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FISCAL POLICY WITH AN INFORMAL SECTOR

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Abstract

Macroeconomic models that omit the shadow economy systematically mis-forecast and mis-measure the effect of fiscal –in particular tax– policy on economic activity and tax revenue. We add an informal sector to the Bank of Greece DSGE model and use the actual package of fiscal consolidation implemented in Greece over the period 2010–2015 to evaluate the role of the black economy. In the data, official Greek GDP declined by about 26%, budget deficits proved larger and more persistent and tax rates increased by much more and tax revenue by much less than predicted. The model replicates the official output decline but implies a true output decline that is less than two thirds of that in recorded output. The discrepancy is even more pronounced for employment. The model also implies that the size of fiscal adjustment and the drop in economic activity could have been considerably milder had the informal sector been curtailed (it instead increased by about 50%). The underground economy seems to have been a key factor in Greece's failure to achieve orderly debt consolidation while avoiding economic depression.

JEL-classification: E26, E32, E62, E65, H26, H68

Keywords: Shadow economy, fiscal consolidation, multipliers, tax revenue, true output

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1 Introduction

The sovereign debt crisis in the Eurozone led to large fiscal consolidation programs in the afflicted countries. The large declines in economic activity have triggered debates on the size of the fiscal multipliers as well as on the implications of austerity. For instance, it has been argued that fiscal multipliers must be much larger than those typically assumed in the literature and which underlined the forecasts made by the official creditors such as the IMF and the European Commission (Blanchard and Leigh, 2013). It has also been argued that severe fiscal consolidation (austerity) programs may be counterproductive and self-defeating (see Corsetti, 2012). The experience of Greece seems consistent with these claims: the recorded recession has proved much deeper and persistent than projected by the official creditors. As a result, the budget targets (projected debt to GDP ratio) have not only been often missed by a wide margin but the debt to GDP ratio has actually increased.

In this paper we make sense of these observations by arguing that the models used to evaluate the effects of fiscal policy in general, and fiscal consolidation in particular, may be severely mis-specified –at least for certain countries– as they do not contain an informal sector.[1] In the standard model, when taxes go up, agents may choose to work and invest less with adverse effects on economic activity and employment. However, in an economy with an informal sector, economic agents have an additional option, namely, to shift their inputs to the underground economy. This possibility has two implications. First, a model that does not include an informal sector will predict multipliers that are systematically smaller than realized ones. The Blanchard and Leigh, 2013, claim about the large errors made in the forecasts of the effects of fiscal consolidation could be very well related to such an omission, rather than to sinister motives. Second, reported multipliers, which are computed on the basis of recorded official output only, and "true" multipliers, which are computed on the basis of total (official and unofficial) output will be different. In particular, the former *overestimate* the latter as they do not include the additional output produced in the informal sector.

Both implications have important consequences. The discrepancy between predicted and actual multipliers matters because it generates over-optimistic projections of tax revenue and deficits and thus of the sustainability of debt. For instance, Greece's recent experience shows that large tax policy interventions during the crisis periods failed to generate tax revenue and debt paths that are even remotely in line with the projections made. This created a vicious circle, with higher tax rates chasing a shrinking tax base.

[1] For the debt crisis afflicted countries, Schneider, 2015, reports values for the shadow economy that range from 11% for Ireland to 23% for Greece.

The discrepancy between reported and true multipliers (overestimation) is consequential for both welfare and policy evaluation. For instance, it makes measured austerity (output growth, unemployment) appear more severe than it actually is.[2]

The objective of this paper is to study quantitatively the behavior of actual and true multipliers as well as of tax revenue in an economy that contains an informal sector. We start by augmenting a standard, simple Neoclassical model with an informal sector which owes its existence to distortionary taxation. Agents can choose to operate in the black sector and evade the relevant tax. But by doing so, they subject themselves to the risk of getting detected and punished (fined). Their choices reflect the balance of these two considerations. We then use the model to derive the properties of tax and spending multipliers. The main findings are as follows: First, as the size of the informal sector increases, so do actual and true multipliers. That is, a larger informal sector implies greater sensitivity to tax and spending changes. Second, both the size of the forecast error and the degree of mismeasurement of the true output are increasing in the share of the black economy: forecasts of the effects of fiscal consolidation that do not take into account the black economy tend to be overoptimistic with regard to both economic activity and tax revenues raised. And third, these effects are more pronounced for tax than for spending changes. Our model thus offers a potential explanation of this difference. Nevertheless, the well known empirical finding (Alesina and Giavazzi, 2015) that tax based consolidations leave a bigger footprint than spending based ones applies to true output also, even if it is less pronounced for true than for recorded output.

The results reported from the baseline model are quite suggestive but contain mostly qualitative information. In order to get a sense about how they play out quantitatively in the real world, we embed the analysis into a richer model of the Greek Economy (a version of the Bank of Greece DSGE model). In order to contribute to the extant, contentious debate regarding the over-optimistic forecasts and the degree of austerity in the recent Greek experience, we feed the actual paths of the fiscal instrument in Greece from 2010–2015 and derive the paths of recorded and true economic activity and tax revenues. As a reference point, note that the recorded decline in GDP was of the order of 26%. The model version without a black economy falls short of the recorded reduction by about a third (17% vs 26%). The version with the unofficial economy implies a reduction of formal GDP of the size actually observed. It also implies that the drop in true output was less than two thirds of that in recorded output. That is, measured austerity is exaggerated by about 50% relative to true austerity. These numbers are extremely robust across different

[2]The fact that economic mayhem in Greece has not led to major social turmoil seems consistent with the view that the contraction in output and employment, while large, may not be as severe as recorded in the official statistics.

specifications of the model as well as under different assumptions about the nature of the expectations about the paths of the values of the fiscal instruments (perfect foresight or perfectly unanticipated). They are also quite striking and reveal the crucial role played by the black economy in distorting not only forecasts but also actual and measured outcomes of economic activity in Greece.

The erroneous forecasts of economic activity go hand in had with mis-forecasts of tax revenue. Tax adjustment in Greece has been enormous. One would like to know how much of the optimistic scenarios on the fiscal front can be attributed to model mis-specification. And also how much damage the black economy inflicted on tax revenue collection. To be more specific, one can ask either how much extra revenue the actual, implemented tax changes would have produced if the black economy were shut down. Or, in a similar spirit, given the amount of tax revenue actually raised, one could ask how much lower the taxes would have been if the black economy became non-operative. We find that: a) the single sector model significantly overestimates projected tax revenue. For instance, it implies an increase of tax revenue as a share of initial GDP of about six percentage points in 2012, while the dual version of the model produces an increase of only one percentage point; b) had the informal sector been shut down –or prevented from growing further– when the fiscal adjustment begun, the same amount of revenue that was raised could have been raised with tax decreases rather than tax increases, a remarkable finding. Note that the distribution of activity shifted significantly as a result of the fiscal package: Our analysis implies that the informal sector grew substantially, from 25% to somewhere between 35% and 40% of GDP.

We draw the following conclusions: First, most of the misjudgment of the size of the effects of fiscal consolidation by the official creditors can be attributed to the standard modeling practice of neglecting the informal sector. Second, much –but not all– of the reduction in official GDP has been offset by the expansion of the underground sector. The economy has suffered greatly but considerably less than what is commonly measured/reported. And third, the existence of the black economy has played havoc with the tax revenue, contributing to both "excessively" high tax rates and weaker economic activity. Had it been contained, fiscal consolidation would have been smaller and the output-employment reduction would have been significantly milder. While the shadow economy has insulated some agents from heavy taxation, it has contributed significantly to a larger burden on those operating in the official sector and has also been deleterious to the economy at large. It seems that much of the failure of the Greek government to meet its revenue targets can be attributed to its inability to tame the black economy.

Literature review: There is no prior work assessing the degree of mismeasurement

of true fiscal multipliers. The closest predecessor to our work is Pappa et al, 2015, which examines how the informal sector as well as corruption matter for the effects of tax and spending changes. Its focus, however, is exclusively on forecast errors: it provides an explanation of why the predicted decline in GDP following fiscal consolidations tends to underestimate the actual one. It cannot address the issue of recorded vs true multipliers, however, as the two coincide in that model. This is due to the fact that while it allows fiscal shocks to lead to a reallocation of employment from the more efficient formal sector to the less efficient informal sector and thus impact on the level of GDP, it assumes that all of the output of the informal sector is used as an intermediate input in the formal sector, so actual and true GDP are always the same; there is no mismeasurement of GDP and of fiscal multipliers. In contrast, in our model, as in the real world, part of GDP is not recorded at all, so true output exceeds measured output and true fiscal multipliers are overestimated. It is precisely this exaggeration that is one of the central subjects of our study.

The rest of the paper is organized as follows. Section 1 describes a simple model of tax evasion and derives qualitative implications for the comparative static effects of arbitrary tax and government spending changes for official and true output as well as for tax revenue. In section 2 we use a version of the DSGE model of the Bank of Greece to perform comparative statics and also to assess the macroeconomic and tax revenue implications of the actual fiscal (tax and spending) consolidation package implemented in Greece during 2010-2015.

2 An illustrative model with tax evasion

In this section we assume that the underground activity centers on the firms' attempt to evade the firm revenue tax. We allow for additional unofficial activities in the more general model of section 2.

2.1 Households

The representative household has preferences over consumption and hours worked captured by the intertemporal utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t\right), \tag{1}$$

where E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, C_t is private consumption and H_t is total hours worked in period t. We specialize the instantaneous

utility function as follows:

$$u\left(C_t, H_t\right) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{H_t^{1+\gamma}}{1+\gamma}, \tag{2}$$

where $\sigma \geq 0$ is the inverse of the intertemporal elasticity of consumption and $\gamma > 0$ is the inverse of the Frisch elasticity of labour supply. Without loss of generality, we will assume that the household is indifferent between the consumption good produced in the formal sector, C_t^p, and that produced informally, C_t^u, so that $C_t = C_t^p + C_t^u$.

The budget constraint of the household in period t is

$$\left(1+\tau_t^c\right) P_t^p C_t^p + P_t^u C_t^u + P_t^p I_t = (1-\tau_t^l) W_t^p H_t^p + W_t^u H_t^u + P_t^p r_t^k K_t + Div_t - T_t \tag{3}$$

where P_t^p and P_t^u are respectively the prices of a unit of formal final good and a unit of informal consumption good. The household earns a wage W_t^p in the formal sector and a wage W_t^u in the informal sector (with H_t^p and H_t^u hours worked, respectively, in these sectors). It also receives capital income from renting out capital services to firms, K_t, at the rate r_t^k. Here, anticipating the assumption that only the formal sector uses capital and in order to lighten the exposition, we are dropping the "p" superscript from investment and capital. Each period, the household saves by investing in physical capital, I_t. Finally, households own the firms in the economy and receive their profits as dividends, Div_t. The government transfers T_t to the household in a lump-sum manner. A consumption tax, $\tau_t^c \in (0,1)$, is levied on the formal consumption good and a labour income tax, $\tau_t^l \in (0,1)$, is levied on the income earned in the formal sector.

The law of motion of the capital stock is:

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{4}$$

where $\delta \in (0,1)$ is the rate of depreciation of capital.

The household chooses consumption of the formal and informal goods, labour supply in the formal and informal sector, and savings in order to maximize utility (1)-(2) subject to (3) and (4). The FOCs are as follows:

$$(1+\tau_t^c) P_t^p \Lambda_t = C_t^{-\sigma} = P_t^u \Lambda_t \tag{5}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1}\left(r_{t+1}^k + 1 - \delta\right)\right] \tag{6}$$

$$\left(1-\tau_t^l\right) W_t^p \Lambda_t = W_t^u \Lambda_t = H^\gamma \tag{7}$$

where Λ_t is the Lagrange multiplier.

Note that (5) implies $P_t^u = (1+\tau_t^c) P_t^p$ and (7) implies $W_t^u = (1-\tau_t^l) W_t^p$.

2.2 Production

Each representative firm has two alternative means of producing a homogeneous good. It can produce the good either formally or informally. For the formal good, Y_t^p, it combines capital, K_t, and labour services, H_t^p. For the informal good, Y_t^u, it uses only labour, H_t^u. More specifically:

$$Y_t^p = A_t^p (H_t^p)^{\alpha} (K_t)^{1-\alpha} \tag{8}$$

$$Y_t^u = A_t^u (H_t^u)^{\zeta} - F \tag{9}$$

where $\alpha, \zeta \in (0,1)$, and A_t^p, $A_t^u > 0$ are the exogenous levels of productivity in the formal and informal sector, respectively. F is the fixed cost of operating in the informal sector.[3]

What distinguishes the formal from the informal good is that the production, sale and consumption of the latter may not always be detected and taxed. In particular, we assume that firms pay a revenue tax $\tau_t^f \in (0,1)$ per unit of formal good produced. For the informal good, the firm can evade the revenue tax if the activity does not get detected. But if the activity gets detected by the tax authorities, an event that occurs with an exogenous probability π, then the producer pays a fine. Without loss of generality, we will assume that this fine takes the form of confiscating a fixed share, ϕ, of the value of the informal output. We also assume that the firm bears the entire burden in the case of detection, the workers who produced (and did not declare their labour income) and the consumers who consumed the informal good do not suffer any consequences. In the next section and for the larger model, we allow fines to be imposed also on other participating agents besides firms.

The producers choose the scale and composition of production in order to maximize expected profits, taking as given final output prices for formal, P_t^p, and informal, P_t^u, goods as well as factor prices, r_t^k, W_t^p, W_t^u:

$$\Pi_t = \max_{K_t^p, H_t^p, H_t^u} \left\{ \left(1 - \tau_t^f\right) P_t^p Y_t^p + (1 - \pi\phi) P_t^u Y_t^u - r_t^k K_t - W_t^p H_t^p - W_t^u H_t^u - J F_t \right\} \tag{10}$$

subject to (8)-(9). J is an indicator function that takes the value of 1 if Y_t^u is positive and

[3] A fixed cost is the simplest way to guarantee that the informal sector only comes into existence due to taxes. Due to the decreasing returns in the informal sector, without the fixed cost there would always be a bit of shadow economy even when taxes were set to zero. Having constant returns to scale also in the informal sector would result in a corner solution for the composition of output between formal and informal output.

zero if Y_t^u is zero. The FOCs are as follows:

$$\begin{aligned} W_t^p &= \left(1-\tau_t^f\right)\alpha P_t^p \frac{Y_t^p}{H_t^p} \\ W_t^u &= (1-\pi\phi)\,\zeta P_t^u \frac{Y_t^u}{H_t^u} \\ r_t^k &= \left(1-\tau_t^f\right)(1-\alpha) P_t^p \frac{Y_t^p}{K_t} \end{aligned}$$

The informal goods are only used for consumption, hence

$$Y_t^u = C_t^u$$

The formal goods are used for consumption, C_t^p, investment, I_t, and public consumption, G_t^c, purposes.

$$Y_t^p = C_t^p + I_t^p + G_t^c$$

2.3 Government

The government budget constraint is

$$\tau_t^c P_t^p C_t^p + \tau_t^l W_t^p H_t^p + \tau_t^f P_t^p Y_t^p + \pi\phi_t P_t^u Y_t^u + T_t = P_t^p G_t^c$$

We assume that when the shadow activity is detected, any amount of the informal good that is confiscated is returned to the households via a lump sum transfer. That is, the government only consumes the formal good. As a result the private consumption of the informal good is equal to the production of that good (net of the fixed cost, F).

2.4 Solution-calibration

We illustrate the main mechanisms at work by focusing on the steady state solution and undertaking comparative statics. The steady state solution is described in the Appendix. We will carry out a full dynamic analysis in the DSGE model of the following section.

