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A CONDITIONAL CAPM; IMPLICATIONS FOR THE ESTIMATION OF SYSTEMATIC RISK

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Abstract

The purpose of this paper is to examine: (i) whether or not, the residuals of the Market Model are conditionally heteroscedastic; (ii) whether or not, there exists an intervalling effect in conditional heteroscedasticity in the residuals of the Market Model; (iii) the effect of conditional heteroscedasticity on the estimation of systematic risk.; as well as to propose a simple data driven conditional CAPM. To this end daily closing price of stocks traded at the Athens Stock Exchange are used. Empirical evidence is provided for the existence of: (a) conditional heteroscedasticity in MM residuals; (b) a pronounced intervalling effect on ARCH in MM residuals; (c) GARCH in mean type of conditional heteroscedasticity for the majority of cases where ARCH was present in MM residuals. These findings in terms of theory are conducive to a conditional CAPM, which takes into account the effect of conditional variance on expected returns, rather than the standard CAPM. Furthermore, in terms of practical implications these findings may lead to better estimates of systematic risk

Keywords: Conditional Capital Asset Pricing Model, Market Model, Conditional Volatility, Systematic Risk, Intervalling Effect, Athens Stock Exchange.

JEL Classification: C10, G10, G11, G12, G32

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1. Introduction

Among the most fundamental issues in finance is the relation between risk and return. There is little doubt that thus far the Capital Asset Pricing Model (CAPM) of Sharp (1964), Lintner (1965) and Black (1972) has played a very important role, as it describes how investors react to risk and value risky assets. More specifically CAPM expresses the expected return of the risky asset *j* $E(Rj)$ as follows:

$$E(R_j) = R_f + \beta_j [E(R_m - R_f)]$$

Where:

R_f is the return on the risk free asset;

R_m is the return on the whole economy, usually approximated by a composite market index;

β_j is the so-called beta or systematic risk coefficient of asset *j*.

As is apparent, according to the CAPM, the expected return of asset *j* is a linear function of its corresponding beta and presumably this one-dimensional expression for risk suffices to describe the cross section of expected returns. There is a voluminous literature on the ability of the CAPM to explain observed asset returns and there are serious specification issues that have been raised regarding its ability to explain much. For instance, Fama and French in a series of papers during the 90s (Fama and French 1992, 1993, 1996) found that the (standard) CAPM does not hold empirically. Fama and French proposed the so-called Three Factor Model (TFM) which reflects the fact that two classes of stocks (those with small capitalization and high book to market value) tend to do better than the market as a whole. TFM has gained recognition in financial management and many studies have shown that the majority of actively managed mutual funds underperform broad indices based on Fama and French's three factors, if classified properly. However, the success of the TFM spurred a considerable debate in the literature and the main reason is that the extra two factors used by Fama and French are just returns on portfolios formed on the same characteristics which lack a clear economic relationship with systematic risk. At the same time, numerous attempts have been made to explore the nature of the anomalies associated with the standard CAPM and correct them. Breen et al

(1989) showed that betas in the CAPM framework are not time-invariant but rather vary over the business cycles, as also shown by Chen (1991). Jaganathan and Wang (1996) have developed a conditional version of the CAPM in which it is assumed that the CAPM holds in a conditional sense allowing beta and the market risk premium to vary over time. A main problem with the conditional CAPMs in general is the choice of conditioning variables and the lack of theory about how to form the relationship between the betas and the conditioning variables.

The present work using the standard static CAPM as a starting point tries to look at the context of a conditional CAPM for a specific simple conditional type of CAPM where the conditioning is data driven, derived by an examination of the residuals of the so-called market model. Such a model may lead to more accurate estimates of systematic risk. The rest of the paper is structured as follows: the methodological approach is explained in section 2; the empirical results are presented and commented upon in section 3; section 4 summarizes and concludes the paper.

2. Data and methodological approach

The data are daily closing prices of all stocks traded on the Athens Stock Exchange for the period from 1/10/1999 to 30/9/2004 and were provided by the Athens Stock Exchange (ASE). From May 31 2001, the ASE joined the mature financial markets according to the classification of Morgan Stanley Capital International. Until that date, the ASE belonged to the European Emerging Markets. Its current total capitalization is about 70 million euro. Further details about the ASE are given in many papers (see for instance Milionis and Papanagiotou, 2009). The Athens General Index is used as the market index. Returns are expressed as logarithmic differences of (adjusted) price relatives.

