OPTIMAL MONETARY POLICY IN THE EURO AREA IN THE PRESENCE OF HETEROGENEITY

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ABSTRACT
This paper examines the optimal design of monetary policy in the European monetary union in the presence of structural asymmetries across union member countries. It derives analytically an optimal interest rate rule under commitment and studies the dependence of its coefficients on the parameters of the structural model of each economy, the central bank's preferences for inflation and output stabilization as shown in its loss function, and the relative size of each country. Based on a two-country, forward-looking, general equilibrium model, which is estimated for two euro area countries (Germany and France), we show that there are gains to be achieved by the ECB taking into account the heterogeneity of economic structures. This finding appears to be robust under alternative weights given by the central bank to the stabilization of the target variables. Although the implementation of the proposed rule involves difficulties relating to data and estimation constraints as well as risks of accommodating structural divergences, it is important that the ECB takes into consideration national characteristics in formulating its monetary policy, especially in view of more countries joining the European monetary union in the future. However, as monetary and financial integration advances, the welfare benefits of monetary policy responding to individual countries' variables may become less significant.

Keywords: Monetary policy rules; Heterogeneous monetary union
JEL classification: E52; E58

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1 Introduction

On 1st January 1999 the European monetary union was established, initially with 11 member countries, followed by Greece from 1st January 2001 and Slovenia from 1st January 2007. Member countries are subject to a centralized monetary policy conducted by the European Central Bank (ECB) and a common currency. According to the mandate of the ECB, as defined in the Maastricht Treaty (article 105 (1)), the primary objective of monetary policy is to maintain price stability over the medium term in the euro area; without prejudice to this objective, the ECB shall support the general economic policies in the Community, which include sustainable and non-inflationary growth. The focus of the ECB is on price stability in the euro area as a whole. Questions of individual country performance do not enter policy decision-making (ECB, 2005).

Research on the monetary policy strategy of the ECB has increased in recent years. Most researchers focus on the specification of the appropriate monetary policy rule and the welfare improvement that can be achieved by using this rule. Little attention, however, has been given to the issue of data aggregation and the importance of national differences for the success of the common monetary policy. Although the dispersion of economic developments across member countries is considered a normal feature of any monetary union related to the convergence process, in the European monetary union it is also, at least to some extent, attributed to diverging national policies and long-lasting structural inefficiencies, such as nominal and real rigidities in product and factor markets. In view of the enlargement of the European monetary union, national differentials are expected to become even larger with potential costs in terms of the union’s economic performance.

Even though the objectives of the ECB are expressed exclusively in union-wide terms, the fact that the economies of the euro area are characterized by structural differences and may be hit by asymmetric shocks can make neglecting national developments very costly. Therefore, it is interesting to examine the benefits for the effectiveness of monetary policy in the euro area from incorporating national information into interest rate decisions, as opposed to reacting solely to aggregate union-wide variables. In order to investigate this claim, we extend the analysis of the studies presented in Section 2 and analytically derive an optimal interest rate reaction function of the monetary union’s central bank by minimising its loss function subject to a multi-country structural model. The paper contributes to the literature on the optimal design of monetary policy in the European monetary union in the presence of structural asymmetries across union member countries by studying the dependence of the coefficients of the interest rate rule on the parameters of the structural model of each economy, the central bank’s preferences for inflation and output stabilization, as shown in its loss function, and the relative size of each country. Furthermore, recognizing the advantages of New-Keynesian models in describing the economy, our analysis adopts a forward-looking perspective in the spirit of Clarida, Galí and Gertler (1999). As an
extension to Benigno (2004) and Lombardo (2002) we allow for more than one type of asymmetry. We evaluate the optimal weights that each country’s economic variables should be assigned by the central bank in its interest rate reaction function using the parameters of the multi-country structural model and we assess the welfare improvement that would be achieved by the implementation of such a rule compared to a rule that focuses only on union-wide variables. The results of our paper suggest that an optimal monetary policy rule should take into consideration not only the relative size of the countries as at present (relative output or population), but also the structural characteristics of the economies.

The baseline model used to derive the optimal monetary policy rule is described in Section 3 and is a dynamic, general equilibrium model, with the aggregate demand equation resulting from the consumer’s utility maximization problem and the New-Keynesian Phillips curve being based on Calvo’s (1983) staggered price setting. Following Svensson (1999), Giannoni and Woodford (2002b) and Svensson (2003), in Sections 4 and 5 we derive the optimal interest rate reaction function subject to the assumption that the union economy is described by an aggregate union-wide model, and alternatively by a disaggregate multi-country model. In Section 6, we estimate the structural equations of both models, the first with data from a hypothetical union between Germany and France, and the second with individual country data for Germany and France. Using these estimates, we calculate in Section 7 the optimal coefficients of the interest rate reaction function, the volatility of the variables of interest, the value of the loss function for both models and the loss ratio in order to compare the relative performance of the two interest rate reaction functions. Our basic conclusion is that welfare can be improved by the response of the common monetary policy rate to individual countries’ variables, although some qualifications on the applicability of this result are offered in Section 8. Section 9 summarizes and presents the main conclusions. The Appendix contains technical and other details.

2 Related literature

The importance of considering national information when the central bank of the European monetary union decides on monetary policy has been studied empirically by De Grauwe (2000), De Grauwe and Piskorski (2001), Angelini et al. (2002) and Monteforte and Siviero (2002). In particular, the authors evaluate, using the framework proposed by Rudebusch and Svensson (1999), the performance (relative loss) of rules targeting national variables as opposed to union-wide variables for calibrated aggregate demand and supply equations and all but De Grauwe and Piskorski (2001) find that the first type of rule may deliver large welfare gains. Further research by Angelini et al. (2007) shows that the ECB is able to align national economic cycles by taking into account the inflation dispersion across member states, but at the cost of a larger variance of union-wide inflation. In a recent study, Jondeau and Sahuc (2006) estimate, using Bayesian techniques, a multi-country model
for three core euro area countries and its area-wide counterpart, compare their welfare performance and find that using an area-wide model may induce relatively significant losses. Finally, Benigno (2004) and Lombardo (2002, 2006), using two-country optimizing models, examine optimal monetary policy in a currency area, like the European monetary union, characterized by asymmetric shocks across countries. According to Benigno (2004), an optimal inflation-targeting policy should attach a higher weight to the inflation of the country with the higher degree of nominal rigidity. The rationale behind this result is that differences in price flexibility affect relative prices across members of a currency union, cause output dispersion and worsen the welfare of the currency area. Similar conclusions are reached by Lombardo (2002, 2006) on the basis of the degree of market competition.