The model is calibrated at an annual frequency. Table 1 presents the calibration. A key parameter in the model is the share of the informal sector in GDP. We have normalized the scale parameter in the production function of the official sector, A^p, to unity and have selected the scale parameters in the informal sector so that the informal sector in the baseline specification is 25% of GDP, the share of the shadow economy in Greece as reported in Schneider and Williams (2013). Naturally, estimating the size of the informal economy represents a big challenge and the estimated share depends very much on the

assumptions and the methodology used. But as will become clear when we present the results of the exercises conducted (see, for instance, Table 2 or Table 7) varying this share, say between 20% and 30%, does not change the main message as well as the quantitative properties of the results.[4]

The values of the tax rates correspond to their real world counterparts in the Greek economy. We obtain these values by computing effective tax rates from the data (see Appendix 9.1 for details). The consumption and labour tax rate values have been set to 0.15 and 0.34 respectively. The corporate revenue tax rate has been set so that, given the average profits to sales ratio, it corresponds to a profit tax[5] of around 25%. The share of government consumption in GDP is set equal to its value in the data. The labour share in the formal sector, α, is computed using data from AMECO. The same value is used for the labour share in the informal sector. The fixed cost parameter in the production function of the informal sector is set so that profits are zero in the steady state. The coefficient of risk aversion and the inverse labour elasticity are both set equal to 1. The discount factor is calibrated given a real interest rate equal to 5% and the depreciation rate is set equal to 0.06 (see Papageorgiou, 2014).

The detection probability, π, is computed from data on the ratio of the number of firms inspected for undeclared workers to the total number of firms. Its value is 0.14.[6] The fine is set equal to a fixed fraction of informal output, namely, 10% so that in the baseline case, detection results in a tax liability that is roughly double of that of the law abiding firms.

[Table 1 about here]

3 Results

[Tables 2-5 about here]

[4] Alternatively, we could select the share in a way that makes the model satisfy some endogenous property. For instance, the share could be selected so that the model can replicate the path of output –or the size of the total decline in output– under the actual path of the tax instruments during the consolidation period.

[5] Following Collard et al, 2017, we employ a revenue rather than a profit tax because due to our assumptions that firms are competitive and production in the formal sector is Cobb-Douglas, a tax on profits does not affect allocations.

[6] We use the ARTEMIS report (September 2013-November 2015) on the (annual average) number of firms inspected for the presence of undeclared and undeclared employees. The total number of firms with employees under private law contracts is retrieved from the ERGANI annual reports (average of 2014, 2015).

Table 2 shows the effects of increases in the tax rate on firm revenue in the dual economy (formal and informal sectors) that result in higher tax receipts of one percentage point of GDP. Let us focus on the column with the benchmark calibration ($\tau^f = 0.04$). Increasing the tax on firms from 3.02% to 4.0% (second row), increases tax revenue from that source by one percentage point of GDP, from 3.35% to 4.35% of GDP (first row). Informal output increases from 23.6% to 25% of formal output (row s^u) and informal employment increases from 31.7% to 33% of formal employment (row sh^u).

Rows $d\tilde{y}^p$ and $d\tilde{y}^T$ report the corresponding effect on official and true output as percentage deviation from the steady state. We can see that recorded output declines by -2.70% and true output by -1.68%, that is, the recorded decline in economic activity exaggerates the true one. Rows $\frac{dTR^f}{d\tau^f}$ and $\frac{dTR^t}{d\tau^f}$ report the effect of the tax increase on narrow (source) and broad (from all sources) tax revenue. Tax collections from firms increase by 0.94 units (which corresponds to about 1% of GDP) while total tax revenue declines by 0.059 units. Total revenue declines because the switch of activity to the informal sector decreases the tax base for the labour income and consumption tax. Note that total tax revenue includes the average value of the fines collected from the tax evading firms when they are detected.

Table 3 performs a similar exercise in the version of the model without an underground economy. As can be seen, in such an economy, going from a 3% to a 4% tax on firm revenue has a weaker negative effect on economic activity (-1.38%), a stronger positive effect on tax collection from that source and a positive (but smaller due the negative effect on labour income and consumption) effect on total tax collection. Comparison of Tables 2 and 3 confirms the point made earlier that projections of the effects of a tax rise on output and tax revenue are bound to be more optimistic when the model does not contain a shadow economy,[7] as they do not take into account the migration of economic activity to the underground economy.

To summarize the main findings from these tables: First, using a single sector model leads to over-optimistic projections about the effects of tax increases on economic activity and tax revenue. Regarding tax revenue, the forecast error is more pronounced regarding total tax revenue collection (in our baseline exercise, a *decline* of 0.059, as contrasted to a projected increase of 0.468) than regarding source tax revenue (an increase of 0.945 instead of the projected increase of 1.166). These patterns are consistent with the recent

[7]Not reported in the tables are the facts that formal output in the dual economy is 68% of the level that obtains in the single (official only) sector model. True output, though is higher, about 82% of the latter. This ratio measures the mis-allocation induced by the existence of the black economy and which is due to the fact that an inferior technology is used to produce a good that could have been produced more efficiently in the official sector.

Greek experience. According to a recent report[8] "...Since 2010, Athens has introduced revenue boosting measures worth almost 37 billion EUR in total but the result is quite disappointing as the European Commission's official data show that state revenues have declined by 9.2 billion EUR in the same period...In the same period GDP has shrunk about 26%" Second, the recorded decline in GDP exaggerates the true one, conceivably by a significant margin. And third, the size of both the forecast error (the difference between Tables 2 and 3 in row $d\tilde{y}^p$) and the exaggeration of the true effect (the difference between rows $d\tilde{y}^p$ and $d\tilde{y}^T$ in Table 2 is increasing in the relative size of the black economy. The same property also characterizes the forecast error in projections of tax revenues (the differences between the –corresponding– last two rows across Tables 2 and 3).

Tables 4 and 5 report the effects of changes in government spending, holding the distortionary tax rates constant and using lump sum taxes to balance the budget. Note that the share of the formal sector increases with an increase in the share of government spending as the latter type of spending falls exclusively on formal goods. The main finding is that while the effects are qualitatively similar across the two types of fiscal consolidation (like a tax hike, lower public spending increases the share of the informal sector), both the forecast and measurement errors are quite small: model misspecification regarding the shadow economy does not distort quantitatively the implications regarding the effects of changes in government spending (spending multipliers). This finding owes much to the specification used, which makes the wage in the formal sector independent of the level of government spending due to the assumption of constant returns to scale and perfect substitutability between formal and informal work effort. As $W^u = (1-\tau^f)W^p$, the wage in the informal rate is constant too, and so are employment and output in that sector. But to the extent that it holds to some extent in more general environments, it suggests that fiscal consolidation is more likely to succeed and its outcome is less uncertain[9] when it relies on spending rather than on tax measures.

Our findings are consistent with and can also provide a possible explanation for the findings reported by Alesina and Giavazzi, 2015. They argue that "...The accumulated evidence from over 40 years of fiscal adjustments across the OECD speaks loud and clear: ... adjustments achieved through spending cuts are less recessionary than those achieved through tax increases...". We find this to be the case not only in a standard model and for official output but also that it characterizes true output in a dual economy with an informal sector. They also argue that "... only spending-based adjustments have eventually led to a permanent consolidation of the budget, as measured by the stabilisation

[8] Hatzinikolaou and Nikas, Kathimerini, November 12, 2016.

[9] It is reasonable to assume that there is more uncertainty about the tax than the spending multipliers as the information for computing the former –the impact on the *shadow* economy– is harder to measure.

(at least) if not reduction of debt-to-GDP ratios...". Our findings reveal how hard it is to achieve satisfactory results regarding budget deficits and debt reduction with tax rises in economies with an informal sector. Even when the supply of labour is inelastic, as is commonly accepted, the existence of a shadow economy and the resulting greater tax evasion make tax-based fiscal consolidations an uphill battle. Consequently, from the point of view of debt sustainability, and to a much larger extent than that predicted by the standard, one sector model, spending reductions are more potent means for improving the fiscal position and restraining debt growth than tax increases. Finally, our framework contains an additional argument in favor of spending adjustments. Namely, that their effects are less likely to be distorted by the presence of a shadow economy, which helps make policy more reliable and effective.

Finally, it is worth reporting that the findings shown in these tables are very robust to changes in the parametarisation of the model. For instance, making the fine for tax evasion a multiple of the actual tax rate (rather than a fixed number as we have assumed in the baseline calibration), changing the probability of detection, or changing any of the other parameters of the model does not have a major effect on any of the results.

4 BOGDSGE

The analysis conducted above has helped illustrate the properties of fiscal policy in the presence of a shadow economy. In particular, it has established that not accounting for the informal sector leads to overoptimistic forecasts regarding the impact of tax rises on economic activity and on tax revenue; and also exaggerates the degree of austerity (output decline) suffered following a tax based fiscal consolidation. How large are these effects in practice? We will attempt to answer this question for a country that has experienced one of the largest fiscal consolidations in recent history, namely Greece, using the DSGE model of the Greek economy developed at the Bank of Greece and also the actual paths of fiscal instruments during the consolidation period 2010-2015. This model captures well many key features of the Greek economy as shown in Papageorgiou, 2014) it provides a suitable vehicle for the type of quantitative exploration we are interested in.

We attempt to answer the following questions: a) How large is the contraction of the official output predicted by the standard model without an informal sector under a fiscal adjustment of the size that has been actually observed in Greece? b) How much does this contraction underestimate the one predicted by the model that includes a shadow economy? c) How do these numbers compare to their real world counterparts? How large is the mismeasurement of austerity? d) What are the corresponding figures for tax

projections? e) How much milder would the fiscal adjustment and the recession have been if the underground economy had been brought under control at the beginning of the consolidation period?

We delegate the presentation of the formal model to the Appendix and offer here only a description of its main features.

Goods

The economy contains firms that operate at different stages of production. In the first stage, we have perfectly competitive firms that use either capital and labour (formal sector) or labour alone (informal sector) to produce a homogeneous, intermediate good. In the second stage, we have imperfectly competitive firms that convert part of the formal, homogeneous intermediate good (the rest being exported) into a formal, *differentiated* intermediate good. At this stage the firms also convert an imported, homogeneous good into a differentiated, foreign intermediate good. In stage three, firms combine the domestic varieties of intermediate goods with the varieties of the imported good in order to produce a homogeneous, formal, final good, which can be used for consumption and investment purposes. The assumption of imperfect competition is made in order to facilitate the introduction of standard price stickiness. The informal, homogeneous, intermediate good is used only for household consumption, assuming away any market power.

Labour markets

They are assumed to be perfectly competitive.[10]

Price setting

All prices are flexible except for the formal, intermediate goods (both domestic and foreign) that are subject to the standard Calvo pricing friction.

Asset markets

The economy is small. Its residents can hold foreign currency bonds that are subject to adjustment costs that depend on actual relative to steady state holdings. The domestic firms are owned by domestic residents.[11]

Government finances

[10]The BOGDSGE model assumes imperfectly competitive labour markets. We dropped this assumption as it had negligible quantitative effects on our computed multipliers. Nonetheless, in the Appendix we report results for the version of the model with distorted labor markets.

[11]The benchmark BOGDSGE model has two types of households, Ricardian and non-Ricardian. The former own all the firms in the economy and receive their profits as dividends. They can save by investing in physical capital and by buying domestic currency government bonds and foreign currency bonds. The latter do not own any assets and consume their current consumable income. We have also carried out the analysis using the distinction between Ricardian and non-Ricardian households. The results are very similar. We discuss them in the section on robustness.

The government raises revenue through four distortionary taxes, namely, taxes on the revenue of the firms in the sector that produces the formal, homogeneous intermediate good (stage 1); taxes on labour and capital income from the same sector; and taxes on consumption of the formal, final good. The government also raises revenue through a lump sum tax, that is used as a residual to cover any discrepancy that arises between distortionary tax revenue and government spending. Government spending takes four forms: government consumption, public investment, wages paid to public employees and government transfers. In spite of the fact that public debt has played an important role in the Greek crisis, we abstract from it for two reasons: The model lacks proper sovereign debt features; and its level has varied significantly due to factors that are completely outside the model (such as the revision to include previously hidden debt, debt restructuring, bank recapitalization, etc.)

Tax evasion

Tax evasion at the firm level is as in the previous section. But we now also introduce a similar feature for labour income. In particular, workers may be caught and fined when they do not declare the income they earned in the informal sector. While the most natural scenario involves a simultaneous detection of undeclared firm revenue and labour income in the informal sector, we have also considered independent detection of firms and workers tax evasion. The results are not affected by the characteristics of the detection scheme.

Monetary Policy

The exchange rate is fixed. The domestic interest rate equals an exogenously given, risk-free, world interest rate.

5 Results

5.1 Calibration

The model is calibrated at an annual frequency. Table 6 presents the calibration of the dual economy model with tax evasion. The standard parameter values are as in Papageorgiou (2014). The data source for the fiscal policy variables is Eurostat. As mentioned before, we obtain estimates for the tax rates by constructing effective tax rates (see Appendix 9.1). We set the fiscal policy variables equal to their values in 2009. The scale parameters in the production functions of the official and informal sectors, A^p,A^u, the labour shares in the two sectors, α,ζ, the fixed cost parameter in the production function of the informal sector, F, the detection probability, π, and the fine for firms, ϕ^F, are calibrated as described in Section 2. The fine for households, ϕ^W, is set equal to 0.5. The

exponent of public capital in production, α_g, is set equal to the average value of the public investment-to-GDP ratio in the data over the period 2000-2009. We calibrate the discount factor β assuming a nominal interest rate equal to 4%. We set the adjustment cost on private capital, ξ^k, at 2.5 and calibrate the value for the adjustment cost parameter on private foreign assets, ξ^f, to the lowest possible value so as to ensure that the equilibrium solution for foreign assets is stationary. The Calvo parameters, $\theta^d, \theta^x, \theta^m$, are set equal to 0.35, implying that firms adjust their prices every about 6 quarters, which is in the range of estimates for the euro area countries (see e.g. Christoffel et al. 2008).

[Table 6 about here]

5.2 Results

5.2.1 Steady-state analysis

Using the steady state of the BOGDSGE model, we repeat the analysis performed in Section 2 for all the fiscal instruments it contains (taxes on firm revenue, capital income, labour income and consumption; public consumption and investment spending and the wage bill in the public sector).[12] The results are summarized below. For the sake of space, we only present the effects associated with an increase in the steady state level of the taxes on firm revenue and on labour income as well as with a decrease in the level of government consumption. The remaining tables are delegated to the Appendix.

[Tables 7-12 about here]

The patterns exhibited in Tables 7-12 are quite similar to those obtained in the small model version of Section 2, so we will not offer any detailed comments on the results of the various exercises conducted. It is worth mentioning, though, that the implied multipliers fall in the range reported in the empirical multiplier literature as well as that suggested by quantitative DSGE models (see Schmidt et al. (2015) for a comparative study of multipliers in the euro area), notwithstanding the large variation in existing estimates. For instance, Mertens and Ravn, 2014, report values in the range of 2 and 3.

We now proceed to the main objective of this paper, namely, to use the actual paths of the fiscal instruments in this model in order to evaluate the effects of the fiscal consolidation package on macroeconomic activity and tax revenue in Greece during the crisis period. Section 5.3 discusses the various exercises conducted.

[12] In this version of the model that abstracts from non-Ricardian households, government transfers are of no consequence. In the full DSGE that draws a distinction between Ricardian and non-Ricardian agents, government transfers matter as shown in the sensitivity analysis.

5.3 Fiscal consolidation in Greece

We solve the model under two alternative, informational assumptions about the paths of the fiscal instruments: perfect foresight; and random walk. Under the former, we start the economy in its steady state and then plug in the model the actual values of the fiscal (tax-spending) instruments for the period 2010-2015 (2015 is the last year with available fiscal data). After 2015, the fiscal instruments are assumed to gradually return to their pre-crisis 2009 values. In particular, we assume that they follow an autoregressive process using as initial values the 2015 values and an autoregressive coefficient equal to 0.9. We allow lump-sum transfers to fill any government financing gaps. Under the random walk informational assumption, we assume that during the consolidation period, people expect the current fiscal policy stance to remain the same in the next period, so any change is perfectly unanticipated. In reality, some of the changes were known as the plans were drawn for more that one year. But at the same time, there were many ex post, unanticipated changes as often the plans had to be revised mid-course due to failure to achieve the deficit-debt paths and new, harsher fiscal measures had to be introduced. So the true expectations may lie somewhere between these two polar extremes. The actual paths of the fiscal instruments are depicted in Figure 7 in the Appendix. A discussion of the computation of the tax rates can be found in Appendix 9.1.

The model is solved in Dynare in a non-linear fashion. The same exercise is carried out in the single sector version of the model.

