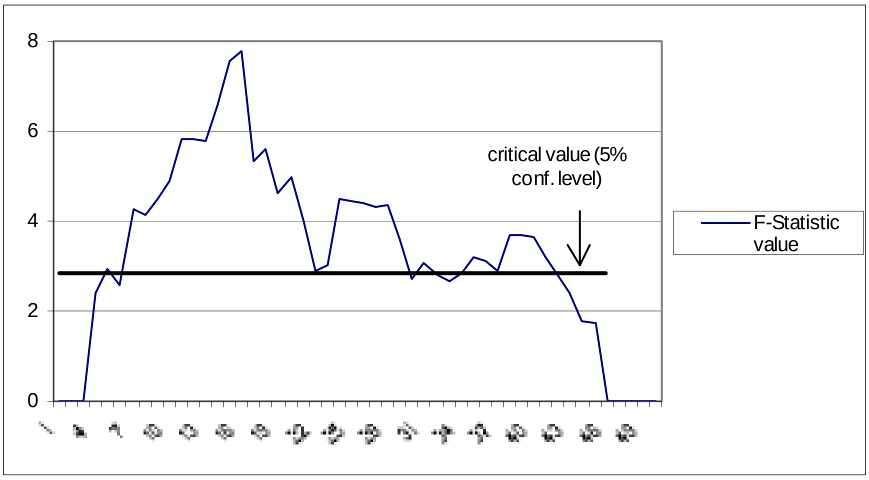


*Diagram 3: Model's statistical significance at various rebalancing periods.*

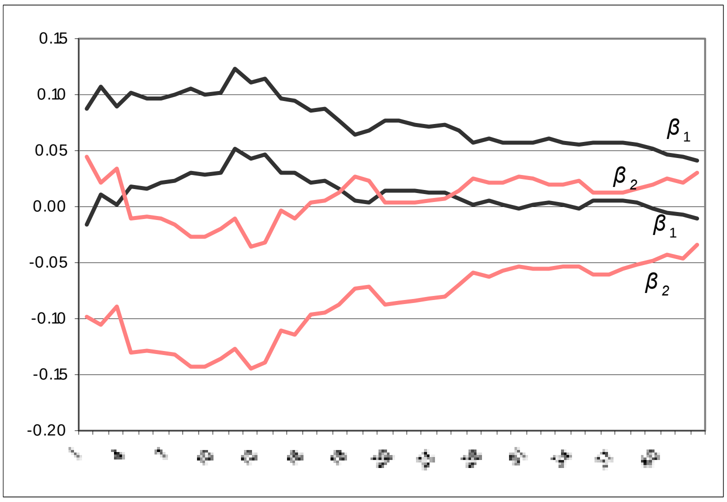


*Diagram 4: Coefficient confidence bands at various rebalancing periods.*

| TABLE 1: Global positions in OTC derivatives markets (Notional amounts, trillions of USD) | | | |
|---|---|---|---|
| **Type** | **1995** | **1998** | **2001** |
| FX derivatives | 13.1 | 22.1 | 20.4 |
| Options (FX) | 2.4 | 5.04 | 2.8 |
| Interest rate derivatives | 26.6 | 48.1 | 75.8 |
| Options (interest rate) | 3.5 | 8.5 | 10.9 |
| Source: BIS | | | |

| TABLE 2: Global positions in the OTC interest rate derivatives markets (Nominal amounts, in trillions USD) | | | |
|---|---|---|---|
| By maturity | **1995** | **1998** | **2001** |
| Up to 1 year | 11.7 | 20.2 | 29.2 |
| Between 1 and 5 years | 11.5 | 19.0 | 29.5 |
| Over 5 years | 3.5 | 8.9 | 17.2 |
| Source: BIS | | | |

| Table 3: Effect on the 5Y spot rate from a change in 3x5 and/or 5x10 forwards | | |
|---|---|---|
| | CAPS or FLOORS | |
| Change in forward | 3x5 | 5x10 |
| Increase | + | - |
| Decrease | - | + |

| TABLE 4: Regression results | | | | | |
|---|---|---|---|---|---|
| Sample period 1993-2002<br>(Number of observations: 512. Standard error is in parenthesis.) | | | | | |
| | Z(-1) | $\Delta F_{3,5}(-1,-17)$ | $\Delta F_{5,10}(-1,-17)$ | $R^2$ | Adjusted $R^2$ |
| <u>Equation (A)</u> using spot and forward rates from German government yield curve | -0.0391<br>(0.0096) | 0.0797<br>(0.0184) | -0.0856<br>(0.0273) | 0.046 | 0.041 |

| TABLE 5: Regression results | | | | | |
|---|---|---|---|---|---|
| Sample period 1994-2002<br>(Number of observations: 458. Standard error is in parenthesis.) | | | | | |
| | Z(-1) | $\Delta F_{3,5}(-1,-16)$ | $\Delta F_{5,10}(-1,-16)$ | $R^2$ | Adjusted $R^2$ |
| <u>Equation (B)</u> using spot and forward rates from German IRS curve | -0.0663<br>(0.0130) | 0.0718<br>(0.0213) | -0.1285<br>(0.0377) | 0.061 | 0.055 |
| <u>Equation (C)</u> using spot BOBL rate and forward rates from German IRS curve | -0.0464<br>(0.0125) | 0.0704<br>(0.0216) | -0.1060<br>(0.0374) | 0.042 | 0.035 |
| <u>Equation (D)</u> using spot rate from IRS curve and forward rates from government yield curve | -0.0470<br>(0.0109) | 0.0845<br>(0.0187) | -0.1233<br>(0.0315) | 0.053 | 0.047 |