The degree of heterogeneity in the European monetary union, examined in a large number of empirical studies, is found to be higher than that observed in other currency areas such as the United States. The relatively persistent inflation and output growth differentials observed in the euro area (ECB, 2003; Benalal et al., 2006; Agresti and Mojon, 2001) result in part from the real convergence process, although this process has reached an advanced stage in recent years. At the same time, these differentials can be partly attributed to structural differences across member countries, for example in price and wage-setting mechanisms and in consumption behavior patterns as reflected in the coefficients of the Phillips and the aggregate demand curves respectively. On the one hand, evidence on the estimated structural parameters of the Phillips curve by Gerberding (2001), Jondeau and Le Bihan (2005), Rumler (2005), Benigno and López-Salido (2006), Korenok et al. (2006) and Leith and Malley (2007) show large heterogeneity of the current member countries which can be explained by divergence in the degree of price stickiness or forward-looking behavior in price setting. For example, the slope of the Phillips curve ranges between 0.0003 and 0.3 in these studies. Even greater heterogeneity is found by Di Bartolomeo et al. (2004) for the then acceding countries, with the slope of the Phillips curve ranging between 0.69 and 1.52. On the other hand, empirical estimates of the forward-looking aggregate demand curve reported by Goodhart and Hofmann (2005), Doménech et al. (2001), Smets (2003) and Leith and Malley (2005) show no significant effect of the real interest rate on the output gap for the euro area countries, as in the United States (see also Fuhrer and Rudebusch, 2004). Other studies analyze the different channels of monetary transmission in the European monetary union as a whole (e.g. Peersman and Smets, 2003 and Angeloni et al., 2003) and in the individual member countries (see Peersman, 2004 and Mojon and Peersman, 2003 for an overview of the literature) and examine country characteristics that may explain divergences, such as the structure of the financial system (financial stability and depth, banks’ concentration, availability of alternative financing, development of capital markets and non-bank financial intermediaries, lending maturities), openness, price and wage rigidity (barriers to entrepreneurship, employment protection legislation), interest rate sensitivity to demand (industrial structure) and households’ and firms’ portfolio

1 See also Angeloni and Ehrlmann (2003), Clements et al. (2001) and Guiso et al. (1999).
composition. The main conclusion is that there is considerable dispersion across countries. However, the studies do not give clear results with respect to the ranking of countries on the basis of monetary policy effectiveness.

3 The baseline model

The New-Keynesian model used is a dynamic, stochastic, general equilibrium model, based on optimizing behavior combined with some form of nominal price rigidity. Early examples of such models include Goodfriend and King (1997), Rotemberg and Woodford (1997, 1999) and McCallum and Nelson (1999). The equations of the model are derived from well-specified optimization problems, i.e. the representative agent’s problem and the pricing decisions of individual firms. Traditional aggregate demand and supply equations are often criticized as being too ad hoc. However, this criticism does not apply to the New-Keynesian framework, since the coefficients in these equations are explicit functions of the underlying structural parameters of the consumer’s utility function, the production function and the price-setting process. Furthermore, both equations contain forward-looking elements and assume rational expectations contrary to the traditional models.

A common and plausible assumption widely used in the literature is that the central bank aims at minimizing a quadratic loss function specified in terms of inflation ($\pi_t$) and the output gap ($\bar{y}_t$), which is defined as the deviation of actual from potential output. Although this assumption may seem rather ad hoc, Woodford (2003) has provided a formal justification for the use of such a loss function, which is derived as a quadratic approximation to the expected utility of the representative household. A useful extension can be obtained if one includes real money balances as an additional argument in the household’s utility function. In this case, considering the welfare consequences of the transactions frictions related to money demand adds an extra term to the loss function, namely the squared deviation of the interest rate from a constant rate$^2 (i_t - \bar{i})$. Thus, the intertemporal loss function to be minimized can be written as:

$$ E_t \sum_{j=0}^{\infty} \delta^j V_{t+j} = \Omega E_t \sum_{j=0}^{\infty} \delta^j \left\{ (\pi_{t+j})^2 + \lambda (\bar{y}_{t+j})^2 + \kappa (i_{t+j} - \bar{i})^2 \right\} + t.i.p. \quad (1) $$

where $\Omega$ includes constant parameters of the welfare maximization problem, $\delta$ denotes the discount rate ($0 < \delta < 1$), $\lambda$ and $\kappa$ are the relative weights on output and interest rate stabilization respectively and $t.i.p.$ denotes terms independent of policy.

The aggregate demand equation is derived from the Euler condition of a representative agent’s optimization problem and relates the output gap to the expected future output gap and the real interest rate. Thus, it shows the sensitivity of output to the monetary

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$^2$This is the interest rate consistent with inflation being equal to the equilibrium level (Woodford, 2003).
policy interest rate. Changes in the latter affect the real interest rate and this alters the optimal time path of consumption. The forward-looking aggregate demand curve is given by:

\[ \bar{y}_t = \bar{y}_{t+1|t} - \frac{1}{\sigma} [i_t - \pi_{t+1|t} - \bar{\tau}] + u_t \]  

(2)

where \( \pi_{t+1|t} \) and \( \bar{y}_{t+1|t} \) represent the expected inflation and output gap respectively for period \( t + 1 \) on the basis of information in period \( t \), \( \sigma \) is the intertemporal elasticity of substitution of consumption, \( \bar{\tau} \) represents the Wicksellian natural rate of interest, which is required to bring output to the flexible-prices level and \( u_t \) denotes a productivity shock.

The source of the real effects of monetary policy in this model is the assumption that prices are adjusted at exogenous random intervals (Calvo, 1983). Calvo’s model assumes that only a fraction \( (1 - \alpha) \) of producers charge a new price at the end of a period, whereas the rest \( (\alpha) \) continue charging the old price. The parameter \( \alpha \) is a measure of the degree of price rigidity. The New-Keynesian Phillips curve relates inflation to expected future inflation, and also to the deviation of output from potential output that could be attained under flexible prices, namely:

\[ \pi_t = \delta \pi_{t+1|t} + \theta \gamma \bar{y}_t + \epsilon_t \]  

(3)

where \( \theta = \frac{(1-\alpha)(1-\alpha)}{\alpha} \), \( \gamma = \eta + \sigma \), with \( \eta \) being the elasticity of real marginal cost with respect to output, and \( \epsilon_t \) represents either changes in tastes that affect leisure or stochastic shifts in the markup of wages over the intertemporal elasticity of substitution between leisure and consumption.

4 An optimal rule based on a union-wide model

Using the baseline model\(^3\) described in the previous section and following Giannoni and Woodford (2002a), we can derive the optimal reaction function of the central bank based on a union-wide model. We start by assuming that the central bank decides on the interest rate for the simplest case of a two-country union, taking into account the aggregate (weighted) variables of both countries. The central bank minimizes a modified loss function, with positive weights \( 1, \lambda \) and \( \kappa \) on the squared deviations of inflation from the inflation target \( (\bar{\pi}) \), squared output gap and squared interest rate deviations from a constant rate \( (\bar{i}) \) consistent with the inflation target, as follows:

\[ \min_{\pi_t} \sum_{j=0}^{\infty} \delta^j L_{t+j} = \min_{i_t} \sum_{j=0}^{\infty} \delta^j \frac{1}{2} \left[ (\pi_{t+j} - \bar{\pi})^2 + \lambda (\bar{y}_{t+j} - \bar{y})^2 + \kappa (i_{t+j} - \bar{i})^2 \right] \]  

(4)

\(^3\)We recognize that this model treats the union like a closed economy and disregards features that are present in currency areas, like terms-of-trade effects, relative-price effects, etc. However, it enables us to get manageable and straightforward solutions and trace the monetary policy implications we want to focus on.
subject to the union’s forward-looking Phillips and aggregate demand curves:

\[ \pi_t^U = \delta \pi_{t+1|t}^U + \rho \tilde{y}_t^U + \epsilon_t \]  

(5)

\[ \tilde{y}_t^U = \tilde{y}_{t+1|t} - \frac{1}{\sigma} (\pi_t^U - \pi_{t+1|t} - \tau) + u_t \]  

(6)

where \( \pi_t^U = w \pi_t + (1 - w) \pi_t^* \), \( \tilde{y}_t^U = w \tilde{y}_t + (1 - w) \tilde{y}_t^* \) and \( w \) is the weight given to each country in the union according to its relative size.