Figures 1 and 3 correspond to the perfect foresight solution and Figures 2 and 4 to the perfectly unanticipated solution. Figures 1-2 plot the path of officially recorded (formal) and true paths of GDP, consumption, private investment and employment in these two cases, along with the paths of the corresponding variables of the one sector model. The dual economy model implies a cumulative decline in GDP by 2015 that is of the order of that observed in the data (26% under perfect foresight and 19% under the random walk hypothesis). The true decline is significantly lower (about 17% and 12%, respectively). Interestingly, the decline predicted by the model that abstracts from the underground economy is fairly close to the true one. The differences across the three measures are greater for employment. The recorded cumulative decrease in employment (hours) is 21%, the one implied by the single sector model is 11% and the true one is only 4% (the numbers under the random walk assumption are 15%, 8% and 2%, respectively). Note that there was nothing in the calibration that targeted the decline in economic activity. Moreover, a situation where the model explains all of the decline is not problematic because the exercise reported is not a variance decomposition. In other words, we do not have any a priori reasons to expect a particular sign and size of the effects of the other –besides the

fiscal– shocks that took place during that period.

Figures 3 and 4 plot the effects of the individual fiscal instruments in the perfect foresight and perfectly unanticipated case, respectively. Figures 3 shows that the labour income tax has the biggest effect, accounting for about one third of the total decrease in GDP (8%). It is followed by the capital income tax and the decrease in the public employment wage bill (4% each). Figure 4 shows that the government wage bill has the biggest effect on GDP (5%), followed by the labour income tax (4%) and government consumption (3%).

Figure 5 plots the implied path of the shadow economy. The higher taxes lead to substantial diversion of activities towards the informal sector. In the perfect foresight case, its share increases by 14 percentage points from 2010 and 2015, while for the random walk case it increases by 11 percentage points. This is consistent with anecdotal evidence that suggests that the black economy has mushroomed during the Greek recession.

Finally, Figure 6 provides a comparison of the associated tax revenue paths for each tax instrument across the versions with and without an underground economy. The differences across the two model versions are striking. For instance, while the single sector model predicts that the effect of the tax package adopted would have increased the tax revenue to GDP share in 2012 by six percentage points, the increase in the dual economy is only one percentage point. This finding implies that much of the failure of the Greek government to meet its revenue targets may be due to its failure to tame the black economy.

In order to see this point from a different angle, it is instructive to consider the following counterfactuals conducted in the perfect foresight version of the model. Let us use the paths of the exogenous fiscal instrument in the solution of the dual economy in order to generate the paths of the corresponding total tax revenues. Then plug these revenue paths in the single sector economy and solve for the combinations of the labour and capital income taxes that can support them. This exercise provides information on how much lower the labour and capital income taxes could have been if Greece had managed to eliminate the black economy at the beginning of the consolidation period. Table 13 gives possible combinations of these two rates. In the Table, consumption taxes have been held constant to their initial level, the capital tax rate has been set to 0% and 10%, respectively, (recall that the baseline value is 20%) and the labour income tax rate has been allowed to vary in order to satisfy the imposed path of total tax revenues. As can be seen in the Table, the resulting average over the period labour income tax rates in these two cases are 13.7 and 17.7%, respectively, much lower than the steady state labor income tax rate of 35%.

[Table 13 about here]

In the second counterfactual, we use the change in tax revenue rather than the total amount in order to avoid the problem associated with the fact that the steady state differs across the dual and single sector economies. In particular, we calculate the paths of the *extra* revenue raised in the dual economy in Greece from 2010 to 2015 (relative to the steady state). We then ask how much labour and capital income taxes would have to increase in each period – relative to their steady state values – in the single economy in order to generate this extra revenue. We carry this exercise using the average annual (about 1 billion EUR) and the maximum annual value (about 2 billion EUR) of the change in the tax revenues in the dual economy observed over the 2010-2015 period. Table 14 reports the corresponding average tax rates in the single sector model as well as the cumulative total change in GDP. As can be seen in the Table, the changes in the tax rates are quite small. For instance, raising the tax rates from $\tau^l = 0.34$ to 0.342, and from $\tau^k = 0.20$ to 0.201 would generate about 1 more billion EUR. The cumulative decline in economic activity would have been 10.2% rather than the –true– decline of 17% in the dual economy. That is, in the absence of an informal sector Greece would have experienced a recession of the order of -2% per year, much less than the -5% recorded.

[Table 14 about here]

These results are striking. They help establish quantitatively that the existence of a large shadow economy made fiscal consolidation in Greece a much more challenging endeavor than it would have otherwise been: it led to "excessive" increases in tax rates that did not translate into large increases in tax revenue, but led instead to a severe and protracted downturn. Had the informal economy been better controlled, the fiscal consolidation could have been milder both in terms of the tax burden for households and firms and in terms of the implied output loss.

[Figures 1-6 about here]

5.4 Sensitivity

We have carried out an extensive set of sensitivity exercises. In particular, we have also considered specifications of the model with the following features: a) Ricardian and non-Ricardian households; b) imperfect competition in the labour markets with labour unions setting wages; c) nominal and real wage rigidity; d) inclusion of the public wage

bill in government consumption; e) variation of the following parameters: (i) value of 0.5 –instead of 0.9– for the persistence parameter in the fiscal instruments autoregressive rules after 2015; (ii) a value of zero for the fine on workers who are caught working in the informal sector (ϕ^W); (iii) a value of two for the coefficient of relative risk aversion σ; (iv) a value of two for the Frisch elasticity γ; (v) we also varied the Calvo parameters, the disutility from working in the informal sector as well as the fine imposed on tax evading firms.

Some of these exercises are reported in the Appendix while the remaining are available from the authors. Figures 8 and 11 and Tables 23 and 26 show the results associated with the model that contains features (a)-(d). This version of the model implies a decline in GDP of about 26% too. One can thus conclude that the quantitative results reported above are very robust to changes in the specification of the model. We also find that the greatest sensitivity is with regard to the coefficient of relative risk aversion.

6 Conclusion

Following the outbreak of the sovereign debt crisis in Greece, the country undertook an ambitious fiscal consolidation program. At the outset of the program, Greece's official creditors were predicting that the adjustment, while substantial, would be manageable and that the resulting recession would be limited and short lived. The actual experience defied these predictions by a wide margin. Tax rates kept on increasing, yet tax revenue grew little with the public debt to GDP ratio exploding. And the economy plunged into a deep and protracted recession.

Our paper has provided an explanation for these facts that centers on the existence of a substantial informal sector in Greece. Failure to account for this sector led to overoptimistic projections about the size of the required fiscal adjustment as well as about the severity of GDP and employment contraction. Had the model underlying the projections contained a dual economy, the predictions would have been quite more pessimistic on the revenue front but also more accurate. Moreover, had the Greek government been able to control the black economy at the outset of the fiscal consolidation program, smaller tax increases would have been required to achieve the deficit reduction targets and the recession would have been milder. It seems that clearer recognition by both creditors and borrower alike of the challenges presented by the black economy to the Greek program would have led to a better designed program. For instance, placing greater emphasis on increasing the tax base by clamping down on tax evasion might have been a more effective means of putting public debt on a sustainable path than increasing tax rates. The recent expe-

rience in Greece following the imposition of capital controls that forced people to switch from cash to credit card payments, thus reducing black market activities, corroborates this argument. Tax revenues jumped up unexpectedly and significantly and the country experienced a budget surplus that significantly exceeded the target (see Hondroyannis and Papaoikonomou, 2017).

At the same time, the black economy seems to have mitigated the effect on economic activity and employment: the model implies that the true level of GDP and employment are significantly above those officially recorded. But the mitigated austerity comes at a large cost, even ignoring the tax revenue and tax rate spiral dimension, as the black economy tends to make inefficient use of resources relative to the formal economy due to the small size of the respective firms (i.e. lack of economies of scale) and saving of unproductive detection evasion effort.

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8 Tables and Figures

Table 1: Parameters

Parameter	Description	Value
A^p	TFP Formal	1
A^u	TFP Informal	0.7
α	Labour share, formal	0.6
ζ	Labour share, informal	0.6
F	Fixed cost, informal	0.06
δ	Cap. Depreciation	0.06
β	Discount factor	0.9524
σ	Risk aversion	1
γ	Inverse labour elasticity	1
π	Prob detection	0.14
ϕ	Fine	0.1
G^c/Y	Govt spending / GDP	0.20
τ^f	Revenue tax rate	0.04
τ^c	Consumption tax rate	0.15
τ^l	Labour income tax rate	0.34

Table 3: Tax on Firm Revenue, Single Sector Economy

TR^f/y^p	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700
τ^f	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700
$d\tilde{y}^p$		-1.3615	-1.3728	-1.3844	-1.3962	-1.4083	-1.4206
$\frac{dTR^f}{d\tau^f}$		1.2344	1.2002	1.1662	1.1325	1.0990	1.0659
$\frac{dTR^t}{d\tau^f}$		0.5198	0.4941	0.4686	0.4434	0.4184	0.3936

Table 4: Government Spending, Dual Economy

G^c/GDP	0.1700	0.1800	0.1900	0.2000	0.2100	0.2200	0.2300
s^u	0.2558	0.2539	0.2519	0.2500	0.2480	0.2461	0.2441
sh^u	0.3358	0.3341	0.3324	0.3307	0.3290	0.3272	0.3254
$d\tilde{y}^p$		0.7535	0.7643	0.7753	0.7867	0.7984	0.8106
$d\tilde{y}^T$		0.6164	0.6261	0.6360	0.6462	0.6568	0.6677
$\frac{dTR^t}{dG^c}$		0.0852	0.0865	0.0879	0.0893	0.0907	0.0921

Note: See note in Table 2. G^c = government spending (consumption).

Table 2: Tax on Firm Revenue, Dual Economy

TR^f/y^p	0.0135	0.0235	0.0335	0.0435	0.0535	0.0635	0.0735
τ^f	0.0106	0.0204	0.0302	0.0400	0.0498	0.0595	0.0693
s^u	0.2096	0.2220	0.2355	0.2500	0.2657	0.2827	0.3011
sh^u	0.2910	0.3037	0.3169	0.3307	0.3451	0.3601	0.3757
$d\tilde{y}^p$		-2.5310	-2.6161	-2.7075	-2.8060	-2.9122	-3.0269
$d\tilde{y}^T$		-1.6357	-1.6615	-1.6885	-1.7169	-1.7465	-1.7777
$\frac{dTR^f}{d\tau^f}$		1.0587	1.0024	0.9453	0.8875	0.8288	0.7692
$\frac{dTR^t}{d\tau^f}$		0.0494	-0.0046	-0.0596	-0.1159	-0.1735	-0.2326

Note: TR^f= tax revenue from tax on firm revenues, τ^f = tax rate, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$: percentage change in output (formal, p, or true, T) for a one percentage point change in the narrow (source) tax revenue as share in GDP, $dTR^f/d\tau^f$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^f$ = change in total tax revenue (from all sources) from change in the tax rate.

Table 5: Government Spending, Single Sector Economy

G^c/GDP	0.1700	0.1800	0.1900	0.2000	0.2100	0.2200	0.2300
$d\tilde{y}^p$		0.8156	0.8291	0.8431	0.8576	0.8725	0.8880
$\frac{dTR^t}{dG^c}$		0.1021	0.1039	0.1057	0.1076	0.1094	0.1114

Table 6: Calibration, DSGE

Parameter	Description	Value
A^p	TFP formal sector	1
A^u	TFP informal sector	0.53
α	Labour share formal sector	0.6
ζ	Labour share informal sector	0.6
α_g	Public capital elasticity in production	0.053
F	Fixed cost informal sector	0.060
δ^p	Private capital depreciation rate	0.069
δ^g	Public capital depreciation rate	0.043
ξ^k	Private capital adjustment cost parameter	2.5
ξ^f	Adjustment cost parameter for froreign assets	0.05
ξ^c	Habit persistence	0.6
ψ	Elasticity of marginal depreciation costs	1.5814
β	Discount factor	0.9615
σ	Risk aversion	1
γ	Inverse of labour elasticity	1
ϑ	Substitutability/complementarity between private and public goods	0.05
π	Probability of detection	0.14
ϕ^F	Fine for firms	0.1
ϕ^W	Fine for households	0.5
$\theta^d, \theta^x, \theta^m$	Calvo parameters	0.35
μ^d	Markup - domestic market	1.35
μ^x	Markup - foreign market	1.1
μ^m	Markup - importing firms	1.35
x_d, x_x, x_m	Indexation parameters	0.256
ω_c	Home bias in the production of consumption goods	0.65
ω_i	Home bias in the production of investment goods	0.3
ε_c	Elasticity of substitution between imported and domestic consumption goods	3.351
ε_i	Elasticity of substitution between imported and domestic investment goods	6.352
ε_x	Elasticity of exports	1.463
$\overline{f}$	Target level of net private foreign assets-to-GDP ratio	0
x	Productivity of public spending on goods and services	0.29
G^c/Y	Govt intermediate cons./GDP	0.1024
G^i/Y	Govt investment /GDP	0.057
$W^g H^g/Y$	Govt wage bill /GDP	0.1307
G^{tr}/Y	Govt transfers/GDP	0.2059
τ^f	Tax rate on revenue	0.04
τ^l	Tax rate on labour	0.34
τ^k	Tax rate on capital	0.20
τ^c	Tax rate on consumption	0.15

Note: For the detection probability we use the ARTEMIS report (September 2013-November 2015) data on the (yearly average) number of firms inspected for the presence of undeclared and underdeclared dependent employment. The total number of firms with employees under private law contracts is retrieved from the ERGANI annual reports (average of 2014, 2015).

Table 7: Tax on Firm Revenue, Dual Economy, DSGE

TR^f/y^p	0.0109	0.0209	0.0309	0.0409	0.0509	0.0609
τ^f	0.0112	0.0256	0.0400	0.0545	0.0691	0.0837
s^u	0.2242	0.2367	0.2500	0.2641	0.2790	0.2949
sh^u	0.3217	0.3349	0.3485	0.3625	0.3768	0.3915
$d\tilde{y}^p$		-2.7325	-2.8065	-2.8829	-2.9614	-3.0421
$d\tilde{y}^T$		-1.8445	-1.8683	-1.8915	-1.9141	-1.9360
$d\tilde{y}^u$		3.0281	2.9928	2.9554	2.9157	2.8737
$\frac{dTR^f}{d\tau^f}$		0.7305	0.6900	0.6498	0.6100	0.5707
$\frac{dTR^t}{d\tau^f}$		-0.0648	-0.0907	-0.1156	-0.1395	-0.1622

Note: TR^f= narrow (source) tax revenues, τ^f = tax rate on firm revenue, s^u= share of informal output to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$: percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in the narrow tax revenues as share of GDP, $dTR^f/d\tau^f$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^f$ = change in total tax revenues from change in the tax rate.

Table 8: Tax on Firm Revenue, Single Sector Economy, DSGE

TR^f/y^p	0.0174	0.0274	0.0374	0.0474	0.0574
τ^f	0.0254	0.0400	0.0547	0.0694	0.0842
$d\tilde{y}^p$		-1.4480	-1.4663	-1.4854	-1.5053
$\frac{dTR^f}{d\tau^f}$		1.0352	1.0051	0.9753	0.9456
$\frac{dTR^t}{d\tau^f}$		0.3116	0.2918	0.2721	0.2526

Table 9: Tax on Labour Income, Dual Economy, DSGE

TR^l/y^p	0.1616	0.1716	0.1816	0.1916	0.2016	0.2116	0.2216
τ^l	0.2893	0.3065	0.3233	0.3400	0.3564	0.3726	0.3884
s^u	0.2117	0.2238	0.2365	0.2500	0.2642	0.2791	0.2948
sh^u	0.3032	0.3179	0.3330	0.3485	0.3644	0.3806	0.3971
$d\tilde{y}^p$		-2.3916	-2.4648	-2.5376	-2.6093	-2.6794	-2.7473
$d\tilde{y}^T$		-1.5408	-1.5691	-1.5956	-1.6201	-1.6421	-1.6614
$d\tilde{y}^u$		3.4076	3.3487	3.2849	3.2163	3.1428	3.0648
$\frac{dTR^l}{d\tau^l}$		0.4128	0.3885	0.3640	0.3396	0.3153	0.2913
$\frac{dTR^t}{d\tau^l}$		0.1206	0.0907	0.0608	0.0313	0.0023	-0.0260

Note: TR^l= narrow (source) tax revenues, τ^l = tax rate, s^u= share of informal to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$ is the percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in narrow tax revenues as a share of GDP, $dTR^l/d\tau^l$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^l$ = change in total tax revenues from change in narrow (source) tax rate.