It must first be noted that methodologically the CAPM is a general equilibrium model; hence, both the return of the each time particular asset and the market return are expected future returns, while the relevant coefficient of systematic risk is the future beta of the each time particular asset. Unfortunately data for expected future returns do not exist. What is feasible is to perform an ex post analysis of ex ante expected returns. In order to replace ex-ante with ex-post realized returns, it has to be assumed that expectations are on average correct and therefore actual events can be considered as proxies for expectations. Hence, the common approach for the estimation of the standard betas is to use the so-called Market Model (henceforth MM) and OLS estimators. In that way, a beta is estimated as the slope coefficient in the regression:

$$R_j = \alpha_j + \beta_j R_m + u_j$$

Where u_j is the stochastic disturbance and α_j is a constant. It is noticeable that the the risk free rate is not included in the MM. For realistic changes in the value of the risk free rate, there is very little difference in the estimated beta values.

Not taking into consideration phenomena related to market microstructure, one would expect that beta estimates would be invariant to changes in the length of the differencing interval over which index and security returns are calculated. However, a plethora of empirical findings suggest that this is not the case. Indeed, it has been found that beta estimates change systematically as the differencing interval over which they are estimated is lengthened (see for instance Corhay, 1991, Cohen et al. 1983a, Scholes and Williams 1977, Dimson, 1979). This phenomenon, known as "intervalling effect", results in a biased beta estimate and consequently in an incorrect assessment of a security's risk. The main reason for this bias is friction in the trading process and the price adjustment delays entailed (Cohen et al., 1983b). The magnitude of the bias decreases as the differencing interval over which returns are calculated (henceforth denoted by l) is increased and it has been proved that (Fung et al., 1985):

$$p\lim_{N\to\infty}(\lim_{l\to\infty}\hat{\beta}_{\mathrm{j}}^{\mathrm{ols}}(l)) = \beta_j \tag{1}$$

where N is the sample size, $\hat{\beta}_{\mathrm{j}}^{\mathrm{ols}}(l)$ is the OLS estimator of beta corresponding to differencing interval l and β_{j} is the true value of the market risk for security j.

Based on what eq. (1) implies in terms of the behaviour of betas as the differencing interval increases without bound, Cohen et al. (1983a) have suggested a methodology leading to unbiased estimates of betas, the so-called asymptotic betas. This methodology has been adopted by several other researchers.

The methodology described above is based exclusively on OLS estimates. Although not explicitly stated in the finance textbooks, it must be noted at this point that among the assumptions for the validity of the MM is that the residual variance should be constant both in the unconditional and conditional sense. Theoretical considerations (e.g. Bollerslev et al., 1992; Nelson 1992) as well as empirical evidence offer support for a kind an “intervalling effect” on autoregressive conditional heteroscedasticity (henceforth ARCH) in stock returns and exchange rates (e.g. Brailsford, 1995; Baillie and Bollerslev, 1989). Therefore, ARCH is expected and indeed has been found to be more pronounced in higher frequency returns. In spite of its importance for the accuracy of the estimates of systematic risk, such an investigation has not been undertaken in the residuals of the classical MM thus far. In the next section the possibility of an intervalling effect in MM residuals is explored and evidence is provided for the existence of such an effect. Moreover, the character of this effect is examined and further analyzed.

