Correlated Risk

- Credit risk anticyclical
- Asymmetric information theory
- Bernanke & Gertler (1988): adverse selection, banks less choosy
- Kiyotaki & Moore (1997): moral hazard
- Carling et al (2007) micro data
- Bond defaults, credit ratings, corporate failure
- Chen (2001) macro feedback (credit crunch)
Financial Contagion

- Epidemiological models of counterparty risk
- Horst (2007): cascade model with contagion & correlation
- Proprietary data only
- No empirical research on contagion
Epidemiology

• Susceptibles (x), infectives (y), removals (z): \( x_t + y_t + z_t = N \)
• Infection rate: \( \alpha = \beta [1 - (1 - \pi)^y] \)
• \( \pi \) = probability of contact with infective
• 1 - \( \beta \) = probability of immunity
• \( \Delta x_t / x_{t-1} = \alpha y_{t-1} \quad \Delta z_t / z_{t-1} = \delta y_{t-1} \)
• Deterministic model: Kermack & McKendrick (1927)
• Stochastic model: Reed & Frost (Abbey 1952)
Economic Contagion

• Contagion is not just about counterparties
• Contagion thru trade
• Between firms
• Between industries (input – output)
• Rumors
• Rational v mechanical contagion theory
• “Keynesian” contagion
• Malign v benign contagion
• Central banks publish data on sectoral credit risk
This Paper

• Bank of Israel data on problem loans
• 1997 q1 - 2010 q3
• 12 sectors for banking system
• Data for individual banks (not by sector)
• Econometric identification of contagion
• Estimate contagion between sectors
• Model simulations
THEORY

Credit Risk Exposure
Endogenous Infection
Correlation & Contagion
Theory: Risk Exposure
Endogenous Contact

- $U_{ik} = F[c_{ik}, H(c_{ik}/Y_k = 1)]$ \( c_{ik} = 1 \)
- $U_{0ik}$: $c_{ik} = 0$
- $c_{ik} = 1$: $d_{ik} = U_{ik} - U_{0ik} > 0$
- $P(c_{ik} = 1) = 1/[1+\exp(-d_{ik})]$
- $f(q) = 1/q$ \( f' < 0 \)
- Endogenous resistance too
- Exposure avoidance
- Investment in resistance
\[ Y_{At} = \alpha_A + \theta Y_{Bt-1} + \lambda Y_{At-1} + \phi z_t + u_{At} \]
\[ Y_{Bt} = \alpha_B + \theta Y_{At-1} + \lambda Y_{Bt-1} + \phi z_t + u_{Bt} \]
\[ r(u_Au_B) = \rho \]

<table>
<thead>
<tr>
<th>Case</th>
<th>( \theta )</th>
<th>( \lambda )</th>
<th>( \sigma^2 )</th>
<th>( \phi )</th>
<th>( \rho )</th>
<th>( r )</th>
<th>( \text{Var}(Y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.3</td>
<td>1.5</td>
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<td>3.065</td>
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<td>0</td>
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<td>0.520</td>
<td>2.604</td>
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<td>4</td>
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<td>0.681</td>
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<td>5</td>
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<td>0.3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.597</td>
<td>2.446</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0.500</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\( \sigma_z = 1 \)
Sources of Correlation

- Common risk factors \((z)\)
- Correlated risk factors \(r(x_k x_J) \neq 0\)
- Correlated shocks \(r(u_n u_m) \neq 0\)
- Contagion \(\theta \neq 0\)
ECONOMETRICS

Reflection Problem
“Spatial” identification
Identification by IV (GMM)
Identification by weak exogeneity
Static Contagion
Cross – section data

\[ y = \alpha C y + \varepsilon \quad \text{N - vector} \]

\[ y_i = 1 \text{ if defaulted} \quad \& \quad 0 \text{ otherwise} \]

linear probability \( y \mod el \)

\( C \) sparse \( N \times N \) \( c_{ij} = 1 \quad i \& \quad j \text{ contact} \quad (0 \text{ otherwise}) \)

\[ y = A \varepsilon \quad A = \left[I_N - \alpha C \right]^{-1} \]

\[ \frac{\partial y_i}{\partial \varepsilon_k} = a_{ik} \]

\[ q_k = \sum_{i=1}^{N} a_{ik} \]

\[ \hat{\pi}_{ML} \quad (SAR) \]

\[ \varepsilon_i = e + \omega_i \]
Contagious Impulses
31 x 31 Rook Lattice

\[ Y = \alpha Cy + \varepsilon \]

- Impulse responses (from epicenter) die away more slowly with larger \( \alpha \).
- Edge effects attenuate impulse responses when \( \alpha = \frac{1}{4} \).
Local Impulses
31 x 31 rook lattice
Instrumental Variables