Table 10: Tax on Labour Income, Single Sector Economy, DSGE

TR^l/y^p	0.1485	0.1585	0.1685	0.1785	0.1885	0.1985	0.2085
τ^l	0.2842	0.3029	0.3215	0.3400	0.3584	0.3768	0.3951
$d\tilde{y}^p$		-1.0756	-1.0980	-1.1213	-1.1456	-1.1709	-1.1974
$\frac{dTR^l}{d\tau^l}$		0.7249	0.7098	0.6945	0.6789	0.6630	0.6467
$\frac{dTR^t}{d\tau^l}$		0.5503	0.5329	0.5151	0.4969	0.4783	0.4593

Table 11: Government Spending, Dual Economy, DSGE

G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
s^u	0.2569	0.2546	0.2523	0.2500	0.2477	0.2454	0.2430
sh^u	0.3539	0.3521	0.3503	0.3485	0.3467	0.3448	0.3430
$d\tilde{y}^p$		0.7619	0.7718	0.7818	0.7919	0.8022	0.8126
$d\tilde{y}^T$		0.6181	0.6273	0.6367	0.6461	0.6557	0.6654
$d\tilde{y}^u$		-0.0676	-0.0673	-0.0670	-0.0668	-0.0667	-0.0665
$\frac{dTR^t}{dG^c}$		0.1306	0.1315	0.1323	0.1331	0.1339	0.1347

Note: G^c = government consumption, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$ percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in government spending as a share of GDP, dTR^t/dG^c = change in total tax revenues from change in government spending.

Table 12: Government Spending, Single Sector Economy, DSGE

G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
$d\tilde{y}^p$		0.6903	0.7009	0.7117	0.7226	0.7336	0.7448
$\frac{dTR^t}{dG^c}$		0.1102	0.1117	0.1131	0.1145	0.1159	0.1172

Table 13: A Counterfactual: Taxes without a shadow economy

Tax Rate	$\tau^k = 10\%$	$\tau^k = 0\%$
τ^l	13.7%	17.7%

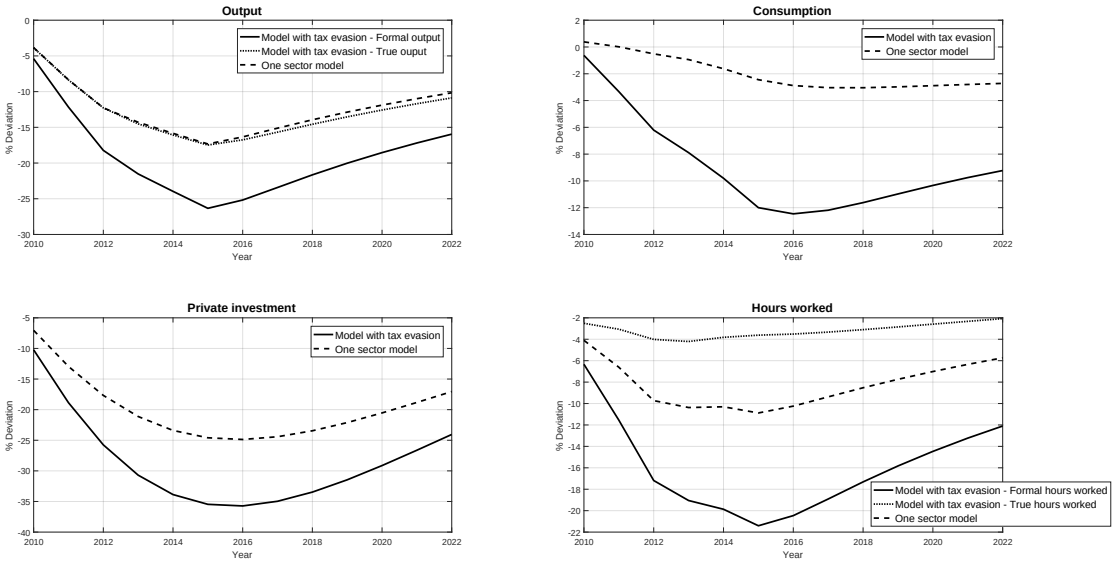
Note: The -average over the period– tax rate on labor income that given the capital tax rate would have generated the amount of tax revenue raised in the dual economy by the actual fiscal instruments.

Table 14: A Counterfactual: Fiscal consolidation without a black economy

		Impact on GDP
1 billion pa	$\tau^l : 0.34 \to 0.342$, $\tau^k : 0.20 \to 0.201$	-10.2%
2 billions pa	$\tau^l : 0.34 \to 0.365$, $\tau^k : 0.20 \to 0.215$	-12.4%

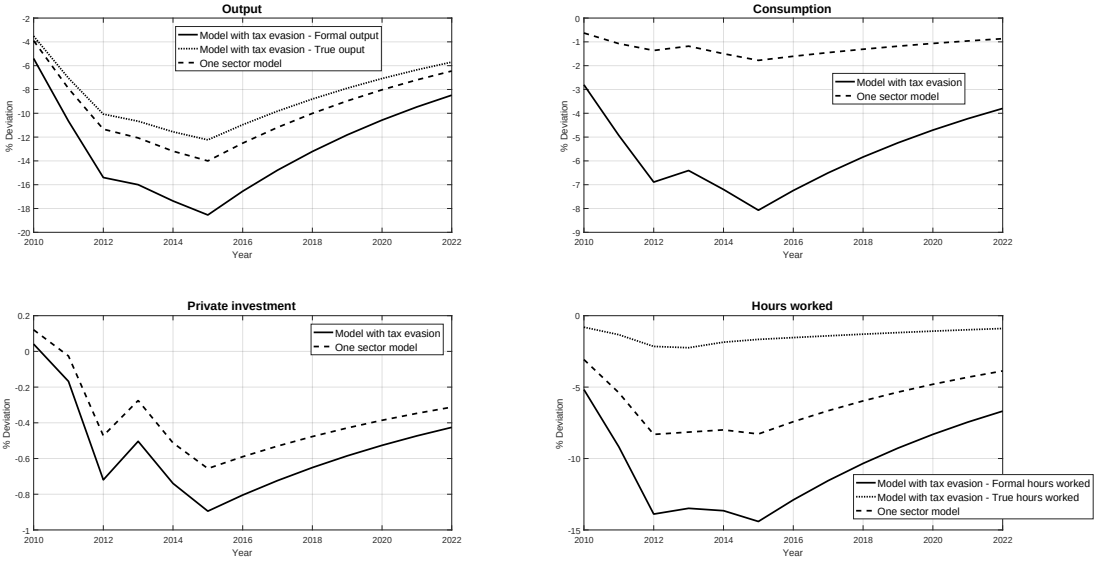
Note: Labor and capital income taxes–relative to their steady state values– needed to raise the amount of tax revenue raised per annum (pa) by the actual Greek fiscal instruments over 2010–2015 in the dual economy.

Figure 1: The effects of the Greek fiscal consolidation package on economic activity: Perfect foresight



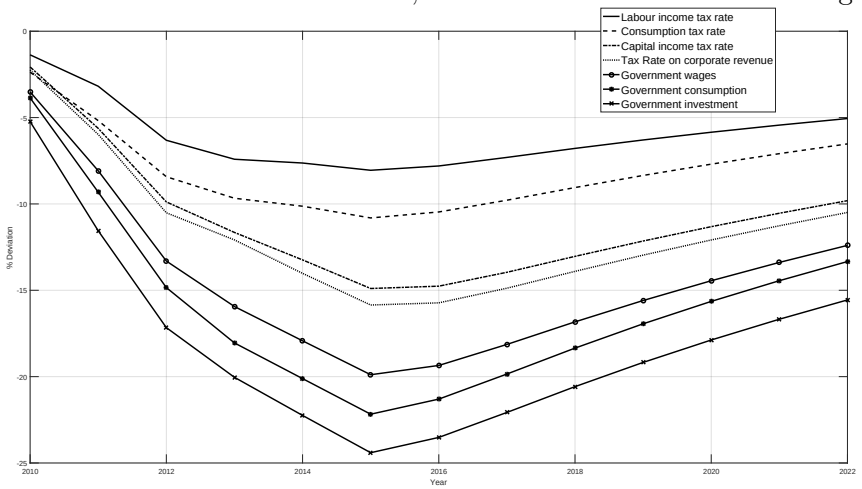
Note: 2010-2015: Actual values of fiscal instruments; 2015 onward: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

Figure 2: The effects of the Greek fiscal consolidation package on economic activity: Random walk



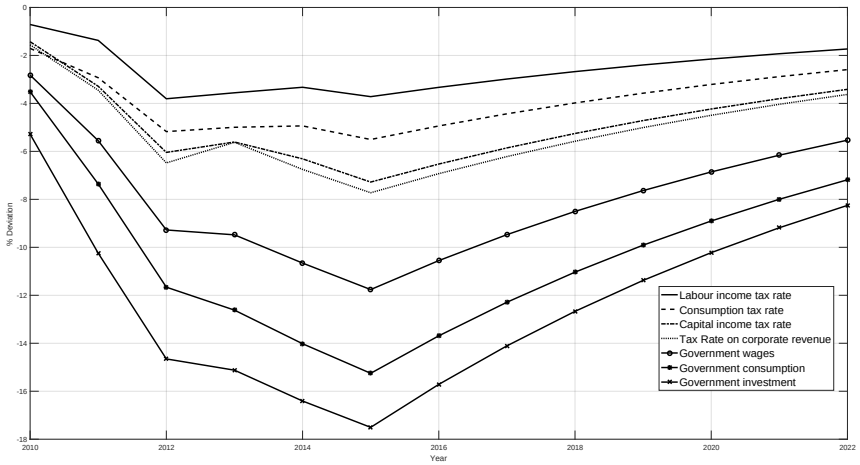
Note: 2010-2015: Actual values of fiscal instruments; 2015 onward: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

Figure 3: Contribution of individual, fiscal instruments: Perfect foresight



Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

Figure 4: Contribution of individual, fiscal instruments: Random walk



Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

Figure 5: The share of informal output to GDP

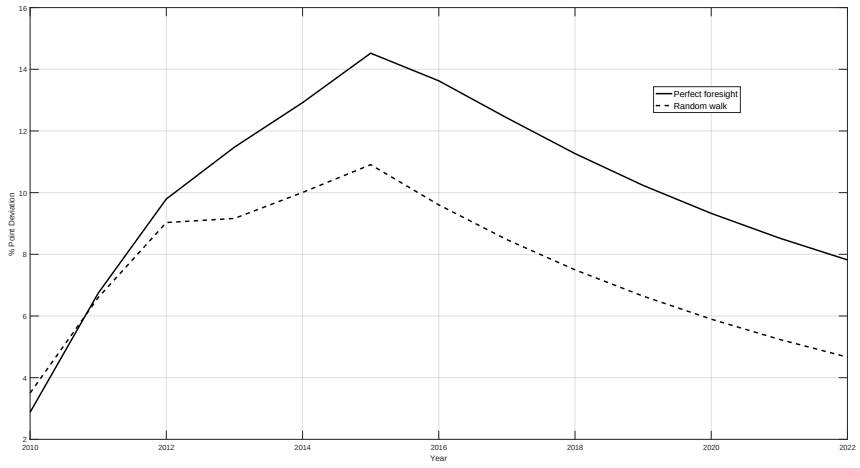
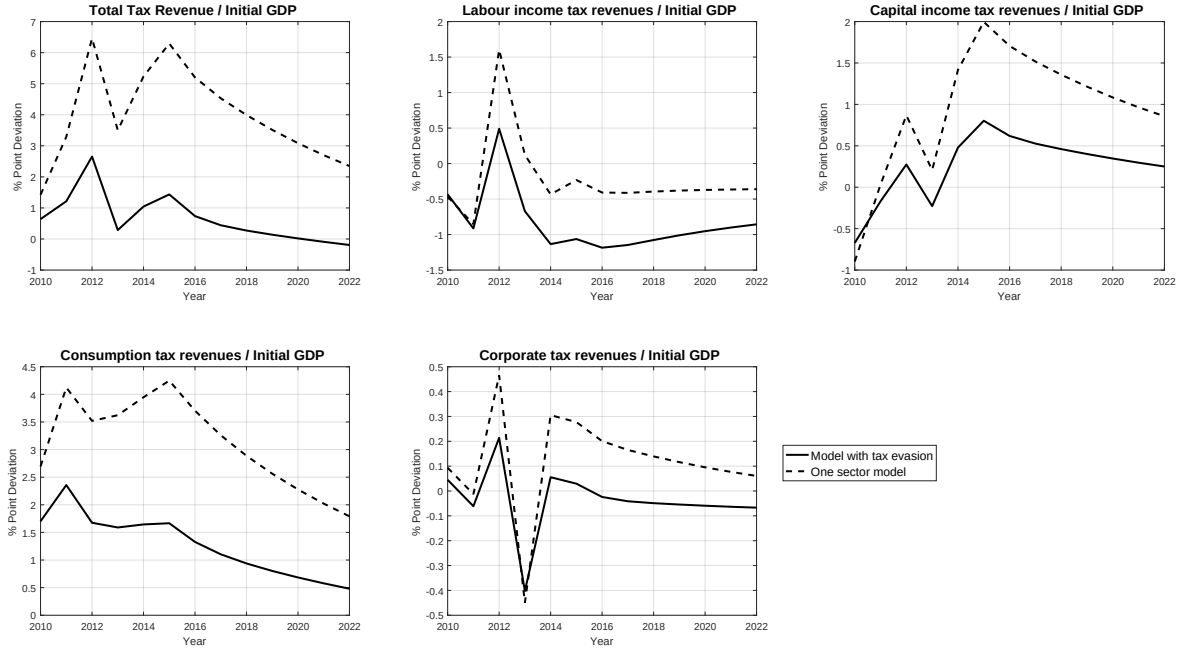


Figure 6: The effects of individual taxes on tax revenue with and without tax evasion: Perfect foresight



Note: In each graph, the path of the corresponding fiscal variable is as described in the footnote of Figure 1, while the remaining fiscal instruments are held at their steady state values.

9 Appendix

9.1 Tax rate computation, 2010-2015

The methodology followed for constructing the effective tax rates is based on the work of Mendoza et al. (1994). Broadly speaking, the effective tax rates are estimated as the ratios between the tax revenues from particular taxes and the corresponding tax bases using information from the National Accounts. The data set comprises of annual data that cover the period 2000-2015. The data source is Eurostat. The macroeconomic variables used for the computation of the effective tax rates are:

- HY: Taxes on individual or household income including holding gains
- $WSSE$: Compensation of employees
- $SSCER$: Employers' actual social security contributions
- $SSCH$: Households' actual social security contributions
- $GOSH$: Gross operating surplus and mixed income of households
- $CFCH$: Consumption of fixed capital of households

- $IYRH$: Interest income received by households
- $IYPH$: Interest income paid by households
- $CORY$: Taxes on the income or profits of corporations including holding gains
- $STAMP$: Stamp taxes
- $TFCT$: Taxes on financial and capital transactions
- TLG: Taxes on winnings from lottery or gambling
- CTC: Current taxes on capital
- CAT: Capital taxes
- OTP: Other taxes on production
- $OTPN$: Other taxes on production n.e.c.
- $NFYT$: Taxes on income paid by non-financial corporations
- FYT: Taxes on income paid by financial corporations
- GOS: Gross operating surplus, Total economy
- CFC: Consumption of fixed capital, Total economy
- TPI: Taxes on production and imports
- GIC: Intermediate consumption, government
- HC: Household and NPISH final consumption expenditure

9.1.1 Tax rate on personal income

Tax revenue data do not provide a breakdown of tax revenue from individual labour and capital income. In order to decompose tax revenue from labour and capital income of households, we follow Mendoza et al. (1994) and compute a personal income tax rate that applies both to labour and capital income of households:

$$\tau^h = \frac{HY}{(WSSE - SSCH - SSCER) + (GOSH - CFCH) + (IYRH - IYPH)} \quad (11)$$

Effective tax rate on employed labour income

The effective tax rate on labour income is computed as the ratio of labour income taxes to the labour income of employees:

$$\tau^l = \frac{\tau^h \left(WSSE - SSCH - SSCER\right) + SSCH + SSCER}{WSSE} \tag{12}$$

Effective tax rate on capital income

The effective tax rate on capital income is computed as the ratio of capital income taxes to the capital income:

$$\tau^k = \frac{\tau^h \left(GOSH - CFCH + IYRH - IPRH\right) + CAPT}{GOS - CFC} \tag{13}$$

where $CAPT = TFCT + CAT + TLG + OTP + STAMP + CTC + OTPN + CORY$ are capital income tax revenue.