The Lagrangian is given by:

\[
\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \left\{ \frac{1}{2} \left[ (\pi_{t+j} - \bar{\pi})^2 + \lambda (\tilde{y}_{t+j}^U)^2 + \kappa (i_{t+j} - i)^2 \right] + \phi_{t+j} \left[ \pi_{t+j} - \delta \pi_{t+1|t}^U - \rho \tilde{y}_{t+j}^U \right] + \psi_{t+j} \left[ \tilde{y}_{t+j}^U - \tilde{y}_{t+1|t}^U + \frac{1}{\sigma} (i_t - \pi_{t+1|t} - \tau) \right] + \delta \phi_{t+1|t} \left[ \pi_{t+1|t} - \delta \pi_{t+2|t} - \rho \tilde{y}_{t+1|t} \right] + \delta \psi_{t+1|t} \left[ \tilde{y}_{t+1|t} - \tilde{y}_{t+2|t} + \frac{1}{\sigma} (i_{t+1|t} - \pi_{t+2|t} - \tau) \right] \right\}
\]

where \( \phi_{t+j} \) and \( \psi_{t+j} \) are the Lagrange multipliers associated with the constraints in period \( t + j \).

Under commitment it is sufficient to minimize the Lagrangian for only two periods:

\[
\min_{i_t} \left\{ \frac{1}{2} \left[ (\pi_t - \bar{\pi})^2 + \lambda (\tilde{y}_t^U)^2 + \kappa (i_t - i)^2 \right] + \phi_t \left[ \pi_t - \delta \pi_{t+1|t}^U - \rho \tilde{y}_t^U \right] + \psi_t \left[ \tilde{y}_t^U - \tilde{y}_{t+1|t}^U + \frac{1}{\sigma} (i_t - \pi_{t+1|t} - \tau) \right] + \delta \phi_{t+1|t} \left[ \pi_{t+1|t} - \delta \pi_{t+2|t} - \rho \tilde{y}_{t+1|t} \right] + \delta \psi_{t+1|t} \left[ \tilde{y}_{t+1|t} - \tilde{y}_{t+2|t} + \frac{1}{\sigma} (i_{t+1|t} - \pi_{t+2|t} - \tau) \right] \right\}
\]

The first-order conditions with respect to \( \pi_{t+1|t}^U, \tilde{y}_{t+1|t}^U \) and \( i_t \) yield respectively:

\[ \delta (\pi_{t+1|t}^U - \bar{\pi}) - \phi_t \delta + \phi_{t+1|t} \delta - \psi_t \frac{1}{\sigma} = 0 \]  

(7)

\[ \delta \lambda (\tilde{y}_{t+1|t}) - \psi_t - \delta \phi_{t+1|t} \rho + \delta \psi_{t+1|t} = 0 \]  

(8)

\[ \kappa (i_t - \hat{i}) + \psi_t \frac{1}{\sigma} = 0 \Rightarrow \psi_t = -\kappa \sigma (i_t - \hat{i}) \]  

(9)

Substituting \( \psi_t \) from the last condition Eq. (9) into the first-order conditions Eqs. (7 and 8), solving the second condition Eq. (8) for \( \phi_t \) and substituting it in equation Eq. (7), we obtain:
\[ i_t = \frac{\rho}{\kappa \sigma} (\pi_t - \bar{\pi}) + \frac{\lambda}{\kappa \sigma} \Delta \tilde{y}_t + \left[ \frac{\rho}{\sigma \delta} + 1 \right] i_{t-1} + \left[ \frac{1}{\delta} \right] (i_{t-1} - i_{t-2}) - i \left[ \frac{\rho}{\sigma \delta} \right] \]  

(10)

This implicit instrument rule, which the central bank commits to follow, involves a positive contemporaneous response of the interest rate to deviations of union inflation from the target and to changes in union output gap. Furthermore, it involves history dependence as the interest rate responds positively to past interest rates. The coefficients of the rule satisfy the generalized Taylor principle of determinacy as proposed by Giannoni and Woodford (2002b).

The response of the interest rate to the aggregate (weighted) variables varies directly with the size of \( \rho \). Thus, the steeper the slope of the Phillips curve, the stronger the interest rate reaction to inflation deviations from target. Note that lower price rigidity (\( \alpha \)), i.e. the fraction of firms not adjusting their prices in every period, implies a stronger effect of output on inflation and a steeper Phillips curve. Therefore, an undesirable change in inflation calls for a more aggressive interest rate adjustment in order to stabilize inflation towards the target. Similarly, the interest rate can be seen to respond to the target variables in proportion to \( \frac{1}{\sigma} \). Thus, when the slope of the aggregate demand curve is steeper, the interest rate reaction to inflation deviations from target, as well as to output gap changes, should be stronger. It should be noted that a lower intertemporal elasticity of substitution of consumption (\( \sigma \)) makes the real interest rate effect on the output gap larger and the aggregate demand curve steeper. Therefore, in case of an output change, the central bank must adjust the interest rate sufficiently to bring the output gap close to zero.

Additionally, the interest rate response to past interest rates depends inversely on the size of \( \delta \). Thus, the more importance consumers attach to the future level of the variables (which in turn implies lower inertia for these variables), the stronger the monetary policy leverage is. Therefore, the interest rate needs to adjust less to past interest rates changes.

Finally, the interest rate response to output gap changes is directly related to \( \lambda \), i.e. the weight given by the central bank to output gap stabilization. In contrast, the interest rate response to inflation deviations from target and output gap changes depends inversely on \( \kappa \); thus, if the central bank is concerned about interest rate variability, it must adjust the interest rate to changes in target variables more smoothly.

5 Deriving the optimal rule based on a multi-country model

Recognizing that asymmetries may exist across member countries of the European monetary union, it is interesting to derive the optimal reaction function of the central bank,

\footnote{The relevant condition is: \( \text{coef}(\pi_t) + \frac{1 - \lambda}{\rho} \text{coef}(\tilde{y}_t) > 1 - \text{coef}(i_{t-1}) \)}
taking into account national information explicitly. For the simplest case of a two-country union, it is assumed that the central bank minimizes the average of individual economies’ loss functions, weighted according to the countries’ relative size ($w$):

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \left[ w L_{t+j} + (1 - w) L^*_{t+j} \right] \quad \text{(11)}$$

The loss functions of the two countries share the same features, namely the discount factor ($\delta$), the inflation target ($\pi^*$) and the relative weights on the output gap ($\lambda$) and on interest rate deviations from a constant rate ($\kappa$), while the monetary policy interest rate ($i_t$) is common for both union member countries.