More specifically, the MM will be estimated using differencing intervals from one up to thirty days. It is noted that for $l > 1$, estimates of beta corresponding to the same l but estimated using a different starting day may be different (Corhay, 1992). To take into account, this effect for differencing intervals of length $l > 1$, l estimates of beta will be obtained, each corresponding to a different starting day within the differencing interval l. The final estimate of beta corresponding to differencing interval l will be the average of these l estimates (see Corhay, 1992 for a detailed discussion on that point). A Ljung-Box test for autocorrelation in the squared residuals of the market model will be performed for each case. For every case that the Ljung–Box test results are significant, i.e. there exists autoregressive conditional heteroscedasticity in the residuals of the MM, the following models of conditional volatility will be estimated:

(a) GARCH(1,1)

$$R_{it} = a + \beta_i R_{mt} + u_{it}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

(b) EGARCH(1,1)

$$R_{it} = a + \beta_i R_{mt} + u_{it}$$

$$\log\left(\sigma_t^2\right) = a_0 + a_1 \frac{|u_{t-1}| + \gamma u_{t-1}}{\sigma_{t-1}} + \beta_1 \log\left(\sigma_{t-1}^2\right)$$

(c) GARCH-M (1,1)

$$R_{it} = a + \beta_i R_{mt} + \gamma_i g(\sigma_t) + u_{it}$$

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

(d) EGARCH-M(1,1)

$$R_{it} = a + \beta_i R_{mt} + \gamma_i g(\sigma_t) + u_{it}$$

$$\log\left(\sigma_t^2\right) = a_0 + a_1 \frac{|u_{t-1}| + \gamma u_{t-1}}{\sigma_{t-1}} + \beta_1 \log\left(\sigma_{t-1}^2\right)$$

In the above models σ_t^2 is the conditional variance, α_1, α_2, β_1, γ_i, δ are constants and g(.) is a function of σ_t for which the following forms will be tried:

$$g(\sigma_t) = \sigma_t$$
$$g(\sigma_t) = \sigma_t^2$$
$$g(\sigma_t) = \ln(\sigma_t^2)$$

If in more than one model of those described above the parameters are statistically significant the selection among rival models will be based upon the value of the Akaike criterion.

The examination of an intervalling effect on conditional volatility in the MM residuals is important in its own right; however, it is also of importance to examine this phenomenon in relation to the estimation of systematic risk. To this end, for each stock, two sets of beta estimates, one in which ARCH is taken into consideration and one without taking it into account, will be obtained. The length of the differencing interval will vary from one to thirty days. It must be noted that the first equation of models (a) and (b) above is tantamount to the MM, but the first equation of models (c) and (d) is not. Indeed, in models (c) and (d) an extra explanatory variable (a function of the conditional variance) has been added in the first equation to express the fact that investors expect a higher return for the extra uncertainty due to the non-constancy of the conditional variance.

3. Results and discussion

The results regarding the dependence of conditional volatility in the MM residuals on the length of the differencing interval are presented in the form of a graph in Figure 1. More specifically, Figure 1 shows the percentage (%) of cases for which the Ljung-Box test in the square of the residuals of the MM rejects the null hypothesis of no autocorrelation (at the 5% significance level) as a function of the length of the differencing interval. As is evident from the results of Figure 1, autoregressive conditional heteroscedasticity is found in 97.4% of the cases for l = 1. This percentage decreases rapidly being less than 7% for differencing intervals greater than about 20-22 days. Overall these results are suggestive of the existence of a strong intervalling effect in MM residuals.

Furthermore, the extent to which autoregressive conditional heteroscedasticity appears in the form a (E)GARCH in mean type of model will be examined (models (c) or (d) in the previous section). This is important since in these cases the CAPM is misspecified and an extra term, related to the conditional variance, must be included in the right hand side of the MM. Consequently, given that the corresponding partial regression coefficient is expected to be positive (see Milionis and Moschos, 2000 for a detailed discussion on that point), with GARCH in mean conditional volatility beta estimates are expected to be reduced. Owing to the existence of this extra term, as explained above, β_j can no longer play the role of the sole risk factor on such occasions and will be called the market beta hereafter.

Figure 2 shows the percentage (%) of cases for which ARCH is of GARCH in mean type in the MM residuals, out of both the total number of cases (triangles) and the number of cases with ARCH (squares). From Figure 2, at first it is evident that, as is the case with conditional volatility in the MM residuals in general, there is a pronounced intervalling effect in the GARCH in mean type of conditional volatility. It is also noted that, despite its fluctuations, GARCH in mean type of volatility as a percentage of the total cases with ARCH does not exhibit any conspicuous intervalling effect and for most differencing intervals represents the majority of ARCH cases.