Effective tax rate on consumption

The effective tax rate on consumption corresponds to the difference between the post-tax consumer price and the pre-tax price at which firms supply the consumption good.

$$\tau^c = \frac{CT}{HC + GIC - CT} \tag{14}$$

where $CT = TPI - TFCT - TLG - OTP$ are total tax revenue from indirect taxation, which by definition are equal to the difference between the nominal value of aggregate consumption at post-tax and pre-tax prices. Note that we deduct the categories $TFCT, TLG$ and OTP, from TPI since these categories include mainly capital and labour income taxes. The denominator is the base of the consumption tax, which is the pre-tax value of consumption.

Tax rate on corporate revenue

In computing the tax rate on corporate revenue (sales) we assume that the tax rate is proportional to the tax rate on corporate income. Specifically we calculate the tax rate on corporate revenue as:

$$\tau^s = \tau^{corp} \frac{profits}{sales} \tag{15}$$

where τ^{corp} is the effective tax rate on corporate income estimated as:

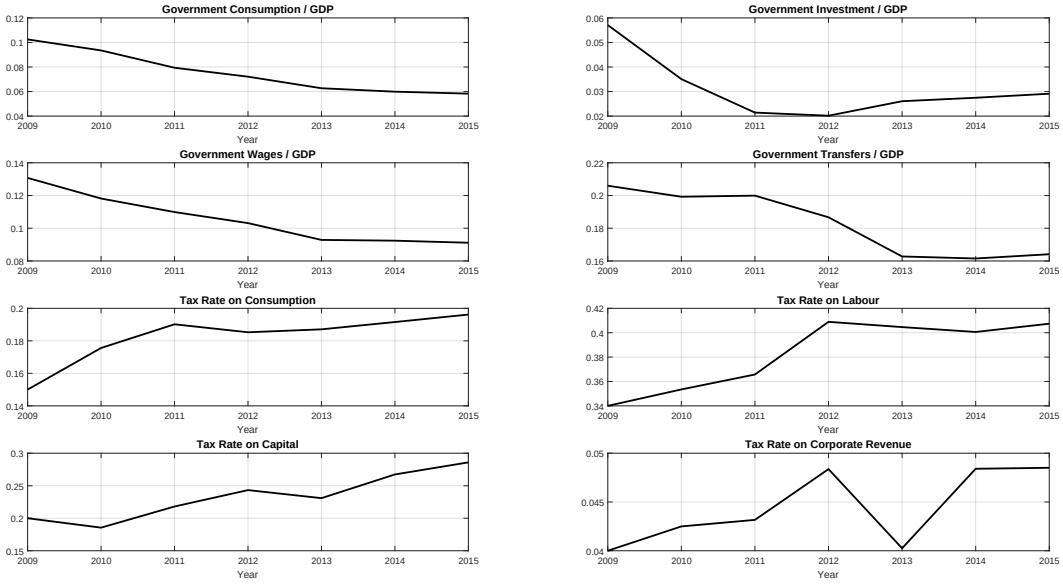
$$\tau^{corp} = \frac{FYT + NFYT}{GOSNFC + GOSFC - CFCNFC - CFCF} \tag{16}$$

The data source for total *profits* (profits before taxes and depreciation) and total *sales* correspond to the aggregate of these measures for theGreek listed firms for the years 2007-2008. The datasource is DataStream.

9.2 Actual Fiscal Instrument Paths, 2009-15

Here we report the actual paths taken by all the fiscal instruments used in the model over the period 2009-2015. The construction of the tax rates follows the description above.

Figure 7: Actual fiscal instrument paths



Note:Government consumption, investment and the public sector wage bill are expressed as shares of the 2009 GDP.

9.3 BOGDSGE: Additional Results, Baseline Version

Table 15: Consumption Tax, Dual Economy, DSGE

TR^c/y^p	0.0692	0.0792	0.0892	0.0992	0.1092	0.1192	0.1292
τ^c	0.1037	0.1191	0.1345	0.1500	0.1656	0.1814	0.1972
s^u	0.2189	0.2290	0.2393	0.2500	0.2610	0.2724	0.2842
sh^u	0.3231	0.3315	0.3400	0.3485	0.3570	0.3655	0.3740
$d\tilde{y}^p$		-1.3899	-1.3939	-1.3979	-1.4017	-1.4055	-1.4091
$d\tilde{y}^T$		-0.8817	-0.8781	-0.8745	-0.8709	-0.8672	-0.8634
$d\tilde{y}^u$		1.8531	1.8090	1.7661	1.7243	1.6836	1.6440
$\frac{dTR^c}{d\tau^c}$		0.6410	0.6204	0.6002	0.5804	0.5611	0.5421
$\frac{dTR^t}{d\tau^c}$		0.4604	0.4437	0.4275	0.4117	0.3963	0.3813

Note: TR^c = narrow (source) tax revenues,τ^c = tax rate on consumption, s^u= share of informal output to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$ is the percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in the narrow tax revenue as share of GDP, $dTR^c/d\tau^c$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^c$ = change in total tax revenues from change in the tax rate.

Table 16: Consumption Tax, Single Sector Economy, DSGE

TR^c/y^p	0.0692	0.0792	0.0892	0.0992	0.1092	0.1192	0.1292
τ^c	0.1043	0.1195	0.1347	0.1500	0.1653	0.1807	0.1960
$d\tilde{y}^p$		-0.5569	-0.5500	-0.5433	-0.5368	-0.5304	-0.5242
$\frac{dTR^c}{d\tau^c}$		0.9938	0.9821	0.9708	0.9597	0.9489	0.9384
$\frac{dTR^t}{d\tau^c}$		0.8711	0.8619	0.8529	0.8441	0.8356	0.8273

Table 17: Capital Income Tax, Dual Economy, DSGE

TR^k/y^p	0.0226	0.0326	0.0426	0.0526	0.0626	0.0726	0.0826
τ^k	0.0849	0.1229	0.1613	0.2000	0.2391	0.2788	0.3189
s^u	0.2045	0.2183	0.2334	0.2500	0.2683	0.2886	0.3111
sh^u	0.3093	0.3217	0.3348	0.3485	0.3630	0.3782	0.3943
$d\tilde{y}^p$		-3.7266	-3.8919	-4.0751	-4.2797	-4.5102	-4.7731
$d\tilde{y}^T$		-2.7450	-2.8192	-2.8992	-2.9863	-3.0817	-3.1878
$d\tilde{y}^u$		3.2066	3.2476	3.2909	3.3371	3.3872	3.4431
$\frac{dTR^k}{d\tau^k}$		0.2735	0.2511	0.2286	0.2061	0.1836	0.1611
$\frac{dTR^t}{d\tau^k}$		0.0086	-0.0144	-0.0373	-0.0600	-0.0826	-0.1051

Note: TR^k = narrow (source) tax revenues, τ^k = tax rate on capital income, s^u = share of informal output to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$ is the percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in the narrow tax revenue as share of GDP, $dTR^k/d\tau^k$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^k$ = change in total tax revenues from change in the tax rate.

Table 18: Capital Income Tax, Dual Economy, DSGE

TR^k/y^p	0.0226	0.0326	0.0426	0.0526	0.0626	0.0726	0.0826
τ^k	0.0852	0.1233	0.1615	0.2000	0.2387	0.2777	0.3170
$d\tilde{y}^p$		-2.4169	-2.4972	-2.5865	-2.6864	-2.7989	-2.9265
$\frac{dTR^k}{d\tau^k}$		0.4054	0.3848	0.3641	0.3430	0.3217	0.2999
$\frac{dTR^t}{d\tau^k}$		0.1702	0.1472	0.1237	0.0996	0.0747	0.0488

Table 19: Government Investment, Dual Economy, DSGE

G^i/y^p	0.0270	0.0370	0.0470	0.0570	0.0670	0.0770
s^u	0.2986	0.2778	0.2623	0.2500	0.2398	0.2311
sh^u	0.3852	0.3700	0.3582	0.3485	0.3402	0.3329
$d\tilde{y}^p$		4.6122	3.6295	3.0293	2.6233	2.3295
$d\tilde{y}^T$		3.1137	2.5176	2.1463	1.8915	1.7048
$d\tilde{y}^u$		-2.9782	-2.3565	-1.9613	-1.6865	-1.4832
$\frac{dTR^t}{dG^i}$		1.1249	0.8677	0.7097	0.6028	0.5254

Note: G^i = government consumption, y^u/y^p = ratio of informal to formal output, h^u/h^p = = ratio of informal to formal employment, $d\widehat{y} = d\log y$ percentage change in output (formal, p, informal, u, or total, T) for a one percentage point change in government spending as a share of GDP, dTR^t/dG^i = change in total tax revenues from change in government spending.

Table 20: Government Investment, Single Sector Economy, DSGE

G^i/y^p	0.0270	0.0370	0.0470	0.0570	0.0670	0.0770
$d\tilde{y}^p$		2.8508	2.3239	1.9993	1.7782	1.6174
$\frac{dTR^t}{dG^i}$		0.7274	0.5730	0.4779	0.4133	0.3663

Table 21: Government Wage Bill, Dual Economy, DSGE

$W^g H^g/y^p$	0.1007	0.1107	0.1207	0.1307	0.1407	0.1507	0.1607
s^u	0.2596	0.2564	0.2532	0.2500	0.2468	0.2436	0.2404
sh^u	0.3491	0.3489	0.3487	0.3485	0.3483	0.3481	0.3478
$d\tilde{y}^p$		1.2283	1.2436	1.2593	1.2755	1.2920	1.3090
$d\tilde{y}^T$		1.0118	1.0267	1.0419	1.0575	1.0735	1.0900
$d\tilde{y}^u$		-0.0076	-0.0077	-0.0079	-0.0081	-0.0083	-0.0085
$\frac{dTR^t}{dW^g H^g}$		0.3547	0.3548	0.3548	0.3548	0.3548	0.3548

Note: $W^g H^g$ = public employment wage bill, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$ percentage change in output (formal, p, informal, u, or total, T) for a one percentage point change in government spending as a share of GDP, $dTR^t/dW^g H^g$ = change in total tax revenues from change in government wage bill.

Table 22: Government Wage Bill, Single Sector Economy, DSGE

$w^g h^g/GDP$	0.1007	0.1107	0.1207	0.1307	0.1407	0.1507	0.1607
$d\tilde{y}^p$		1.2192	1.2343	1.2499	1.2658	1.2821	1.2988
$\frac{dTR^t}{dW^g H^g}$		0.3525	0.3526	0.3526	0.3526	0.3526	0.3526

9.4 Sensitivity Analysis

9.4.1 All inclusive BOGDSGE: Ricardian and non Ricardian agents, labour unions, wage rigidity

Table 23: Tax on Labour Income, Dual Economy

TR^l/y^p	0.1616	0.1716	0.1816	0.1916	0.2016	0.2116	0.2216
τ^l	0.2890	0.3062	0.3232	0.3400	0.3566	0.3729	0.3890
s^u	0.2127	0.2245	0.2369	0.2500	0.2638	0.2783	0.2934
sh^u	0.3039	0.3184	0.3332	0.3485	0.3641	0.3801	0.3963
$d\tilde{y}^p$		-2.3059	-2.3659	-2.4256	-2.4845	-2.5453	-2.5984
$d\tilde{y}^T$		-1.4695	-1.4869	-1.5029	-1.5171	-1.5924	-1.5396
$\frac{dTR^l}{d\tau^l}$		0.3497	0.3315	0.3133	0.2952	0.2773	0.2597
$\frac{dTR^t}{d\tau^l}$		0.1031	0.0815	0.0601	0.3090	0.0184	-0.0016

Note: TR^l= tax revenue from labor income,τ^l = tax rate on labor income, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$: percentage change in output (formal, p, or true, T) for a one percentage point change in the narrow (source) tax revenue as share in GDP, $dTR^l/d\tau^l$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^l$ = change in total tax revenue (from all sources) from change in the tax rate.

Table 24: Tax on Labour Income, Single Sector Economy

TR^l/y^p	0.1485	0.1585	0.1685	0.1785	0.1885	0.1985	0.2085
τ^l	0.2743	0.2956	0.3175	0.3400	0.3631	0.3870	0.4116
$d\tilde{y}^p$		-1.3125	-1.3752	-1.4453	-1.5243	-1.6142	-1.7175
$\frac{dTR^l}{d\tau^f}$		0.5975	0.5830	0.5679	0.5522	0.5356	0.5182
$\frac{dTR^t}{d\tau^l}$		0.4336	0.4177	0.4010	0.3836	0.3651	0.3456

Table 25: Government Spending, Dual Economy

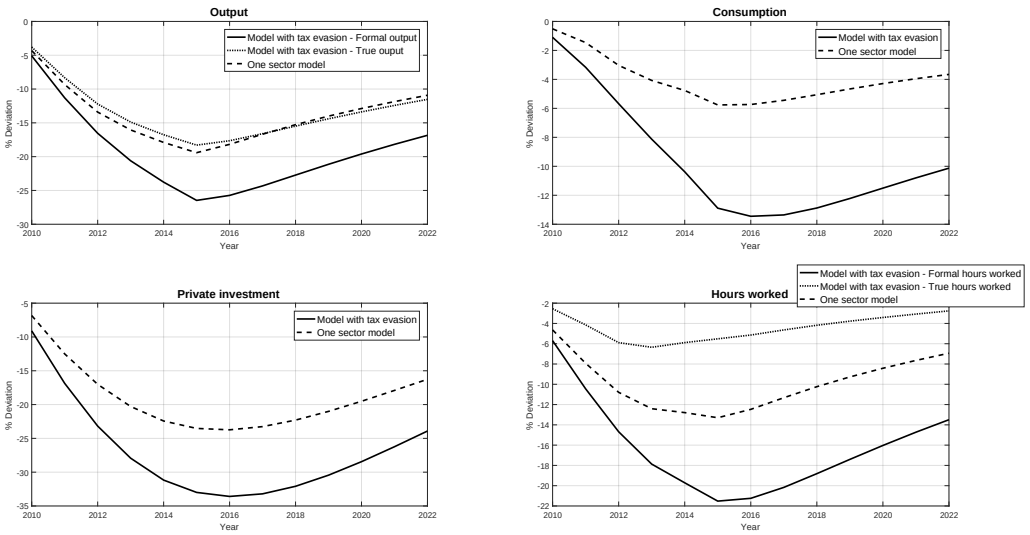
G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
s^u	0.2571	0.2548	0.2524	0.2500	0.2476	0.2452	0.2428
sh^u	0.3540	0.3522	0.3504	0.3485	0.3466	0.3447	0.3428
$d\tilde{y}^p$		0.8491	0.8608	0.8726	0.8846	0.8969	0.9094
$d\tilde{y}^T$		0.6990	0.7100	0.7211	0.7325	0.7441	0.7559
$\frac{dTR^t}{dG^c}$		0.1568	0.1577	0.1585	0.1594	0.1602	0.1610

Note: See note in Table 23. G^c = government spending (consumption).