The loss functions of the two countries are given respectively by:

$$L_t = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda \tilde{y}_t^2 + \kappa (i_t - \bar{i})^2 \right] \quad \text{(12)}$$

and

$$L^*_t = \frac{1}{2} \left[ (\pi^*_t - \pi^*)^2 + \lambda \tilde{y}^*_t^2 + \kappa (i_t - \bar{i})^2 \right] \quad \text{(13)}$$

Next, we assume heterogeneous forward-looking Phillips and aggregate demand curves for both countries, i.e.:

$$\pi_t = \delta \pi_{t+1|t} + \beta \tilde{y}_t + \varepsilon_t \quad \text{(14)}$$

$$\tilde{y}_t = \tilde{y}_{t+1|t} - \frac{1}{\tau} \left( i_t - \pi_{t+1|t} - \bar{r} \right) + \upsilon_t \quad \text{(15)}$$

$$\pi^*_t = \delta \pi^*_{t+1|t} + \beta^* \tilde{y}^*_t + \varepsilon^*_t \quad \text{(16)}$$

$$\tilde{y}^*_t = \tilde{y}^*_{t+1|t} - \frac{1}{\tau^*} \left( i_t - \pi^*_{t+1|t} - \bar{r} \right) + \upsilon^*_t \quad \text{(17)}$$

The Lagrangian for two periods is given by:

$$\min_{i_t} \left\{ \frac{1}{2} \left[ w \left[ (\pi_{t+1|t} - \pi^*)^2 + \lambda \tilde{y}^*_{t+1|t} \right] + (1 - w) \left[ (\pi^*_{t+1|t} - \pi^*)^2 + \lambda \tilde{y}^*_{t+1|t} \right] + \kappa (i_t - \bar{i})^2 \right] \\
+ \phi_t \left[ \pi_t - \delta \pi_{t+1|t} - \beta \tilde{y}_t \right] + \chi_t \left[ \tilde{y}_t - \tilde{y}_{t+1|t} + \frac{1}{\tau} \left( i_t - \pi_{t+1|t} - \bar{r} \right) \right] \\
+ \psi_t \left[ \pi^*_t - \delta \pi^*_{t+1|t} - \beta^* \tilde{y}^*_t \right] + \omega_t \left[ \tilde{y}^*_t - \tilde{y}^*_{t+1|t} + \frac{1}{\tau^*} \left( i_t - \pi^*_{t+1|t} - \bar{r} \right) \right] \\
+ \frac{\delta}{2} \left[ w \left[ (\pi_{t+1|t} - \pi^*)^2 + \lambda \tilde{y}^*_{t+1|t} \right] + (1 - w) \left[ (\pi^*_{t+1|t} - \pi^*)^2 + \lambda \tilde{y}^*_{t+1|t} \right] + \kappa (i_t - \bar{i})^2 \right] \\
+ \delta \phi^*_{t+1|t} \left[ \pi^*_{t+1|t} - \delta \pi^*_{t+2|t+1} - \beta^* \tilde{y}^*_{t+1|t} \right] + \delta \chi^*_{t+1|t} \left[ \tilde{y}^*_{t+1|t} - \tilde{y}^*_{t+2|t+1} + \frac{1}{\tau^*} \left( i_{t+1|t} - \pi^*_{t+2|t+1} - \bar{r} \right) \right] \\
+ \delta \psi^*_{t+1|t} \left[ \pi^*^*_{t+1|t} - \delta \pi^*^*_{t+2|t+1} - \beta^* \tilde{y}^*^*_{t+1|t} \right] + \delta \omega^*_{t+1|t} \left[ \tilde{y}^*^*_{t+1|t} - \tilde{y}^*^*_{t+2|t+1} + \frac{1}{\tau^*} \left( i_{t+1|t} - \pi^*^*_{t+2|t+1} - \bar{r} \right) \right] \right\}$$

where $\phi_t$, $\chi_t$, $\psi_t$ and $\omega_t$ are the multipliers associated with the constraints.
The first-order conditions with respect to \( \pi_{t+1|t}, \pi^*_{t+1|t}, \bar{y}_{t+1|t}, \bar{y}^*_{t+1|t} \) and \( i_t \) are respectively:

\[
\delta w \left( \pi_{t+1|t} - \hat{\pi} \right) - \delta \phi_t - \frac{1}{\tau} \chi_t + \delta \phi_{t+1|t} = 0 \tag{18}
\]

\[
\delta (1 - w) \left( \pi^*_{t+1|t} - \hat{\pi} \right) - \delta \psi_t - \frac{1}{\tau^*} \omega_t + \delta \psi_{t+1|t} = 0 \tag{19}
\]

\[
\delta w \lambda \bar{y}_{t+1|t} - \chi_t - \delta \beta \phi_{t+1|t} + \delta \chi_{t+1|t} = 0 \tag{20}
\]

\[
\delta (1 - w) \lambda \bar{y}^*_{t+1|t} - \omega_t - \delta \beta^* \psi_{t+1|t} + \delta \omega_{t+1|t} = 0 \tag{21}
\]

\[
\kappa (i_t - \hat{i}) + \frac{1}{\tau} \chi_t + \frac{1}{\tau^*} \omega_t = 0 \tag{22}
\]

Solving Eqs. (18) and (20) to eliminate \( \phi_t \) and Eqs. (19) and (21) to eliminate \( \psi_t \), and substituting \( \chi_t \) and \( \omega_t \) into Eq. (22) we get the following rule:

\[
i_t = \frac{\beta w}{\kappa \tau} (\pi_t - \hat{\pi}) + \frac{\lambda w}{\kappa \tau} \Delta \bar{y}_t + \frac{\beta^* (1 - w)}{\kappa \tau^*} (\pi^*_t - \hat{\pi}) + \frac{\lambda (1 - w)}{\kappa \tau^*} \Delta \bar{y}^*_t + \left[ 1 + \frac{1}{\delta} \right] i_{t-1} - \frac{1}{\delta} i_{t-2} + \frac{1}{\delta} \left[ \frac{\beta}{\tau} + \frac{\beta^*}{\tau^*} \right] (i_{t-1} - \hat{i}) \tag{23}
\]

In contrast to the rule based on the union-wide model, in the above rule the interest rate adjusts to individual countries’ variables beyond that justified by the relative size of the economies. In particular, the interest rate responds differently to each country’s target variables, depending on that country’s structural parameters, namely the degree of price rigidity and the intertemporal elasticity of substitution of consumption as reflected in the slopes of the Phillips and the aggregate demand curves respectively. The central bank is thus reacting more aggressively to macroeconomic developments in the country with lower price rigidity (steeper Phillips curve) and / or a lower intertemporal elasticity of substitution of consumption (steeper aggregate demand curve). The reason is that this country experiences larger effects of changes in the output gap on inflation and in the real interest rate on the output gap. In case of an undesirable deviation of inflation or the output gap from their respective targets, the central bank must react more aggressively to stabilize this country’s variables in order to avoid large fluctuations and minimize welfare losses. Similar results are found in De Grauwe and Piskorski (2001) and Angelini et al. (2002) who derive numerically the optimal coefficients of the interest rate rule by minimizing the central bank’s loss function subject to the estimated structural models for the countries forming the union.

\[^6\text{Note that the interest rate response to inflation deviations from target relates to both these structural parameters, while the response to output gap changes depends only on the intertemporal elasticity of substitution.}\]
Finally, given our assumptions about a common discount factor ($\delta$) and central bank relative preferences for output gap and interest rate stabilization (parameters $\kappa$ and $\lambda$), the dependence of the interest rate on these parameters does not vary across countries. Thus, the relevant conclusions of the previous section also hold here.