It must be clarified that the above findings do not imply a rejection of the CAPM. Indeed, as noted in the previous section, the CAPM assumes a constant variance in the deviations from equilibrium. The previous empirical results indicate that this condition does not hold for the majority of cases. Hence, models (c) and (d) must be seen as generalizations of the CAPM under more general conditions i.e. allowing for the presence of (E)GARCH in mean conditional heteroscedasticity. It must also be noted that even using models (a) and (b), where no extra term is added to the MM model, the estimation of beta using these models is more efficient than the corresponding OLS estimate.

It is of much importance to investigate the alleged influence of conditional volatility on the estimation of systematic risk. Figure 3 shows the variation of average OLS beta and average market beta with the length of the differencing interval. From the character of this variation, as shown in Figure 3, several interesting comments can be made. At first it is noted that the value of average betas is greater than one in all cases. This is not surprising as betas of individual stocks are not weighted by the corresponding capitalization values for the calculation of the average beta. Further, average beta (and average market beta for the GARCH cases) increases with the length of the differencing interval with both OLS and GARCH estimation, confirming the existence of an intervalling effect in betas. However, estimates of (market) betas which take into consideration autoregressive conditional heteroscedasticity are always lower than the corresponding (for the same differencing interval) betas estimated purely with OLS. This difference takes its maximum values for the shortest differencing intervals (one to four days) confirming the point made previously.

Further insight about the magnitude of the deviation of beta estimates using the two methods is provided in Figure 4. This figure shows the variation of the value of the mean absolute percentage error (MAPE) across the differencing interval. This statistic is calculated by the formula:

$$MAPE_{GARCH-OLS} = \frac{1}{J}\sum_{j=1}^{J}\left(\frac{\left|\hat{\beta}_j^{OLS} - \hat{\beta}_j^{GARCH}\right|}{\hat{\beta}_j^{GARCH}} \cdot 100\right)$$

where J is the total number of securities.

As is evident from Figure 4, as a result of the intervalling effect on conditional volatility in the MM residuals, the difference in the beta estimates due to the method of estimation is much more substantial for the shorter differencing intervals, (exceeding 15% for differencing intervals of 2 and 3 days) as compared to the longest differencing intervals where this difference is only of the order of 2% to 3%.

4. Summary and conclusions

Despite the criticism of the standard CAPM, as described in the introduction, for a variety of reasons (see for instance Jagatnathan and Wang, 1996, Elton et al., 2007) betas are still the most widely used measure of systematic risk by analysts and financial managers.

In this work empirical evidence is provided for the existence of: (a) conditional heteroscedasticity in MM residuals; (b) a pronounced intervalling effect on ARCH in MM residuals; and (c) GARCH in mean type of conditional heteroscedasticity for the majority of cases where ARCH was present in MM residuals.

As a consequence of (a), the use of simple OLS for the estimation of betas is not justified. As a consequence of (b), autoregressive conditional heteroscedasticity affects unevenly the OLS estimates of betas. As a consequence of (c), an extra term related to the conditional variance must be added to the right hand side of CAPM. This simply reflects the fact that the variance of returns is not constant in the conditional sense, as implied in the standard CAPM. It is important to mention that the development of autoregressive conditional heteroscedasticity models had not been developed when the CAPM was introduced; hence, given the evidence provided in this work, the CAPM should be augmented to allow for autoregressive conditional heteroscedasticity. Under such conditions a conditional type CAPM is more appropriate. The highest influence on OLS beta estimates is for differencing intervals up to five days for which the corresponding market beta estimates are lower by more than 13%. These findings may help towards better estimation of systematic risk. Methods which have been developed for the estimation of systematic risk based on data of such frequencies (e.g. the so-called inferred asymptotic betas, see Cohen et al., 1983b) must necessarily take into account the intervalling effect on ARCH in MM residuals. Hence, the approach followed in this work provides a relatively simple way to improve the accuracy of systematic risk estimates.

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Appendix

Figure 1. Percentage (%) of cases for which the Ljung-Box test in the square residuals of the Market Model rejects the null hypothesis of no autocorrelation.