Table 26: Government Spending, Single Sector Economy

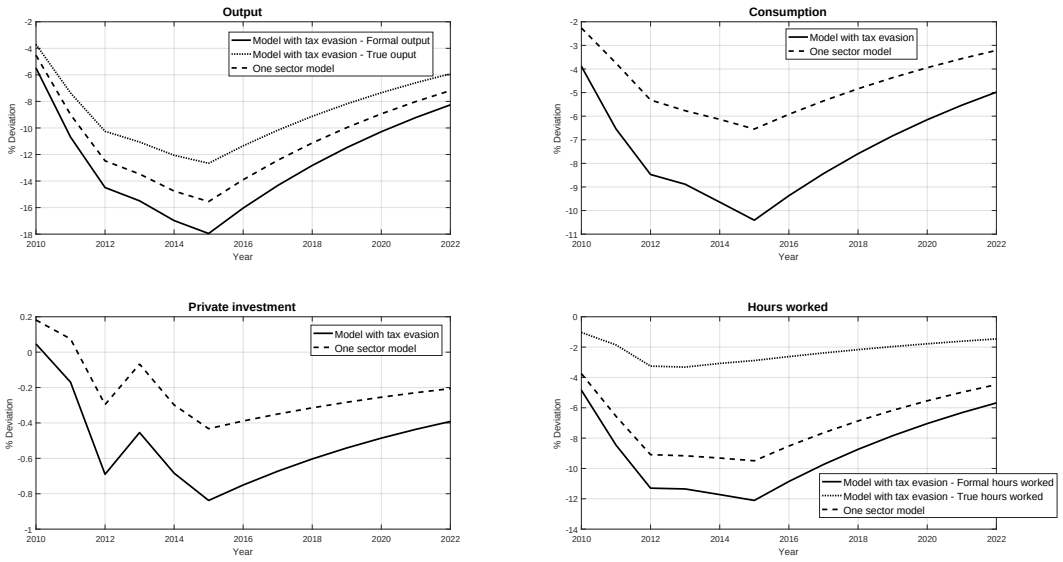
G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
$d\tilde{y}^p$		0.7937	0.8066	0.8197	0.8331	0.8468	0.8608
$\frac{dTR^t}{dG^c}$		0.1408	0.1424	0.1439	0.1454	0.1469	0.1484

Figure 8: The effects of the Greek fiscal consolidation package on economic activity: full BOGDSGE model, Perfect foresight



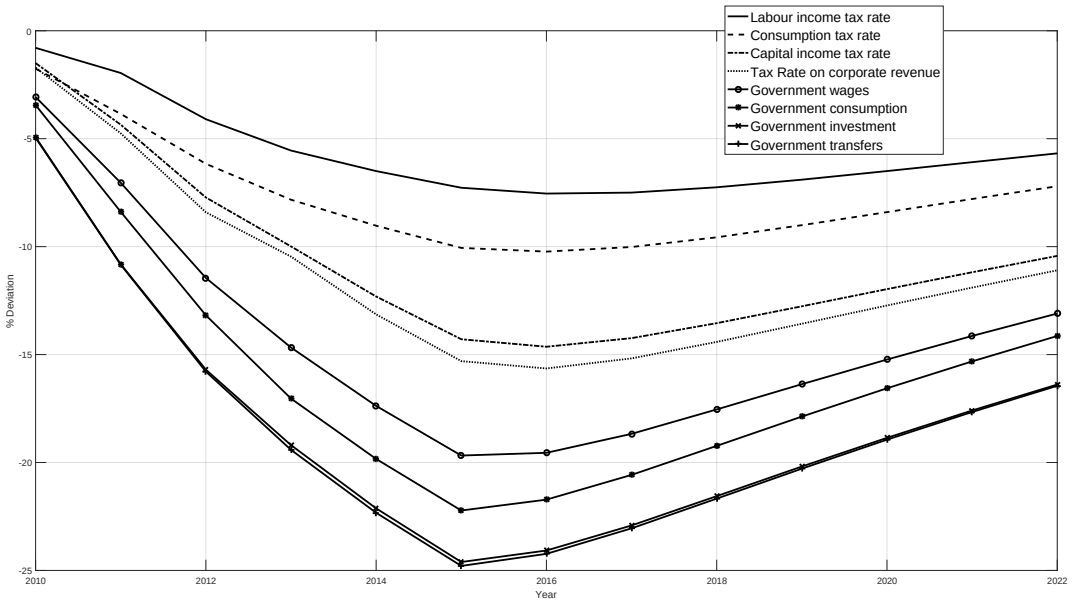
Note: 2010-2015: Actual values of fiscal instruments; 2015-2022: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

Figure 9: The effects of the Greek fiscal consolidation package on economic activity: full BOGDSGE model, Random walk



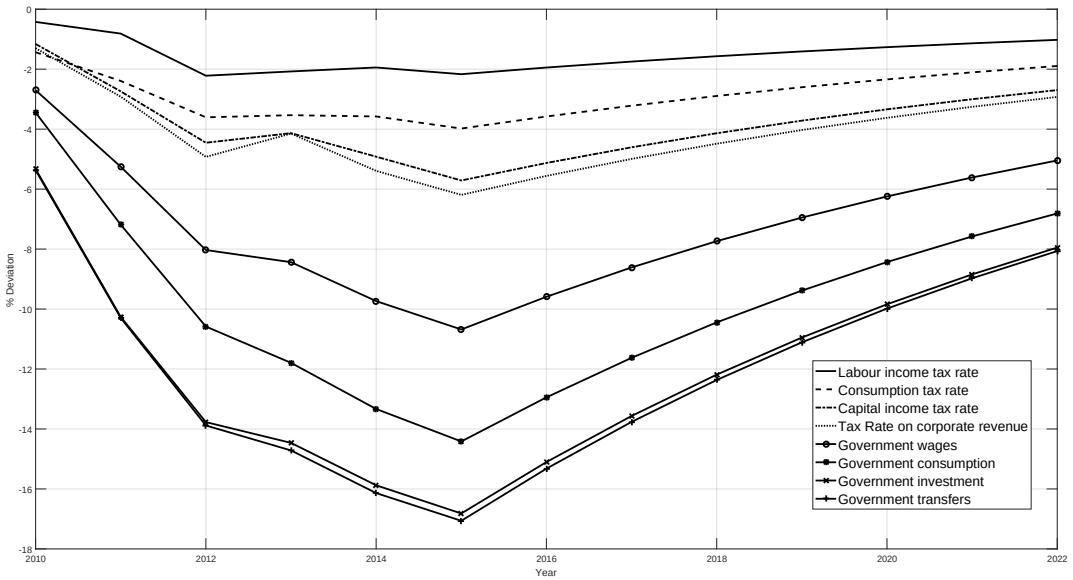
Note: 2010-2015: Actual values of fiscal instruments; 2015-2022: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

Figure 10: Contribution of individual, actual fiscal instruments: full BOGDSGE model, Perfect foresight



Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

Figure 11: Contribution of individual, actual fiscal instruments: full BOGDSGE model, Random walk



Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

9.4.2 Robustness with regard to variation in the model parameters

We consider variation in the values of various parameters (see the box in the first gigure of each row). Figures 12-13 report the results.

Figure 12: Sensitivity analysis with respect to various parameters (perfect foresight)

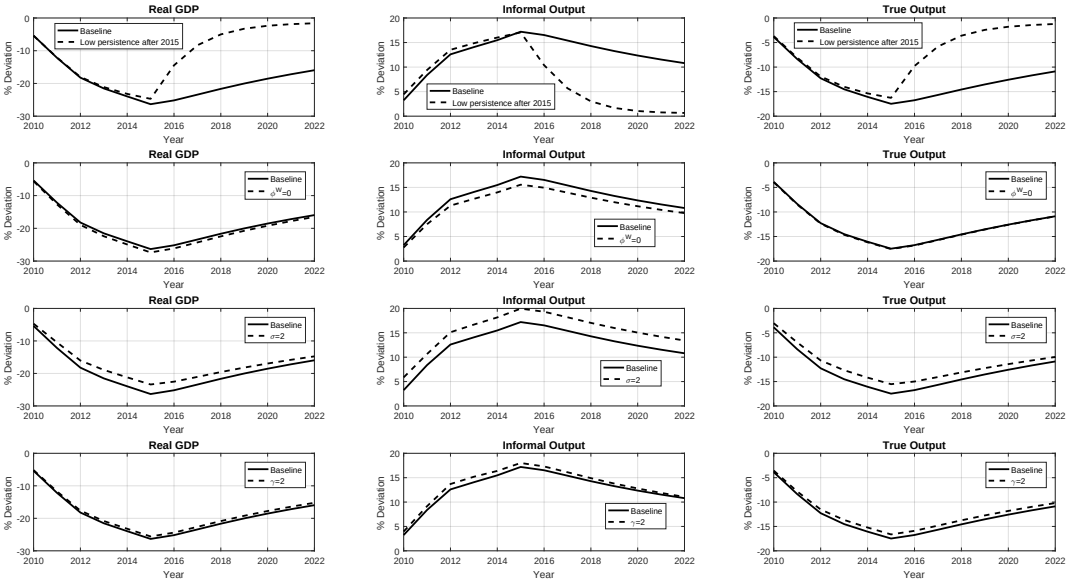
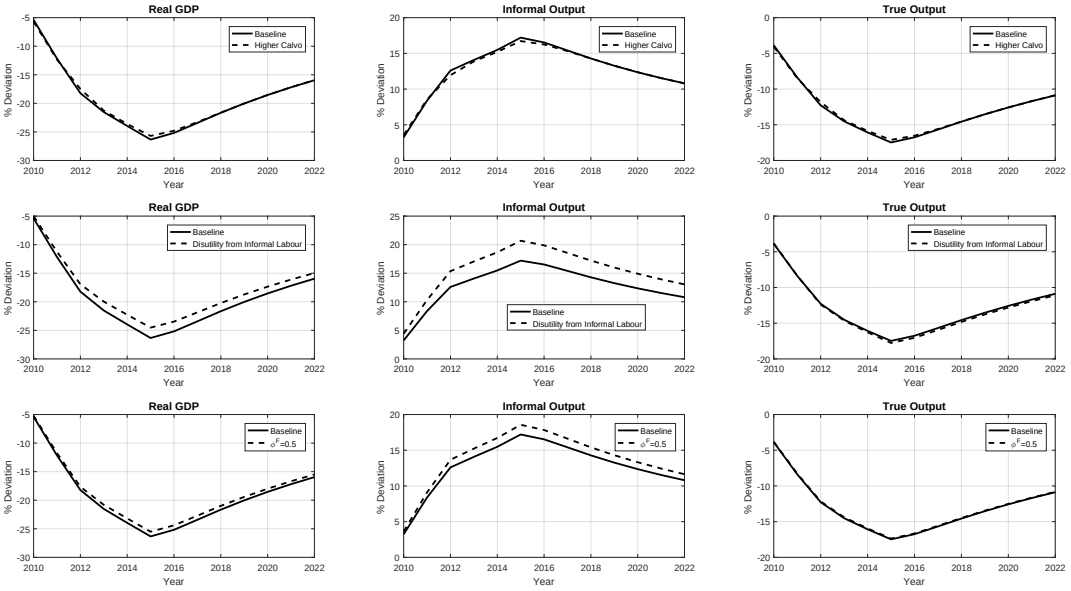


Figure 13: Sensitivity analysis with respect to various parameters (perfect foresight)



9.5 Small Model: Steady State Solution

Let the formal final good be the numeraire so $P^p = 1$. The steady state solution for the variables $\{H^p, H^u, C^p, C^u, Y^p, Y^u, K, I, C, r^k, W^p, W^u, P^u\}$ is the solution to the following to set of equations:

$$
\begin{aligned}
W^u &= W^p(1-\tau^l) \\
P^u &= (1+\tau^c)P^p = (1+\tau^c) \\
Y^p &= A^p(H^p)^\alpha K^{1-\alpha} \\
Y^u &= A^u(H^u)^\zeta - F \\
W^p &= (1-\tau^f)\alpha A^p(H^p)^{\alpha-1}K^{1-\alpha} \\
W^u &= (1-\pi\phi)\zeta A^u(H^u)^{\zeta-1} \\
r &= (1-\tau^f)(1-\alpha)A^p(H^p)^\alpha K^{-\alpha} \\
r &= 1/\beta - (1-\delta) \\
I &= \delta K \\
C^u &= Y^u \\
C^p &= Y^p - \delta K - G \\
C &= C^p + C^u \\
\frac{1-\tau^l}{1+\tau^c} W^p C^{-\sigma} &= (H^p + H^u)^\gamma
\end{aligned}
$$

We define true output, Y^T as the sum of official GDP, Y^p, and shadow output, Y^u.

9.6 The DSGE Model

9.6.1 Households

The preferences for each household i are given by the utility function:

$$u\left(C_{i,t} - \xi^c \overline{C}_{t-1}, H_{i,t}\right) \tag{17}$$

where $C_{i,t}$ is effective consumption (defined below), $H_{i,t}$ is total hours worked, $\xi^c \in [0,1)$ is a parameter that measures the degree of external habit formation in consumption and $\overline{C}_{t-1}$ is the average (across households) lagged once private consumption. The instantaneous utility function is of the form:

$$u\left(C_{i,t} - \xi^c \overline{C}_{t-1}, H_{i,t}\right) = \frac{\left(C_{i,t} - \xi^c \overline{C}_{t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \kappa \frac{H_{i,t}^{1+\gamma}}{1+\gamma}, \tag{18}$$

where $\kappa > 0$ is a preference parameter. Effective consumption is defined as

$$C_{i,t} = C_{i,t}^p + C_{i,t}^u + \vartheta Y_t^g$$

where $C_{i,t}^p$ are consumption goods produced in the formal sector, $C_{i,t}^u$, are those produced in the informal sector and Y_t^g are the per-capita public goods and services produced by the government. We assume that the household is indifferent between the formally and informally produced private consumption goods. The parameter $\vartheta \in [-1,1]$ governs the impact of public goods and services on household utility.

The household's budget constraint in period t is

$$\begin{aligned} & (1+\tau_t^c)\, C_{i,t}^p + \frac{P_t^u}{P_t^c} C_{i,t}^u + \frac{P_t^i}{P_t^c} I_{i,t} + \frac{B_{i,t+1}}{P_t^c} + \frac{S_t F_{i,t+1}}{P_t^c} = \\ = \; & (1-\tau_t^l)(W_t^p H_{i,t}^p + W_t^g H_{i,t}^g) + (1-\pi\phi^W) W_t^u H_{i,t}^u + (1-\tau_t^k) r_t^k u_{i,t} K_{i,t}^p + \\ & + Div_{i,t} + R_{t-1} \frac{B_{i,t}}{P_t^c} + R_{t-1}^H \frac{S_t F_{i,t}}{P_t^c} - T_t - \Gamma_{i,t}^h, \end{aligned} \tag{19}$$

where P_t^c and P_t^u are respectively the prices of a unit of formal and informal consumption good; P_t^i is the price of a unit of investment good and S_t is the nominal exchange rate expressed in terms of the domestic currency per unit of foreign currency.

In each period t, the household earns labour income by working $H_{i,t}^p$ hours in the formal sector at the real wage rate W_t^p, by working $H_{i,t}^g$ hours in the public sector at the real wage rate W_t^g and $H_{i,t}^u$ hours in the informal sector at the real wage rate W_t^u. It also receives capital income from renting capital services to firms, $u_{i,t} K_{i,t}^p$, where $u_{i,t}$ is the capital utilization rate and $K_{i,t}^p$ is the physical private capital stock; r_t^k is the rental rate

of capital. The household can save by investing in physical capital, $I_{i,t}$, and by buying domestic government bonds, $B_{i,t+1}$, that pay a gross nominal interest R_t at time $t+1$ and foreign bonds, $F_{i,t+1}$, that pay a nominal gross interest R_t^H. Households own all firms in the economy and receive their profits as dividends, $Div_{i,t}$. A consumption tax, $\tau_t^c \in (0,1)$ is levied on the formal consumption good, while a labour income tax, $\tau_t^l \in (0,1)$, and a capital tax, $\tau_t^k \in (0,1)$, are levied on the income earned in the formal sector. The household may evade the tax on the part of the labour income that is earned by working in the informal sector. It is detected with the constant probability π, in which case it pays out a fine that is a fixed share ϕ^W of the labour income earned informally. Finally, T_t denotes lump-sum taxes/transfers.

The households face costs when they adjust their private foreign asset holdings, $\Gamma_{i,t}^h$, if the private foreign assets-to-GDP ratio, $\frac{S_t F_{i,t+1}}{P_t^y Y_t^{GDP}}$, deviates from its long-run level, $\overline{f} \geq 0$. In particular:

$$\Gamma_{i,t}^h = \frac{\xi^f}{2} \frac{P_t^y Y_t^{GDP}}{P_t^c} \left(\frac{S_t F_{i,t+1}}{P_t^y Y_t^{GDP}} - \overline{f} \right)^2, \tag{20}$$

where Y_t^{GDP} is the economy's real GDP, P_t^y is the GDP deflator, and $\xi^f \geq 0$ is an adjustment cost parameter.