6 Model estimation

In order to evaluate the performance of the optimal interest rate reaction functions described in the previous sections, we need to have estimates of the structural parameters of the Phillips and the aggregate demand equations. For the first model (Eqs. 5 and 6), we assumed the existence of a hypothetical monetary union between Germany and France and estimated it using aggregate data for the two countries, weighted according to the OECD weighting scheme, for the period 1965:1-1998:4. The second model, which consists of the individual country equations (Eqs. 14 to 17), was estimated using data for Germany and France over the same period. Of course, the calculation of the coefficients of the interest rate reaction function depends on the choice of the countries, on the empirical estimates of the model’s parameters and on the assumptions about the relative preferences of the central bank as shown in the loss function. A more complete model for the euro area should include all member countries and would presumably result in more pronounced asymmetries (see e.g. Jondeau and Sahuc, 2006). However, our results are based on two core, relatively similar economies and are likely to give us a lower bound to the welfare improvement that can be achieved if the central bank of a monetary union focuses on the structural characteristics of each member country, and not only on union-wide variables. This would also hold true, if we were to include the future members of the union.

All data series used for the estimation of the Phillips and aggregate demand curves in both models have a quarterly frequency and are drawn from the OECD Economic Outlook database. Inflation is measured in terms of the Harmonized Index of Consumer Prices. The output gap is given by the deviation of real GDP from its potential level, the calculation of which is based on a production function approach. The nominal interest rate is the 3-month interbank rate. The parameters were estimated using the Generalized Method of Moments (GMM), which is widely used in models with forward-looking variables.
The instruments chosen are lagged values of the explanatory variables, so that they are predetermined at the time the central bank decides on the level of the interest rate. Furthermore, they are uncorrelated with the residuals, but strongly correlated with the forward-looking variables.

Table 1. GMM estimates of the Phillips and the aggregate demand curves at individual country level and union level

<table>
<thead>
<tr>
<th>Germany</th>
<th>France</th>
<th>Monetary union of Germany and France</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.04</td>
<td>$\beta^*$</td>
</tr>
<tr>
<td>(0.026)</td>
<td>(0.073)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>$\frac{1}{\tau}$</td>
<td>0.04</td>
<td>$\frac{1}{\tau}^*$</td>
</tr>
<tr>
<td>(0.019)</td>
<td>(0.011)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>$\text{adj} R^2 (PC)$</td>
<td>0.85</td>
<td>$\text{adj} R^2 (PC)$</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\text{adj} R^2 (AD)$</td>
<td>0.87</td>
<td>$\text{adj} R^2 (AD)$</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$SE (PC)$</td>
<td>0.70</td>
<td>$SE (PC)$</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$SE (AD)$</td>
<td>0.48</td>
<td>$SE (AD)$</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$J\text{-stat}$</td>
<td>0.11</td>
<td>$J\text{-stat}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J\text{-stat p-value}$</td>
<td>0.52</td>
<td>$J\text{-stat p-value}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are heteroskedasticity and autocorrelation consistent standard errors.

Our estimates show an almost flat aggregate demand curve in all cases, close to Goodhart and Hofmann’s (2005) findings (0.01 - 0.03), and similar to those obtained by Smets (2003) and Leith and Malley (2005) for the whole euro area (0.06). Also, the Phillips curve estimates show high values of the discount factor ($\delta$), as reported in most empirical studies. Furthermore, we obtain low estimates for the slope of the Phillips curve for Germany and the Union ($\beta$ and $\rho$), which are significant only at the 10 percent level. In contrast, the Phillips curve for France (with slope $\beta^*$) is found to be comparatively steeper, revealing a stronger effect of the output gap on inflation for that country. The difference between the slopes of the Phillips curve in the two countries can be attributed to the higher price

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12 Depending on the model estimated, we used 3 to 5 lags of inflation, 2 to 5 lags of the output gap and 4 lags of the interest rate.
13 Doménech et al. (2001) also find that interest rate moves were not the driving force behind output gap changes before 1999.
14 In these papers the authors estimate a modified output Euler equation which includes both past and future values of the output gap.
15 Jondeau and Le Bihan (2005) estimate a hybrid Phillips curve, which includes a lag and a lead of inflation in addition to the output gap and report values between 0.09 and 0.12 for its slope. Although the hybrid version has been widely tested in empirical literature, most theoretical studies on monetary policy decision-making use baseline models involving only forward-looking Phillips and aggregate demand curves (see Clarida, Galí and Gertler, 1999; Svensson, 2003; Giannoni and Woodford, 2002a, 2002b; Woodford, 2003) which enable the derivation of straightforward and relatively simple interest rate reaction functions. Nevertheless, as a robustness check, we estimated the model consisting of the hybrid Phillips curve and the aggregate demand curve. Both unconstrained estimates of the hybrid Phillips curve and estimates under the restriction that the coefficients of expected future inflation and lagged inflation sum to unity (see e.g. Galí et al., 2005) show that the pattern of heterogeneity in the model’s parameters remains the same as in the results reported in this paper. Namely, the slope of the Phillips curve for France is considerably larger than Germany’s, while the slope of the aggregate demand curve for France is smaller relative to Germany.
level stickiness in Germany relative to France. This is also supported by Leith and Malley (2007), who estimate the New Keynesian Phillips curve for the G7 countries and provide evidence that in Germany the time required for price adjustment by all firms in the economy is the longest (close to two years) compared with just over 9 months in France. Similar evidence is given in Rumler (2005) and Peersman (2004) who find higher price persistence in Germany than in France. The values of the \( J \)-statistic (for the system of the Phillips and the aggregate demand curves) verify the validity of the instruments used and the values of \( \text{adj} R^2 \) are reasonably high.

7 Monetary policy performance under the alternative rules and sensitivity analysis

In order to evaluate the relative performance of the optimal interest rate reaction functions derived in Sections 4 and 5, we need to compare the value of the loss generated under each model. Both models are written in state space form and are solved numerically under commitment following Söderlind (1999) (see Appendix) and using the parameters estimated in the previous section. We can then derive the dynamics of the system and in particular the interest rate reaction function. We can also calculate the variance of the target variables, namely the inflation deviations from the target, the output gap and the interest rate deviations from the constant rate, in order to estimate the expected value of the loss function. According to Rudebusch and Svensson (1999), the value of the intertemporal loss function approaches the infinite sum of unconditional means of the period loss function, which is given by the weighted sum of the unconditional variances of the target variables:

\[
E(L_t) = \frac{\delta}{1-\delta} \text{trace}(K \Sigma_{YY})
\]

where \( K \) is a diagonal matrix, the diagonal elements of which are the parameters that represent the relative preferences of the central bank for the stabilization of inflation, the output gap and the interest rate in its loss function (\( 1, \lambda \) and \( \kappa \) respectively), and \( \Sigma_{YY} \) denotes the covariance matrix of the target variables.\(^{16}\) The discount factor (\( \delta \)) was estimated at 0.99 in both models, which is in line with the literature.