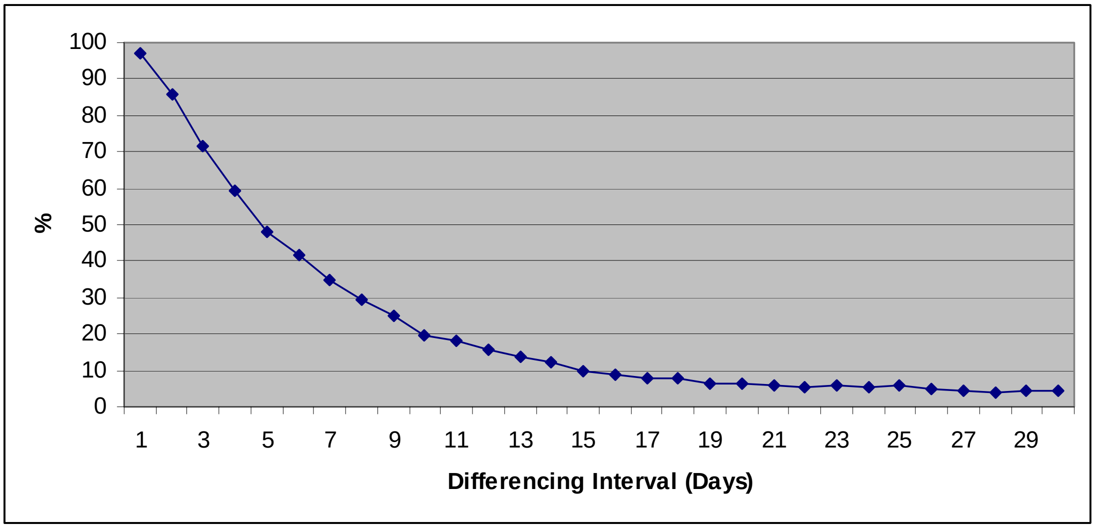


Figure 2. Percentage of GARCH in mean as a function of the differencing interval

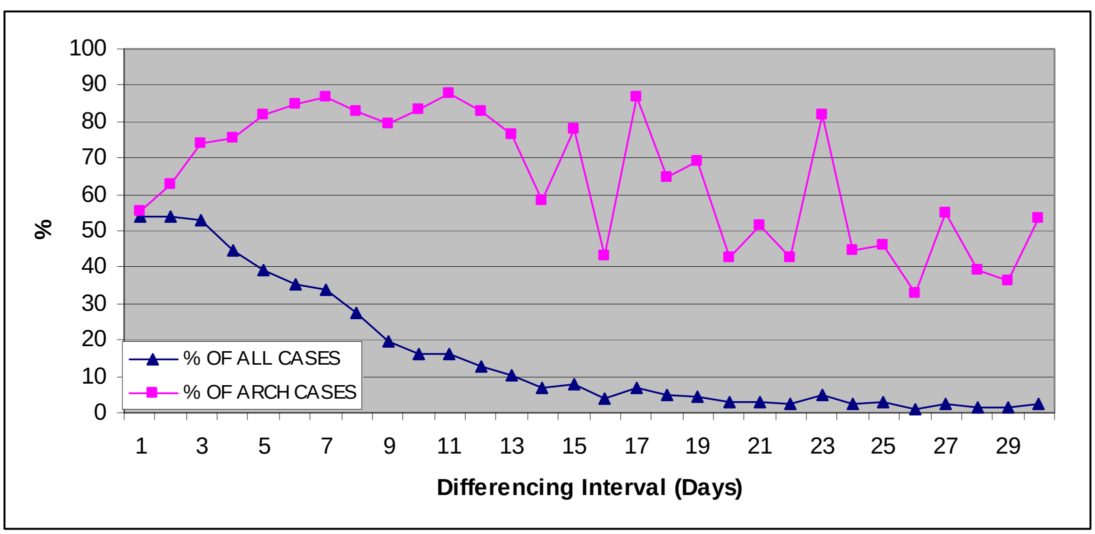


Figure 3. Variation of average beta with the length of the differencing interval