The law of motion of the private capital stock is:

$$K_{i,t+1}^p = \left(1 - \delta^p\left(u_{i,t}\right)\right) K_{i,t}^p + \left[1 - \Psi\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t}, \tag{21}$$

where Ψ is a convex adjustment cost function, $\Psi(0) = 0, \Psi' > 0, \Psi'' > 0$ of the form proposed by Christiano et al. (2005). In particular, we assume

$$\Psi_{i,t} = \frac{\xi^k}{2} \left(\frac{I_{i,t}}{I_{i,t-1}} - 1 \right)^2, \tag{22}$$

where $\xi^k \geq 0$ is an adjustment cost parameter. We assume that the depreciation rate of private capital depends on the rate of capacity utilization according to a convex function that satisfies $\delta^{p\prime} > 0$, $\delta^{p\prime\prime} > 0$. The depreciation function is of the form: $\delta^p\left(u_{i,t}\right) = \delta^p u_{i,t}^{\psi}$, where $\delta^p \in (0,1)$ and $\psi > 0$ are, respectively, the average rate of depreciation of private capital and the elasticity of marginal depreciation costs.

The household chooses consumption of formal and informal good, labour supply in the formal and informal sectors, savings in physical capital, domestic and foreign bonds and capital utilization rate to maximize expected lifetime utility subject to (19)-(22). The FOCs are:

$$\Lambda_{i,t} = \frac{\left(C_{i,t} - \xi^c \overline{C}_{t-1}\right)^{-\sigma}}{1 + \tau_t^c} = \frac{\left(C_{i,t} - \xi^c \overline{C}_{t-1}\right)^{-\sigma}}{P_t^u / P_t^c} \tag{23}$$

$$\Lambda_{i,t} = \beta E_t \left[\Lambda_{i,t+1} \frac{R_t}{\Pi_{t+1}^c} \right] \tag{24}$$

$$\Lambda_{i,t}\left[1+\xi^{f}\left(\frac{S_{t}F_{i,t+1}}{P_{t}^{y}Y_{t}^{GDP}}-\overline{f}\right)\right]=\beta E_{t}\left[\Lambda_{i,t+1}\frac{R_{t}^{H}}{\Pi_{t+1}^{c}}s_{t+1}\right] \tag{25}$$

$$\left(1-\tau_{t}^{k}\right)r_{t}^{k}=q_{i,t}\delta^{p\prime}\left(u_{i,t}\right) \tag{26}$$

$$\begin{aligned}\frac{P_{t}^{i}}{P_{t}^{c}} &= q_{i,t}\left[1-\Psi_{i,t}-\Psi_{i,t}^{\prime}\frac{I_{i,t}}{I_{i,t-1}}\right]+\\ &\quad+\beta E_{t}\left[q_{i,t+1}\frac{\Lambda_{i,t+1}}{\Lambda_{i,t}}\Psi_{i,t}^{\prime}\left(\frac{I_{i,t+1}}{I_{i,t}}\right)^{2}\right]\end{aligned} \tag{27}$$

$$q_{i,t}=\beta E_{t}\frac{\Lambda_{i,t+1}}{\Lambda_{i,t}}\left[\left(1-\tau_{t+1}^{k}\right)r_{t+1}^{k}u_{i,t+1}+q_{i,t+1}\left(1-\delta^{p}\left(u_{i,t+1}\right)\right)\right] \tag{28}$$

$$\left(1-\tau_{t}^{l}\right)W_{t}^{p}=W_{t}^{u}=\frac{\kappa H^{\gamma}}{\Lambda_{i,t}} \tag{29}$$

where β is the discount factor, $\Lambda_{i,t}$ is the marginal utility of a unit of consumption good, $\Pi_{t}^{c}=P_{t}^{c}/P_{t-1}^{c}$ is the gross rate of the consumption price index (CPI) inflation of the formal consumption good, $s_{t}=S_{t}/S_{t-1}$ is the gross growth rate of the nominal exchange rate and $q_{i,t}=Q_{i,t}/\Lambda_{i,t}$ is the shadow price of a unit of capital.

9.6.2 Production

Homogeneous intermediate good firms

The homogeneous intermediate goods sector is composed of a continuum of perfectly competitive intermediate good firms indexed by $j\in[0,1]$. Each firm j can produce the intermediate good either formally or informally. The formal good, $Y_{j,t}^{p}$, is produced by using as inputs capital, $K_{j,t}$, and labour services, $H_{j,t}^{p}$, and makes use of the average public capital $\overline{K}_{t}^{g}$. The informally produced good, $Y_{j,t}^{u}$, requires only labour, $H_{j,t}^{u}$. In particular,

$$Y_{j,t}^{p} = A_{t}^{p}\left(H_{j,t}^{p}\right)^{\alpha}\left(K_{j,t}\right)^{1-\alpha}\left(\overline{K}_{t}^{g}\right)^{\alpha_{g}} \tag{30}$$

$$Y_{j,t}^{u} = A_{t}^{u}\left(H_{j,t}^{u}\right)^{\zeta}-F \tag{31}$$

where $\alpha,\zeta,\alpha_{g}\in(0,1)$ and $A_{t}^{p},A_{t}^{u}>0$ are the exogenous levels of productivity in formal and informal production, respectively. Public capital provides production externalities, so that the production function of the formal sector exhibits increasing returns to scale with respect to all inputs.

Firms pay a corporate tax $\tau_t^f \in (0,1)$ per unit of sales of the formal good but evade the tax for the informally produced good. With –a constant– probability π the producer is caught when tax evading and has to pay a fine ϕ^F per unit of informal good sales. Without loss of generality, we assume the same probability of tax evasion for firms and workers. The producers choose the scale of production taking as given the prices for the formal, $P_{j,t}^p$, and informal, $P_{j,t}^u$, goods as well as factor prices, r_t^k, W_t^p, W_t^u in order to maximize expected profits:

$$\Pi_{j,t} = \max_{K_{j,t}^p, H_{j,t}^p, H_{j,t}^u} \left\{ \left(1-\tau_t^f\right) \frac{P_{j,t}^p}{P_t^c} Y_{j,t}^p + \left(1-\pi\phi^F\right) \frac{P_{j,t}^u}{P_t^c} Y_{j,t}^u - r_t^k K_{j,t}^p - W_t^p H_{j,t}^p - W_t^u H_{j,t}^u \right\} \tag{32}$$

subject to (30)-(31). The FOCs are:

$$\begin{aligned} W_t^p &= \left(1-\tau_t^f\right) \alpha \frac{P_{j,t}^p}{P_t^c} \frac{Y_{j,t}^p}{H_{j,t}^p} \\ W_t^u &= \left(1-\pi\phi^F\right) \zeta \frac{P_{j,t}^u}{P_t^c} \frac{Y_{j,t}^u}{H_{j,t}^u} \\ r_t^k &= \left(1-\tau_t^f\right) (1-\alpha) \frac{P_{j,t}^p}{P_t^c} \frac{Y_{j,t}^p}{K_{j,t}^p} \end{aligned}$$

The absence of market power ensures that $P_{j,t}^p = P_t^p$ and $P_{j,t}^u = P_t^u$ and that all producers choose the same level of production in both goods, i.e. $Y_{j,t}^p = Y_t^p$, $H_{j,t}^p = H_t^p$ and $Y_{j,t}^u = Y_t^u$, $H_{j,t}^u = H_t^u$.

Differentiated Intermediate Goods

We have imperfectly competitive domestic firms, indexed $f \in [0,1]$, that convert the formal, homogeneous good into formal, differentiated intermediate goods that are either used for the domestic final good production, or are exported. There are also importing firms, indexed $f^m \in [0,1]$, that convert the imported, homogeneous good into a differentiated, foreign intermediate good. The assumption of imperfect competition is made in order to facilitate the introduction of standard price stickiness.

We assume price stickiness la Calvo (1983) for the formal, differentiated intermediate goods, whether produced domestically or abroad. In particular, in each period t, a firm f can optimally reset its price with a constant probability $1-\theta^d$ when it sells domestically formal, differentiated goods and $1-\theta^x$ when it exports these goods. Similarly, for importing goods there is a constant probability $1-\theta^m$ that the importing firm f^m optimally resets its price in each period. An intermediate good firm that cannot reoptimize its price, will partially index their prices to the aggregate past inflation according to the following price

indexation schemes if it produces domestic, exported, or imported goods, respectively:

$$\begin{aligned} P_{f,t}^{d} &= P_{f,t-1}^{d}\left(\Pi_{t-1}^{d}\right)^{x_d}, \\ P_{f,t}^{x} &= P_{f,t-1}^{x}\left(\Pi_{t-1}^{x}\right)^{x_x}, \\ P_{f^m,t}^{m} &= P_{f^m,t-1}^{m}\left(\Pi_{t-1}^{m}\right)^{x_m}, \end{aligned}$$

where $\Pi_t^d = P_t^d/P_{t-1}^d$, $\Pi_t^x = P_t^x/P_{t-1}^x$ and $\Pi_t^m = P_t^m/P_{t-1}^m$ and the indexation parameters $x_d, x_x, x_m \in [0,1]$ determine the weights given to past inflation (unit value denotes full indexation).

The price of a firm producing formal, differentiated goods that are sold domestically and has not been able to reoptimize for τ periods in the domestic market is $P_{f,t+\tau}^d = P_{f,t}^d \Pi_{s=1}^{\tau}\left(\Pi_{t+s-1}^d\right)^{x_d}$. Each firm that reoptimizes its price in the domestic market in period t, chooses the optimal price $P_{f,t}^{*d}$ to maximize the discounted sum of expected real profits (in terms of the formal, final consumption good P_t^c), by taking aggregate domestic demand Y_t^d and the aggregate price index in the domestic market, P_t^d, as given. Thus, each firm maximizes:

$$\max_{P_{f,t}^d} E_t \sum_{t=0}^{\infty}\left(\beta\theta^d\right)^{\tau}\frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\frac{P_{t+\tau}^d}{P_{t+\tau}^c}\left[\Pi_{s=1}^{\tau}\left(\Pi_{t+s-1}^d\right)^{x_d}\frac{P_{f,t}^d}{P_{t+\tau}^d} - mc_{t+\tau}^d\right]Y_{f,t+\tau}^d\right\} \tag{33}$$

subject to

$$Y_{f,t}^d = \left(\Pi_{s=1}^{\tau}\left(\Pi_{t+s-1}^d\right)^{x_d}\frac{P_{f,t}^d}{P_{t+\tau}^d}\right)^{-\frac{\mu_{t+\tau}^d}{\mu_{t+\tau}^d-1}} Y_{t+\tau}^d \tag{34}$$

where $mc_t^d = P_t^c mc_t^p/P_t^d$ is the average real marginal cost in terms of the domestic price index of the formal, differentiated goods, $mc_t^p = P_t^p/P_t^c$ is the marginal cost expressed in terms of the formal consumption good, $\Lambda_{t+\tau}/\Lambda_t$ is the intertemporal marginal rate of substitution of the households according to which firms value future profits and $\frac{\mu_t^d}{\mu_t^d-1} > 1$ is the elasticity of substitution between the varieties of formal intermediate goods. Given that all firms face the same marginal cost and take aggregate variables as given it follows that they set the same optimal price $P_{f,t}^{*d} = P_t^{*d}$. Thus, the first order condition of the above problem is:

$$E_t\left\{\left(\beta\theta^d\right)^{\tau}\frac{\Lambda_{t+\tau}}{\Lambda_t}\left[\Pi_{s=1}^{\tau}\frac{\left(\Pi_{t+s-1}^d\right)^{x_d}}{\Pi_{t+s}^d}\frac{P_t^{*d}}{P_t^d}\right]^{-\frac{\mu_{t+\tau}^d}{\mu_{t+\tau}^d-1}} Y_{t+\tau}^d \frac{P_{t+\tau}^d}{P_{t+\tau}^c}\Pi_{s=1}^{\tau}\frac{\left(\Pi_{t+s-1}^d\right)^{x_d}}{\Pi_{t+s}^d}\frac{P_t^{*d}}{P_t^d} - \right.$$

$$\left. - \mu_{t+\tau}^d mc_{t+\tau}^d\right\} = 0 \tag{35}$$

According to (35), firms set nominal prices so as to equate the average future expected marginal revenues to average future expected costs. To facilitate solution, we express (35) in recursive form, so that we define:

$$\begin{aligned} g_t^{d_1} &= \mu_t^d mc_t^d Y_t^d \Lambda_t \frac{P_t^d}{P_t^c} + \beta\theta^d E_t \left[\frac{\left(\Pi_t^d\right)^{x_d}}{\Pi_{t+1}^d} \right]^{-\frac{\mu_t^d}{\mu_t^d - 1}} g_{t+1}^{d_1} \\ g_t^{d_2} &= \Pi_t^{*d} Y_t^d \Lambda_t \frac{P_t^d}{P_t^c} + \beta\theta^{pd} E_t \left[\frac{\left(\Pi_t^d\right)^{x_d}}{\Pi_{t+1}^d} \right]^{1-\frac{\mu_t^d}{\mu_t^d - 1}} \frac{\Pi_t^{*d}}{\Pi_{t+1}^{*d}} g_{t+1}^{d_2} \\ g_t^{d_3} &= g_t^{d_2} \end{aligned}$$

where $\Pi_t^{*d} = P_t^{*d}/P_t^d$.

The aggregate domestic index evolves according to:

$$P_t^d = \left\{ \left(1-\theta^d\right) \left(P_t^{*d}\right)^{\frac{1}{1-\mu_t^d}} + \theta^d \left[P_{t-1}^d \left(\Pi_{t-1}^d\right)^{x_d} \right]^{\frac{1}{1-\mu_t^d}} \right\}^{1-\mu_t^d}$$

The maximization problem for the part of the formal, differentiated intermediate goods that are exported is defined in a similar manner. Each firm maximizes:

$$\max_{P_{f,t}^x} E_t \sum_{t=0}^{\infty} \left(\beta\theta^x\right)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left\{ \frac{P_{t+\tau}^x}{P_{t+\tau}^c} \left[\Pi_{s=1}^\tau \left(\Pi_{t+s-1}^x\right)^{x_x} \frac{P_{f,t}^x}{P_{t+\tau}^x} - mc_{t+\tau}^x \right] Y_{f,t+\tau}^x \right\} \tag{36}$$

subject to

$$Y_{f,t}^x = \left(\Pi_{s=1}^\tau \left(\Pi_{t+s-1}^x\right)^{x_x} \frac{P_{f,t}^x}{P_{t+\tau}^x} \right)^{-\frac{\mu_{t+\tau}^x}{\mu_{t+\tau}^x - 1}} Y_{t+\tau}^x \tag{37}$$

where $mc_t^x = P_t^c mc_t^p / P_t^x$ is the average real marginal cost in terms of the aggregate export price index and $\frac{\mu_t^x}{\mu_t^x - 1} > 1$ is the elasticity of substitution between the varieties of formal intermediate goods that are exported. The first order condition of the above problem is:

$$E_t \left\{ \left(\beta\theta^x\right)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left[\Pi_{s=1}^\tau \frac{\left(\Pi_{t+s-1}^x\right)^{x_x}}{\Pi_{t+s}^x} \frac{P_t^{*x}}{P_t^x} \right]^{-\frac{\mu_{t+\tau}^x}{\mu_{t+\tau}^x - 1}} Y_{t+\tau}^x \frac{P_{t+\tau}^x}{P_{t+\tau}^x} \Pi_{s=1}^\tau \frac{\left(\Pi_{t+s-1}^x\right)^{x_x}}{\Pi_{t+s}^x} \frac{P_t^{*x}}{P_t^x} - \right.$$

$$\left. - \mu_{t+\tau}^x mc_{t+\tau}^x \right\} = 0$$

and the aggregate domestic index is:

$$P_t^x = \left\{ \left(1-\theta^x\right) \left(P_t^{*x}\right)^{\frac{1}{1-\mu_t^x}} + \theta^x \left[P_{t-1}^x \left(\Pi_{t-1}^x\right)^{x_x} \right]^{\frac{1}{1-\mu_t^x}} \right\}^{1-\mu_t^x}$$