As a robustness check, we calculated the loss generated under the alternative interest rate rules assuming that the parameters in the loss function (\( \lambda \) and \( \kappa \)) take values ranging from 0.1 (in which case the central bank focuses almost exclusively on inflation) to 1 (where the central bank attaches a high cost to deviations of actual output from potential and to interest rate deviations from a constant rate).\(^{17}\) The resulting loss ratio, i.e. the loss

---

\(^{16}\)For comparison reasons, we solved both models using the parameters that were empirically estimated for each model, but by imposing the same covariance matrix, which was derived by averaging the variance of the residuals from the multi-country model.

\(^{17}\)This reflects the fact that the weights given to the stabilization of the output gap and the interest rate are not set above that of inflation, which is the primary objective of the ECB.
associated with the interest rate rule based on the union-wide model (Eq. 10) relative to that from the rule based on the multi-country model (Eq. 23) is shown in Figure 1. Table 2 presents the coefficients of the interest rate reaction function, the variance of the target variables, the value of the loss function and the loss ratio under the alternative interest rate rules for specific combinations of $\lambda$ and $\kappa$.

The estimated coefficients of the optimal interest rate reaction functions presented in Table 2 exhibit positive responses to inflation and output gap changes and pronounced interest rate smoothing, which can be attributed to the inclusion of the interest rate changes in the loss function. A further and rather obvious result is that increasing the weight ($\lambda$) given to output gap stabilization in the loss function leads to a higher interest rate response to output gap changes, while increasing the weight ($\kappa$) given to deviations of the interest rate from the rate consistent with the inflation target reduces the interest rate reaction both to inflation and to output gap changes.

The comparison of the alternative rules indicates that there are gains to be had if the central bank takes into account national structural characteristics, as presented in Section 5. In particular, the rule based on the union-wide model (Eq. 10) responds to each country’s variables according to their weight in the aggregate variable. In contrast, the rule based on the multi-country model (Eq. 23) suggests an adjustment to each country’s variables which depends on the structural parameters. This is why the interest rate in the case of the two-country model adjusts more to inflation in France and to the output gap in Germany. The fact that the Phillips curve is relatively steeper in France justifies a stronger (almost double) response of the single interest rate to French inflation, taking also into account that the aggregate demand curve is steeper in Germany and that Germany’s weight is higher. On the other hand, the relatively steeper aggregate demand curve in Germany, combined with the higher weight given to Germany’s variables, calls for a much stronger (three times as high) response to German output gap changes. This finding is at odds with the conclusion reached by Benigno (2004) that an inflation targeting policy in a two-country currency union, which attaches higher weight to the inflation of the country with the higher degree of nominal rigidity, is optimal. One possible explanation is that in this paper we allow for more than one type of asymmetry among countries, namely in the degree of price rigidity and in the intertemporal elasticity of substitution of consumption. As also indicated by Lombardo (2006), the result in Benigno (2004) could be altered if, in addition to nominal rigidity, the degree of competition differs.
Table 2: Coefficients and variances of the target variables and loss value under alternative models

<table>
<thead>
<tr>
<th>λ</th>
<th>κ</th>
<th>Model</th>
<th>Coefficient of</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Loss Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\pi_t^U$</td>
<td>$\tilde{y}_t^U$</td>
<td>$\pi_t$</td>
<td>$\tilde{y}_t$</td>
<td>$\pi_t^*$</td>
<td>$\tilde{y}_t^*$</td>
<td>$\pi_t^{i-\hat{i}}$</td>
<td>$\tilde{y}_t^{i-\hat{i}}$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>Union-wide</td>
<td>0.0168</td>
<td>0.0336</td>
<td>0.0090</td>
<td>0.0210</td>
<td>0.0188</td>
<td>0.0076</td>
<td>0.4335</td>
<td>0.3324</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two-country</td>
<td></td>
<td></td>
<td>0.0018</td>
<td>0.0042</td>
<td>0.0038</td>
<td>0.0015</td>
<td>0.4528</td>
<td>0.3048</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>Union-wide</td>
<td>0.0019</td>
<td>0.0037</td>
<td>0.0010</td>
<td>0.0023</td>
<td>0.0021</td>
<td>0.0008</td>
<td>0.4492</td>
<td>0.3012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two-country</td>
<td></td>
<td></td>
<td>0.0010</td>
<td>0.0023</td>
<td>0.0021</td>
<td>0.0008</td>
<td>0.4544</td>
<td>0.3021</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>Union-wide</td>
<td>0.0168</td>
<td>0.1680</td>
<td>0.0090</td>
<td>0.1050</td>
<td>0.0188</td>
<td>0.0380</td>
<td>0.4497</td>
<td>0.2626</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two-country</td>
<td></td>
<td></td>
<td>0.0018</td>
<td>0.0210</td>
<td>0.0038</td>
<td>0.0076</td>
<td>0.4598</td>
<td>0.2724</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>Union-wide</td>
<td>0.0019</td>
<td>0.0187</td>
<td>0.0010</td>
<td>0.0117</td>
<td>0.0021</td>
<td>0.0042</td>
<td>0.4559</td>
<td>0.2726</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two-country</td>
<td></td>
<td></td>
<td>0.0010</td>
<td>0.0117</td>
<td>0.0021</td>
<td>0.0042</td>
<td>0.4604</td>
<td>0.2731</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>Union-wide</td>
<td>0.0168</td>
<td>0.3024</td>
<td>0.0090</td>
<td>0.1890</td>
<td>0.0188</td>
<td>0.0684</td>
<td>0.4590</td>
<td>0.2428</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two-country</td>
<td></td>
<td></td>
<td>0.0018</td>
<td>0.0378</td>
<td>0.0038</td>
<td>0.0137</td>
<td>0.4624</td>
<td>0.2594</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>Union-wide</td>
<td>0.0019</td>
<td>0.0336</td>
<td>0.0010</td>
<td>0.0210</td>
<td>0.0021</td>
<td>0.0076</td>
<td>0.4623</td>
<td>0.2602</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two-country</td>
<td></td>
<td></td>
<td>0.0010</td>
<td>0.0210</td>
<td>0.0021</td>
<td>0.0076</td>
<td>0.4666</td>
<td>0.2613</td>
</tr>
</tbody>
</table>

Note: The coefficient of $i_{t-1}$ is equal to 2.0118 under union-wide model and 2.0129 under multi-country model and that on $i_{t-2}$ is -1.0101 under both models.
More importantly, for any combination of $\lambda$ and $\kappa$ reported in Table 2, but also in Figure 1, the loss ratio generated from the monetary policy decisions based on the union-wide interest rate rule relative to the two-country interest rate rule remains above unity. This provides evidence that it can be welfare improving if the central bank commits to follow an interest rate rule that focuses on the individual countries’ variables taking into account their structural characteristics, especially in case there are sizeable differences.