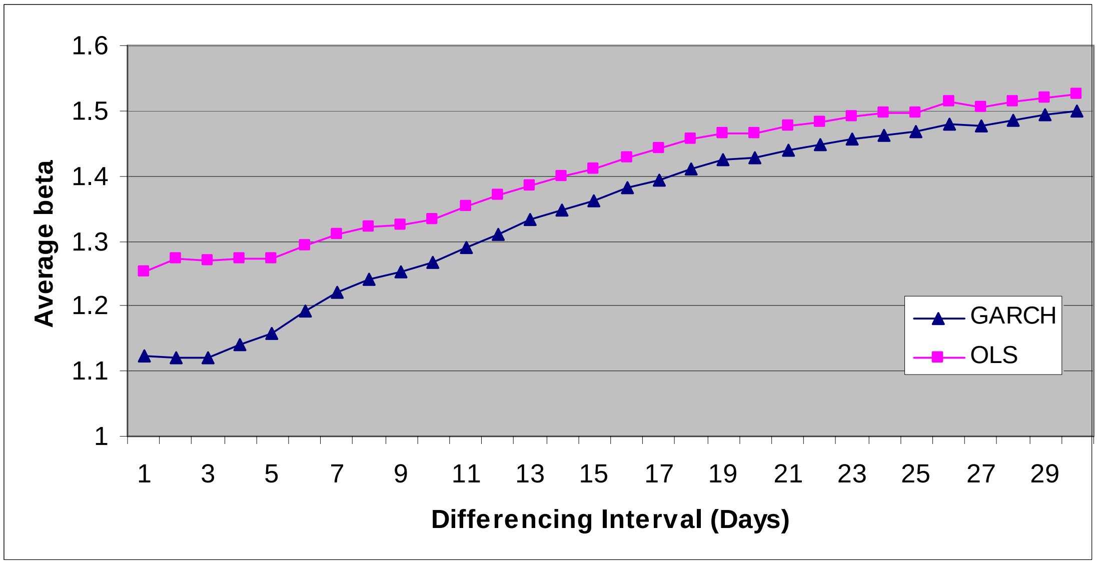
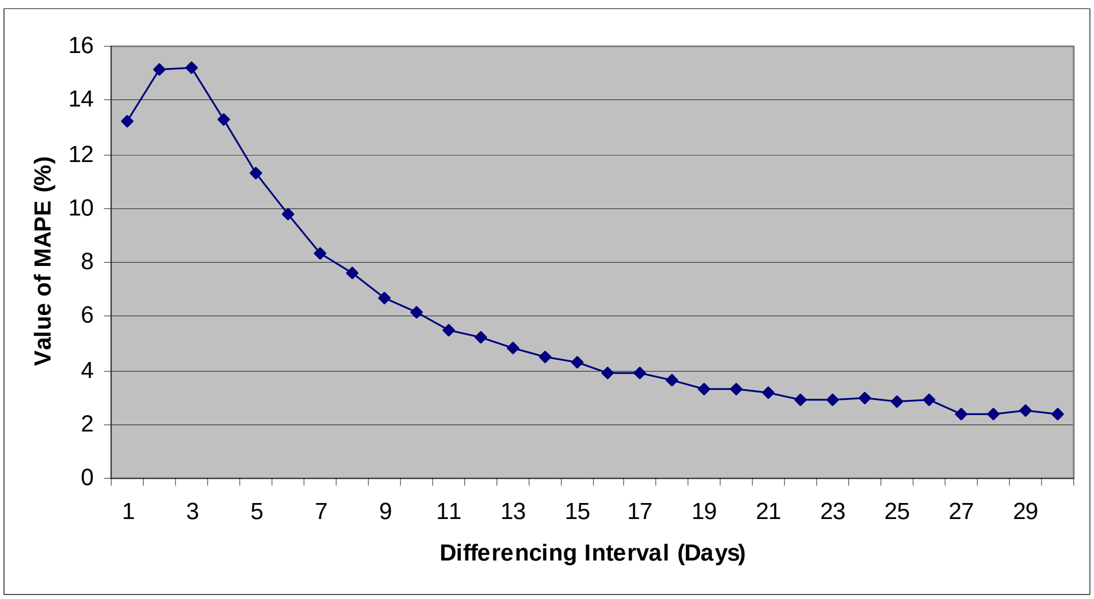


Figure 4. Variation of MAPE with the differencing interval



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