Finally, each importing firm f^m buys differentiated intermediate goods at the international price $S_t P_t^{*y}$ and sells them domestically at a price $P^m_{f^m,t}$. Each firm maximizes:

$$\max_{P^m_{f^m,t}} E_t \sum_{t=0}^{\infty} (\beta\theta^m)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left\{ \frac{P^m_{t+\tau}}{P^c_{t+\tau}} \left[\Pi^\tau_{s=1} \left(\Pi^m_{t+s-1} \right)^{x_m} \frac{P^m_{f^m,t}}{P^m_{t+\tau}} - mc^m_{t+\tau} \right] Y^m_{f^m,t+\tau} \right\} \tag{38}$$

subject to

$$Y^m_{f^m,t} = \left(\Pi^\tau_{s=1} \left(\Pi^m_{t+s-1} \right)^{x_m} \frac{P^m_{f^m,t}}{P^m_{t+\tau}} \right)^{-\frac{\mu^m_{t+\tau}}{\mu^m_{t+\tau}-1}} Y^m_t \tag{39}$$

where $mc^m_t = S_t P^{*y}_t / P^m_t$ is the average real marginal cost and $\frac{\mu^m_t}{\mu^m_t - 1} > 1$ is the elasticity of substitution between the varieties of imported intermediate goods. The first order condition is:

$$E_t \Bigg\{ (\beta\theta^m)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left[\Pi^\tau_{s=1} \frac{\left(\Pi^m_{t+s-1} \right)^{x_m}}{\Pi^m_{t+s}} \frac{P^{*m}_t}{P^m_t} \right]^{-\frac{\mu^m_{t+\tau}}{\mu^m_{t+\tau}-1}} Y^m_{t+\tau} \frac{P^m_{t+\tau}}{P^m_{t+\tau}} \Pi^\tau_{s=1} \frac{\left(\Pi^m_{t+s-1} \right)^{x_m}}{\Pi^m_{t+s}} \frac{P^{*m}_t}{P^m_t} - \\ - \mu^m_{t+\tau} mc^m_{t+\tau} \Bigg\} = 0$$

The aggregate domestic index evolves according to:

$$P^m_t = \left\{ (1-\theta^m) \left(P^{*m}_t \right)^{\frac{1}{1-\mu^m_t}} + \theta^m \left[P^m_{t-1} \left(\Pi^m_{t-1} \right)^{x_m} \right]^{\frac{1}{1-\mu^m_t}} \right\}^{1-\mu^m_t} \tag{40}$$

Final good firms

Perfectly competitive final good firms combine domestic and imported formal, differentiated intermediate goods to produce final consumption and investment goods. The informally produced homogeneous intermediate good is used for consumption purposes only, so that $Y^u_t = C^u_t$.

The representative producer of the private consumption good combines a bundle of formal, differentiated intermediate consumption goods, $C^d_t = \left(\int_0^1 \left(C^d_{f,t} \right)^{\frac{1}{\mu^d_t}} df \right)^{\mu^d_t}$, with a bundle of differentiated intermediate consumption goods that are imported, $C^m_t = \left(\int_0^1 \left(C^m_{f^m,t} \right)^{\frac{1}{\mu^m_t}} df^m \right)^{\mu^m_t}$, to generate a composite consumption final good, C^p_t, by using a constant elasticity of substitution (CES) production function:

$$C^p_t = \left[\omega_c^{\frac{1}{\varepsilon_c}} \left(C^d_t \right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + (1-\omega_c)^{\frac{1}{\varepsilon_c}} \left(C^m_t \right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \right]$$

where $\omega_c \in [0,1]$ measures the home bias in the production of the final consumption good that determines the degree of openness in the long run, and $\varepsilon_c > 0$ is the elasticity of substitution between domestic and imported consumption goods.

The final consumption good producer solves a two-stage problem. In the first stage, he takes as given the retail prices of the varieties of the domestic formal intermediate goods, $P^d_{f,t}$, and imported intermediate goods, $P^m_{f^m,t}$, and optimally chooses his demand of differentiated varieties $C^d_{f,t}$, $C^m_{f^m,t}$ to minimize expenditures: $\int_0^1 P^d_{f,t} C^d_{f,t} df + \int_0^1 P^m_{f^m,t} C^m_{f^m,t} df^m$, subject to the respective CES bundle technologies. Optimal demand satisfies $C^d_{f,t} = \left(\frac{P^d_{f,t}}{P^d_t}\right)^{-\frac{\mu^d_t - 1}{\mu^d_t}} C^d_t$ and $C^m_{f^m,t} = \left(\frac{P^m_{f^m,t}}{P^m_t}\right)^{-\frac{\mu^m_t - 1}{\mu^m_t}} C^m_t$, where $P^d_t = \left(\int_0^1 \left(P^d_{f,t}\right)^{\frac{1}{1-\mu^d_t}} df\right)^{1-\mu^d_t}$ and $P^m_t = \left(\int_0^1 \left(P^m_{f^m,t}\right)^{\frac{1}{1-\mu^m_t}} df^m\right)^{1-\mu^m_t}$ are the aggregate price indexes of the domestic formal and imported, differentiated intermediate goods, respectively. In the second stage, the final consumption good producer chooses his output, C^c_t, and inputs, C^d_t, C^m_t to maximize profits $\Pi^c_t = P^c_t C^p_t - P^d_t C^d_t - P^m_t C^m_t$, taking aggregate prices P^d_t, P^m_t, and P^c_t, as given, which implies:

$$\begin{aligned} C^d_t &= \omega_c \left(\frac{P^d_t}{P^c_t}\right)^{-\varepsilon_c} C^p_t, \\ C^m_t &= (1-\omega_c) \left(\frac{P^m_t}{P^c_t}\right)^{-\varepsilon_c} C^p_t \end{aligned}$$

and

$$P^c_t = \left[\omega_c \left(P^d_t\right)^{1-\varepsilon_c} + (1-\omega_c)\left(P^m_t\right)^{1-\varepsilon_c}\right]^{\frac{1}{1-\varepsilon_c}}.$$

The production of the final private investment good is modeled in an analogous manner. In particular, the representative producer combines a bundle of domestically produced intermediate investment varieties, $I^d_t = \left(\int_0^1 \left(I^d_{f,t}\right)^{\frac{1}{\mu^d_t}} df\right)^{\mu^d_t}$, with a bundle of imported intermediate varieties, $I^m_t = \left(\int_0^1 \left(I^m_{f^m,t}\right)^{\frac{1}{\mu^m_t}} df^m\right)^{\mu^m_t}$, to generate a composite final private investment good, I^p_t, using the CES technology:

$$I^p_t = \left[\omega_i^{\frac{1}{\varepsilon_i}} \left(I^d_t\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} + (1-\omega_i)^{\frac{1}{\varepsilon_i}} \left(I^m_t\right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}}\right]$$

where $\omega_i \in [0,1]$ measures home bias in the production of the final investment good, $\varepsilon_i > 0$ is the elasticity of substitution between domestic and imported investment goods. The demand functions for the bundles of the domestically produced intermediate goods and imported intermediate goods are respectively $I^d_t = \omega_i \left(\frac{P^d_t}{P^c_t}\right)^{-\varepsilon_i} I^p_t$, and $I^m_t = (1-\omega_i)\left(\frac{P^m_t}{P^c_t}\right)^{-\varepsilon_i} I^p_t$. The aggregate price index of the final investment good is $P^i_t = \left[\omega_i \left(P^d_t\right)^{1-\varepsilon_i} + (1-\omega_i)\left(P^m_t\right)^{1-\varepsilon_i}\right]^{\frac{1}{1-\varepsilon_i}}$.

Foreign firms and export market

The exporting firm takes the prices of the exported differentiated goods $P^x_{f,t}/S_t$ as given, and chooses the optimal amounts of differentiated inputs to minimize the total input costs, $\int_0^1 \left(P^x_{f,t}/S_t\right) Y^x_{f,t} df$, subject to the technology that bundles the differentiated varieties of intermediate goods exported. The solution of the cost minimization problem gives the demand for each input $Y^x_{f,t} = \left(\frac{P^x_{f,t}}{P^x_t}\right)^{-\frac{\mu^x_t}{\mu^x_t - 1}} Y^x_t$, and the aggregate price index of the exported domestic intermediate goods $P^x_t = \left(\int_0^1 \left(P^x_{f,t}\right)^{\frac{1}{1-\mu^x_t}} df\right)^{1-\mu^x_t}$, where Y^x_t is total foreign demand for domestic intermediate goods. The latter is assumed to be given by an equation analogous in structure to the demand equations for the domestic and imported intermediate goods

$$Y^x_t = \left(\frac{P^x_t/S_t}{P^{x*}_t}\right)^{-\varepsilon_x} Y^*_t,$$

where P^{x*}_t the price of foreign competitors in the export markets and Y^*_t is a measure of aggregate foreign demand.

9.6.3 Government

The government levies taxes on consumption, on income from labour and capital earnings, on corporate income and lump-sum taxes and issues one-period government bonds in the domestic bond market, B_{t+1}. Total tax revenues together with the issue of new government bonds are used to finance government purchases of goods and services, G^c_t, government investment, G^i_t, government transfers, G^{tr}_t, and the wage bill for public sector employees, $W^g_t H^g_t$. Moreover, the government pays interest payments on past domestic public debt, R_t. The within-period government budget constraint is:

$$\frac{B_{t+1}}{P^c_t} + \tau^c_t C^p_t + \tau^l_t (W^p_t H^p_t + W^g_t H^g_t) + \tau^f_t \frac{P^p_t}{P^c_t} Y^p_t + \pi(\phi^F \frac{P^u_t}{P^c_t} Y^u_t + \phi^W W^u_t H^u_t) +$$
$$+\tau^k_t r^k_t u_t K^p_t + T_t = \frac{P^d_t}{P^c_t} G^c_t + \frac{P^d_t}{P^c_t} G^i_t + G^{tr}_t + W^g_t H^g_t + R_{t-1} \frac{B_t}{P^c_t}$$

Thus, the government has eleven policy instruments: $\tau^c_t, \tau^k_t, \tau^l_t, \tau^f_t, T_t, H^g_t, W^g_t, G^c_t, G^i_t, G^{tr}_t, B_{t+1}$. We assume that the government debt level is zero, $B_{t+1} = B_t = 0$, and lump-sum taxes adjust to ensure that the budget constraint is satisfied in every period. For convenience, regarding spending policy instruments, we will work in terms of their shares of steady state GDP, $S^c_t = \frac{P^d_t G^c_t}{P^y Y^{GDP}}$, $S^i_t = \frac{P^d_t G^i_t}{P^y Y^{GDP}}$, $S^{tr}_t = \frac{P^c_t G^{tr}_t}{P^y Y^{GDP}}$, $S^w_t = \frac{W^g_t H^g_t}{P^y Y^{GDP}}$.

In addition, the government produces the public good, Y^g_t, by combining public spending on goods and services, G^c_t, and public employment, H^g_t:

$$Y^g_t = A^g_t \left(G^c_t\right)^{\chi} \left(H^g_t\right)^{1-\chi}$$

where $\chi \in (0,1)$ is a technology parameter and $A_t^g > 0$.

The law of motion of public capital is:

$$K_{t+1}^g = (1-\delta^g)K_t^g + G_t^i$$

where $\delta^g \in (0,1)$ is the depreciation rate for public capital stock and $K_0^g > 0$ is given.

9.6.4 Monetary policy regime

The domestic economy is modeled as a member of a currency union in the sense that the nominal exchange rate, S_t, is exogenously set and there is no monetary policy independence. We assume that that the domestic nominal interest rate, R_t^H, equals an exogenously given risk-free world interest rate, R_t^*.

9.6.5 Market clearing conditions

$$\begin{aligned}
H_t &= H_t^p + H_t^u + H_t^g \\
K_t &= u_t K_t^p \\
Y_t^p &= u_t^d Y_t^d + u_t^x Y_t^x \\
Y_t^d &= C_t^d + I_t^d + G_t^c + G_t^i \\
Y_t^u &= C_t^u \\
M_t &= u_t^m Y_t^m \\
Y_t^m &= C_t^m + I_t^m \\
p_t^y Y_t^{GDP} &= p_t^y Y_t^p + W_t^g H_t^g \\
p_t^y Y_t^{GDP} &= p_t^d Y_t^d + p^x Y_t^x
\end{aligned}$$

where prices in small letters denote prices in terms of the formal consumption good, e.g. $p_t^y \equiv P_t^y / P_t^c$, and u_t^d, u_t^x and u_t^m are price dispersion measures, defined as:

$$\begin{aligned}
u_t^d &= \int_0^1 \left(\frac{P_{f,t}^d}{P_t^d} \right)^{-\frac{\mu_t^d}{\mu_t^d - 1}} df \\
u_t^x &= \int_0^1 \left(\frac{P_{f,t}^x}{P_t^x} \right)^{-\frac{\mu_t^x}{\mu_t^x - 1}} df \\
u_t^m &= \int_0^1 \left(\frac{P_{f^m,t}^m}{P_t^m} \right)^{-\frac{\mu_t^m}{\mu_t^m - 1}} df^m
\end{aligned}$$

that evolve respectively according to:

$$u_t^d = \left(1-\theta^d\right)\left(\Pi_t^{*d}\right)^{-\frac{\mu_t^d}{\mu_t^d-1}} + \theta^d\left[\frac{\left(\Pi_{t-1}^d\right)^{x_d}}{\Pi^d}\right]^{-\frac{\mu_t^d}{\mu_t^d-1}} u_{t-1}^d$$

$$u_t^x = \left(1-\theta^x\right)\left(\Pi_t^{*x}\right)^{-\frac{\mu_t^x}{\mu_t^x-1}} + \theta^x\left[\frac{\left(\Pi_{t-1}^x\right)^{x_x}}{\Pi^x}\right]^{-\frac{\mu_t^x}{\mu_t^x-1}} u_{t-1}^x$$

$$u_t^m = \left(1-\theta^m\right)\left(\Pi_t^{*m}\right)^{-\frac{\mu_t^m}{\mu_t^m-1}} + \theta^m\left[\frac{\left(\Pi_{t-1}^m\right)^{x_m}}{\Pi^m}\right]^{-\frac{\mu_t^m}{\mu_t^m-1}} u_{t-1}^m$$

Exports market equilibrium is:

$$Y_t^x = \left(\frac{p_t^x/p_t^y}{q_t^{ex}p_t^{x*}}\right)^{-\varepsilon_x} Y_t^*$$

where $q_t^{ex} = \frac{S_t P_t^{*y}}{P_t^y} = q_{t-1}^{ex}\frac{s_t \Pi_t^{*y}}{\Pi_t^y}$ is the real effective exchange rate.

The evolution of net foreign assets includes the adjustment costs suffered by the domestic agents on their foreign asset holdings. The balance of payments is given by the following equation:

$$\frac{S_t F_{t+1}}{P_t^c} = R_{t-1}^H \frac{S_t F_t}{P_t^c} + \frac{P_t^x}{P_t^c}Y_t^x - q_t^{ex}\frac{P_t^y}{P_t^c}M_t - \frac{\xi^f}{2}\frac{P_t^y Y_t^{GDP}}{P_t^c}\left(\frac{S_t F_{t+1}}{P_t^y Y_t^{GDP}} - \overline{f}\right)^2$$

Finally, we define true output as

$$Y_t^T \equiv Y_t^{GDP} + Y_t^u$$

Profits of the intermediate good producers, retailers and importing firms are allocated as dividends:

$$\begin{aligned}
Div_t &= \Xi_t + \Xi_t^d + \Xi_t^m \\
\Xi_t &= \Pi_t = \left(1-\tau_t^f\right)p_t^p Y_t^p + (1-\pi\phi)\,p_t^u Y_t^u - r_t^k K_t - W_t^p H_t^p - W_t^u H_t^u \\
\Xi_t^d &= Y_t^d\left(p_t^d - mc_t^p\right) + Y_t^x\left(p_t^x - mc_t^p\right) \\
\Xi_t^m &= p_t^m Y_t^m - q_t^{ex} p_t^y M_t
\end{aligned}$$

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