**Figure 1: Loss ratio under the union-wide model relative to the two-country model**

Furthermore, the welfare loss increases in proportion to the relative importance the central bank attaches to the goals of output gap and interest rate stabilization compared to inflation stabilization. This can be explained by the fact that increasing the weights of the stabilization of the output gap and the interest rate deviations from a constant rate, weakens inflation targeting and increases welfare losses. A result evident in all cases is the higher volatility of the common interest rate under the rule based on the union-wide model compared to the rule based on the two-country model. The intuition behind this result is

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18 The loss ratio ranges from 1.0001 to 1.012, which corresponds to a potential welfare improvement of up to 1.2%.
that the rule which responds to national variables attaches relatively higher importance to
the country which is characterized by inflation and the output gap being more responsive
to interest rate movements and is thus better off with smaller shifts in the common interest
rate. As a result, the interest rate from the rule based on the two-country model adjusts
less. Similar conclusions are reached by De Grauwe and Piskorski (2001).

8 Challenges for the European Central Bank

In the previous section we showed that if the ECB’s monetary policy takes into account
the heterogeneity observed in the economic structures of the European monetary union
countries, then there are gains to be had. The variation in the structural parameters of the
Phillips and the aggregate demand curves is attributed to country-specific characteristics
as described in Section 2, such as the degree of stickiness and forward-looking behavior
in the price-setting mechanism. This dispersion can be significantly destabilizing for the
national economies and will probably be aggravated as more EU countries join the monetary
union in the future. According to Camacho, Pérez-Quirós and Saiz (2006), the divergence,
measured by business cycles comovements, between the new and the current members of the
European monetary union is found larger than the divergence which the current members
exhibited prior to the establishment of the union. Therefore, setting the interest rate in
response to individual countries’ variables could give rise to substantial welfare gains.\

There may, however, be difficulties in the implementation of such a rule, as identified in
ECB (2005). First, there are problems to precisely estimate the differential impact of the
common monetary policy on the individual countries of the union. Second, possible mea-
surement constraints, especially those regarding the estimation of potential output and the
output gap, are likely to be compounded when disaggregate data are considered. Both
these factors could introduce uncertainty into the conduct of monetary policy and affect
negatively the transparency and accountability of the central bank. Third, a differentiated
response by the central bank taking into account national structural characteristics would
imply that monetary policy is accommodating the existing structural asymmetries, thus
creating disincentives for reform in euro area countries. The first two of the above difficul-
ties could be overcome by better measurement and estimation methods, while the last one
should be addressed by national economic policies, given that the ECB’s single monetary
policy can only be expected to indirectly influence such divergences.

Despite the welfare improvement which can be achieved by the response of the common
monetary policy to individual countries’ developments, in the future there may be less
scope for such a differentiated response as monetary and financial integration advances

This would, by no means, imply that the governors of the national central banks should adopt a nationalistic perspective
as in Aksoy, De Grauwe and Dewachter (2001, 2002). Instead, in our multi-country model all the members of the ECB’s
Governing Council set the policy rate in order to minimize the average of all countries’ loss functions, weighted according to
their relative size.
further. Although divergences across the union members are currently significant, it is expected that they will narrow over time as a result of the common monetary policy and the single financial market (see e.g. Mihov, 2001 and Guiso et al., 1999). Some of the possible factors working in this direction are the following: the financial system is likely to progressively become less segmented and more homogeneous; deregulation and common legislation may foster integration and further development of capital markets; increased banking competition can make the pass-through of changes in policy rates to lending rates more similar; trade linkages are expected to be enhanced and country-specific shocks to fade out, both contributing to higher correlation of economic activity; wage bargaining by labor unions will probably decline due to higher labor mobility, making wage and price adjustments more flexible; intra-industry trade may possibly strengthen, leading to convergence of industrial structures. Finally, for prospective European monetary union members, the shift to a more credible and systematic monetary policy conducted by the ECB can significantly reduce uncertainty and volatility, since monetary policy is factored in private sector’s expectations and forward-looking economic decisions.

9 Conclusions

This paper studied the optimal design of monetary policy in a monetary union in the presence of structural asymmetries among countries by deriving analytically an optimal interest rate rule under commitment and examining the dependence of its coefficients on the parameters of the structural model of each country, the central bank’s preferences in the loss function and the relative size of each economy. We provided empirical evidence on the gains to be achieved by taking into account the heterogeneity in the structure of the economies using data from two core countries, Germany and France. In particular, according to our results, the ratio of the loss generated by an interest rate reaction function based on the union-wide model over that based on the multi-country model, stands above unity. Thus, our exercise suggests that the interest rate should be adjusted so as to stabilize more the variables of the country with the lower nominal rigidity and lower intertemporal elasticity of substitution of consumption. Otherwise, monetary policy decisions may cause large variability of the target variables in this country and generate welfare losses. This finding appears to be robust under alternative values of central bank preferences for the stabilization of the target variables. Although the implementation of the proposed rule involves difficulties relating to data and estimation constraints and risks accommodating structural divergences, taking into consideration national characteristics in formulating ECB monetary policy would be welfare improving, especially in view of more countries joining the European monetary union in the future. However, as monetary and financial integration advances, the welfare benefits of monetary policy responding to individual countries’ variables may become less significant.
References


A Appendix

A.1 State space representation of the union-wide model

The structural equations of the model (Eqs. 5 and 6) can be written as:

\begin{align*}
\pi_{t+1}^U &= \frac{1}{\delta} \pi_t^U - \frac{\rho}{\sigma} \tilde{y}_t^U - \frac{1}{\delta} \epsilon_t \\
\tilde{y}_{t+1}^U &= \tilde{y}_t^U + \frac{\rho}{\sigma \delta} \tilde{y}_t^U + \frac{1}{\sigma} i_t - \frac{1}{\sigma \delta} \pi_t^U - \frac{\eta}{\sigma} + \frac{1}{\sigma \delta} \epsilon_t - u_t
\end{align*}

and their state-space representation is:

\[ X_{t+1}^U = A^U X_t^U + B^U i_t + V_t^U \quad \Rightarrow \]

\[
\begin{bmatrix}
\epsilon_{t+1} \\
u_{t+1} \\
\tilde{y}_{t+1}^U \\
i_t \\
l_{t-1} \\
E_t \pi_{t+1}^U \\
E_t \tilde{y}_{t+1}^U
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{\sigma} & 0 & 0 & 0 & 0 & \frac{\eta}{\sigma} & -\frac{\rho}{\sigma} & 1 + \frac{\rho}{\sigma \delta} \\
\frac{1}{\sigma \delta} & -1 & 0 & 0 & 0 & -\frac{\eta}{\sigma} & -\frac{1}{\sigma \delta} & \frac{\eta}{\sigma}
\end{bmatrix}
\begin{bmatrix}
\epsilon_t \\
u_t \\
\tilde{y}_{t-1}^U \\
i_{t-1} \\
l_{t-2} \\
\pi_t^U \\
\tilde{y}_t^U
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
1 \\
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\epsilon_{t+1} + 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

The target variables are given by:

\[ Y_t^U = C_t^U X_t + C_t^U i_t \quad \Rightarrow \]

\[
\begin{bmatrix}
\pi_t^U - \tilde{\pi} \\
\tilde{y}_t^U \\
i_t - \tilde{i}
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & -\tilde{\pi} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -\tilde{i} & 0 & 0
\end{bmatrix}
* 
\begin{bmatrix}
\epsilon_t \\
u_t \\
\tilde{y}_{t-1}^U \\
i_{t-1} \\
l_{t-2} \\
\pi_t^U \\
\tilde{y}_t^U
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\epsilon_{t}
\]