| TABLE 6: Regressions with large and small changes in forward rates | | |
|---|---|---|
| | Large Changes $\mid \Delta F_{3,5} \mid \geq 38bp$ or $\mid \Delta F_{5,10} \mid \geq 38bp$ | Small Changes $\mid \Delta F_{3,5} \mid < 38bp$ and $\mid \Delta F_{5,10} \mid < 38bp$ |
| Z(-1) | -0.042<br>(0.0134) | -0.0344<br>(0.0140) |
| $\Delta F_{3,5}(-1,-17)$ | 0.1197<br>(0.0353) | 0.0639<br>(0.0226) |
| $\Delta F_{5,10}(-1,-17)$ | -0.1159<br>(0.0439) | -0.0732<br>(0.0390) |
| $R^2$ | 0.055 | 0.034 |
| Adjusted $R^2$ | 0.044 | 0.023 |
| Observations | 259 | 253 |

Note: Sample period 1993-2002. Spot and forward rates from German government yield curve. Standard error is in parenthesis.

| TABLE 7: Regression for periods of large changes in the five-year rate | |
|---|---|
| | Large leading two-month change $\lvert r_5(8) - r_5(-1) \rvert \geq 61$ bp |
| Z(-1) | -0.1096<br>(0.0320) |
| $\Delta F_{3,5}(-1,-17)$ | 0.2030<br>(0.0523) |
| $\Delta F_{5,10}(-1,-17)$ | -0.2412<br>(0.0920) |
| $R^2$ | 0.31 |
| Adjusted $R^2$ | 0.26 |
| Observations | 49 |

Note: The sample is the largest 10% of two-month changes in the BOBL yield in the period 1993-2002. Forward rates are also from German government yield curve. Standard error is in parenthesis.

Table A1. Unit Root tests

| Rates | *BOBLi* | $GovF_{3,5}$ | $GovF_{5,10}$ | *irs5Y* | $irsF_{3,5}$ | $irsF_{5,10}$ |
|---|---|---|---|---|---|---|
| ADF-test statistic | -2.007 | -2.298 | -1.940 | -2.062 | -2.327 | -2.185 |
| First differences | -9.081 | -10.130 | -11.459 | -10.140 | -11.683 | -13.694 |
| *Note: Tests were carried out with an intercept, trend and 4 lags. The 5% (1%) critical values is -3.4193 (-3.9778). The null hypothesis of a unit root is rejected against the one-sided alternative if the t statistic is less than the critical value. In the above examples the test fails to reject the null hypothesis of a unit root in the levels of the series at any of the reported significance levels.* | | | | | | |

Table A2. Johansen cointegration tests on rates from German Government Curve
(Johansen test statistics)
Sample: 1993 -2002

| Null hypothesis: Cointegration rank *r* | Likelihood ratio | Critical values (5%) | Critical values (1%) |
|---|---|---|---|
| R=0 | 37.11 | 29.68 | 35.65 |
| R≤1 | 9.26 | 15.41 | 20.04 |
| R≤2 | 2.51 | 3.76 | 6.65 |
| *Note: Based on Johansen trace statistic. The test procedure consists of estimating a VAR with a linear combination of the n-1 variables. The test determined the number of cointegrating vectors together with the coefficient estimates of the cointegrating equations. The VAR employs 4 lags. The estimation procedure assumes linear trend in the data and an intercept but no trend in the cointegrating equation. Critical values are from Osterwald-Lenum (1992).* | | | |

Table A3. Johansen cointegration tests on rates from the German IRS curve
(Johansen test statistics)
Sample 1994 – 2002 (Total No of observations 458)

| Null hypothesis: Cointegration rank *r* | Likelihood ratio | Critical values (5%) | Critical values (1%) |
|---|---|---|---|
| R=0 | 34.67 | 29.68 | 35.65 |
| R ≤ 1 | 4.47 | 15.41 | 20.04 |
| R ≤ 2 | 0.92 | 3.76 | 6.65 |
| *See note in Table A2* | | | |

Table A4. Johansen cointegration tests on rates from the German IRS curve
(Johansen test statistics)
Sample 1/6/1994 – 10/10/2002 (Total No of observations 457)

| Null hypothesis: Cointegration rank *r* | Likelihood ratio | Critical values (5%) | Critical values (1%) |
|---|---|---|---|
| R=0 | 36.31 | 29.68 | 35.65 |
| R ≤ 1 | 5.47 | 15.41 | 20.04 |
| R ≤ 2 | 0.68 | 3.76 | 6.65 |
| *See note in Table A2* | | | |

Table A5. Johansen cointegration tests on rates from the German IRS curve
(Johansen test statistics)
Sample 1/1/1994 – 10/11/2002 (Total No of observations 458)

| Null hypothesis: Cointegration rank *r* | Likelihood ratio | Critical values (5%) | Critical values (1%) |
|---|---|---|---|
| R=0 | 31.32 | 29.68 | 35.65 |
| R ≤ 1 | 5.12 | 15.41 | 20.04 |
| R ≤ 2 | 1.47 | 3.76 | 6.65 |
| *See note in Table A2* | | | |

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