The loss function can be written in state-space form as:

\[ L_t = Y_t^{U^T} K Y_t^U \]

\[
\begin{bmatrix}
\pi_t^U - \tilde{\pi} \\
\tilde{y}_t^U \\
i_t - \tilde{i}
\end{bmatrix}
' 
= 
\begin{bmatrix}
1 & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \kappa
\end{bmatrix}
* 
\begin{bmatrix}
\pi_t^U - \tilde{\pi} \\
\tilde{y}_t^U \\
i_t - \tilde{i}
\end{bmatrix}
\]
A.2 State space representation of the multi-country model

The structural equations (Eqs. 14 to 17) can be written as:

\[
\pi_{t+1} = \frac{1}{\delta} \pi_t - \frac{\beta}{\delta} \tilde{y}_t - \frac{1}{\delta} \varepsilon_t
\]

\[
\tilde{y}_{t+1} = \tilde{y}_t + \frac{\beta}{\delta_T} \tilde{y}_t + \frac{1}{\tau} i_t - \frac{1}{\delta_T} \pi_t - \frac{1}{\delta} \varepsilon_t - \nu_t
\]

\[
\pi^*_{t+1} = \frac{1}{\delta} \pi^*_{t} - \frac{\beta^*}{\delta} \tilde{y}^*_{t} - \frac{1}{\delta} \varepsilon^*_{t}
\]

\[
\tilde{y}^*_{t+1} = \tilde{y}^*_{t} + \frac{\beta^*}{\delta_{T^*}} \tilde{y}^*_{t} + \frac{1}{\tau^*} i_t - \frac{1}{\delta_{T^*}} \pi^*_{t} - \frac{1}{\delta} \varepsilon^*_{t} - \nu^*_{t}
\]

and represented in state-space form as:

\[
X^T_{t+1} = A^T X^T_t + B^T i_t + V^T_t
\]

\[
\begin{bmatrix}
\varepsilon_{t+1} \\
v_{t+1} \\
\varepsilon^*_{t+1} \\
v^*_{t+1} \\
\tilde{y}_t \\
\tilde{y}^*_t \\
i_t \\
i^*_{t-1} \\
1 \\
E_t \pi_{t+1} \\
E_t \tilde{y}_{t+1} \\
E_t \pi^*_{t+1} \\
E_t \tilde{y}^*_{t+1}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{\delta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & 0 \\
\frac{1}{\delta_{T^*}} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\delta_{T^*}} & -\frac{1}{\delta_{T^*}} \\
0 & 0 & -\frac{1}{\delta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
The target variables are given by:

\[ Y_t^T = C_t^T X_t + C_t^T i_t \]

The loss function is then given by:

\[ L_t = Y_t^T K_t Y_t^T = \left[ \begin{array}{c} \pi_t - \hat{\pi} \\ \tilde{y}_t \\ \pi_t^* - \hat{\pi} \\ \tilde{y}_t^* \\ i_t - \hat{i} \end{array} \right]' \cdot \left[ \begin{array}{cccccc} w & 0 & 0 & 0 & 0 & 0 \\ 0 & w\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-w) & 0 & 0 \\ 0 & 0 & 0 & (1-w)\lambda & 0 & \kappa \\ 0 & 0 & 0 & 0 & \kappa & \end{array} \right] \cdot \left[ \begin{array}{c} \pi_t - \hat{\pi} \\ \tilde{y}_t \\ \pi_t^* - \hat{\pi} \\ \tilde{y}_t^* \\ i_t - \hat{i} \end{array} \right] + \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right]' \cdot \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right] \]
A.3 Estimation of the loss

The evaluation of the loss function relies on the solution of rational expectations models proposed by Söderlind (1999). Under commitment, the problem of the central bank is to minimize the loss function, by choosing an optimal sequence of the policy instrument \( i_t \):

\[
\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j L_t
\]

where the period loss function is:

\[
L_t = Y_t' K Y_t = \begin{bmatrix} C_X' \\ C_I' \end{bmatrix} K \begin{bmatrix} C_X & C_I \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix} = X_t' C_X K C_X X_t + X_t' C_I K C_I i_t + i_t' C_I K C_I X_t + i_t' C_I K C_I i_t
\]

subject to the structural constraints:

\[
X_{t+1} = A X_t + B i_t + V_t
\]

The structural variables can be distinguished into predetermined or backward-looking \( X_{1t} \) and non-predetermined or forward-looking \( X_{2t} \) variables. Therefore, the structural equations are written as:

\[
\begin{bmatrix} X_{1t+1} \\ E_t X_{2t+1} \end{bmatrix} = A \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + B i_t + \begin{bmatrix} e_{1t} \\ 0 \end{bmatrix}
\]

The Lagrangian is given by:

\[
J_0 = E_t \sum_{j=0}^{\infty} \delta^j \left[ X_t' C_X K C_X X_t + X_t' C_I K C_I i_t + i_t' C_I K C_I X_t + i_t' C_I K C_I i_t \right] + 2 \xi_{t+1} [A X_t + B i_t + V_t - X_{t+1}]
\]

where \( \xi \) denotes the Lagrange multipliers.

Under commitment, the solution is given by:

\[
k_{1t+1} = \begin{bmatrix} X_{1t+1} \\ \xi_{2t+1} \end{bmatrix} = M_c \begin{bmatrix} X_{1t} \\ \xi_{2t} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ 0 \end{bmatrix}
\]

\[
k_{2t+1} = \begin{bmatrix} X_{2t+1} \\ i_t \\ \xi_{1t+1} \end{bmatrix} = N_c \begin{bmatrix} X_{1t} \\ \xi_{2t} \end{bmatrix}
\]

where \( M_c \) and \( N_c \) are functions of the submatrices resulting from the generalized Schur decomposition of the first-order conditions.

As in Leitemo and Söderström (2005), the covariance matrix of \( k_{1t+1} \) is given:
\[ \text{vec}(\Sigma_{k_1}) = [I - M_c \otimes M_c]^{-1} \text{vec}(\Sigma_{VV}) \]

where \( \Sigma_{VV} \) is the variance-covariance matrix of disturbances.

Stacking \( k_{1t+1} \) and \( k_{2t+1} \), the covariance matrix of all variables \((k \equiv k_1 + k_2)\) is given by:

\[
\Sigma_{kk} = \begin{bmatrix}
I \\
N_c
\end{bmatrix} \Sigma_{k1} \begin{bmatrix}
I' \\
N_c'
\end{bmatrix}
\]

Picking out the covariance matrix of the structural variables and the instrument \((\Sigma_{X_i})\), the covariance matrix of the target variables \((Y_t)\) can be written as:

\[
\Sigma_{YY} = \begin{bmatrix}
C_X & C_I
\end{bmatrix} \Sigma_{X_i} \begin{bmatrix}
C_X' \\
C_I'
\end{bmatrix}
\]

Therefore, the expected value of the loss function can be estimated as:

\[
E(L_t) = \frac{\delta}{1 - \delta} \text{trace}(K \Sigma_{YY})
\]

42. Christl, J., “Regional Currency Arrangements: Insights from Europe”, including comments by Lars Jonung and the concluding remarks and main findings of the workshop by Eduard Hochreiter and George Tavlas, June 2006.