Cross-section data

\[ y = \alpha Cy + \beta z + \epsilon \]

\[ y = A(\beta z + \epsilon) \]

\[ A = I + \pi C + \pi^2 C^2 + ... \]

\[ \tilde{z} = Cz, \quad \tilde{\tilde{z}} = C^2 z \]

\[ \hat{y} = \beta [z + \pi \tilde{z} + \pi^2 \tilde{\tilde{z}} + ..] \]

\[ y = \alpha C \hat{y} + \beta z + \epsilon \]
Dynamic Contagion
Panel data

\[ y_t = \alpha C y_{t-1} + \varepsilon_t \]
\[ y_{t-1} = \alpha C y_{t-2} + \varepsilon_{t-1} \]
\[ y_t = \alpha^2 C^2 y_{t-2} + \varepsilon_t + \alpha C \varepsilon_{t-1} = \varepsilon_t + \alpha C \varepsilon_{t-1} + \alpha^2 C^2 \varepsilon_{t-2} + \ldots \]

\[
\frac{\partial y_{it}}{\partial \varepsilon_{kt-p}} = \alpha^p c_{ik}^p \quad (d o \ m i n o \ c o n t a g i o n) \\
\frac{dy_{it}}{d\varepsilon_{it-p}} = \alpha^p c_{ii}^p \quad (b o o m e r a n g \ c o n t a g i o n) \\
\]

\[ p \lim \hat{\alpha} = \alpha \; i f \; y_{t-1} \; w e a k l y \; e x o g e n o u s \]
Sectoral Var Model

x: sectoral factors
z: macroeconomic factors
u: innovations
Data: Israel, 7 sectors, 1997 Q1 – 2010 Q3
Not panel VAR!
Identification through weak exogeneity @ t-1

\[ Y_{nt} = \alpha_n + \sum_{i=1}^{p} \lambda_{ni} Y_{nt-1} + \sum_{h=1}^{H} \sum_{i=0}^{b} \phi_{nhi} z_{ht-i} + \sum_{k=1}^{K} \sum_{i=0}^{d} \beta_{nkt} x_{nkt-i} + \sum_{j\neq n}^{J} \sum_{i=1}^{c} \theta_{nji} Y_{jt-i} + u_{nt} \]
Weak Exogeneity
(Granger causality v weak exogeneity)

\[ Y_t = \beta X_{t-1} + \lambda Y_{t-1} + u_t \]
\[ u_t = \rho u_{t-1} + \delta v_t + \varepsilon_t \]
\[ X_t = \theta Y_{t-1} + v_t \]
\[ E(X_{t-1}u_t) = E[(\theta Y_{t-2} + v_{t-1})(\rho u_{t-1} + \delta v_t + \varepsilon_t)] \]
\[ u_{t-1} = \rho u_{t-2} + \delta v_{t-1} + \varepsilon_{t-1} \]
\[ E(X_{t-1}u_t) = \theta \rho^2 \sigma_u^2 + \rho \delta \sigma_v^2 + \delta E(v_t v_{t-1}) \]
DATA

Israel 1997 Q1 – 2010 Q3
7 bank credit sectors
“Problem Loans”

• Bank of Israel definition
• Includes loans under rescheduling
• Broader than FDIC definition
• Role of credit swaps
Rate of Problem Loans
Loan Loss Provisions
(% of Problem Loans)
Problem Credit Rates
## Unit Root Tests

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry Transport &amp; Storage</th>
<th>Communications &amp; Computer services</th>
<th>Financial services</th>
<th>Business services</th>
<th>Persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-1.47</td>
<td>-1.22</td>
<td>-2.07</td>
<td>-1.90</td>
<td>-1.72</td>
<td>-1.67</td>
<td>-2.08</td>
<td>-3.67</td>
</tr>
<tr>
<td>PP</td>
<td>-1.28</td>
<td>-1.90</td>
<td>-1.42</td>
<td>-1.47</td>
<td>-1.55</td>
<td>-1.49</td>
<td>-1.90</td>
<td>-2.77</td>
</tr>
<tr>
<td>KPSS</td>
<td>0.47</td>
<td>0.54</td>
<td>0.62</td>
<td>0.49</td>
<td>0.38</td>
<td>0.39</td>
<td>0.44</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Notes: ADF is the 4th order augmented Dickey-Fuller statistic and PP is the Phillips-Perron statistic with bandwidth 1-4 with critical value -2.9 at p = 0.05. KPSS is the 5th order KPSS statistic with critical value 0.463.
# Correlation Matrix

## Problem Credit Rate

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport &amp; Storage</th>
<th>Communications &amp; Computer services</th>
<th>Financial services</th>
<th>Business services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commerce</td>
<td></td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hostelry</td>
<td></td>
<td></td>
<td>0.90</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transport &amp; Storage</td>
<td>-0.16</td>
<td>0.00</td>
<td></td>
<td>0.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Communications &amp; Computer services</td>
<td>0.05</td>
<td>0.74</td>
<td>0.13</td>
<td>-0.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial services</td>
<td>0.01</td>
<td>0.65</td>
<td>0.03</td>
<td>0.65</td>
<td>0.25</td>
<td>0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Business services</td>
<td>0.66</td>
<td>-0.27</td>
<td>0.74</td>
<td>-0.36</td>
<td>-0.15</td>
<td>-0.32</td>
<td>-0.32</td>
<td></td>
</tr>
<tr>
<td>Persons</td>
<td>0.70</td>
<td>-0.37</td>
<td>0.77</td>
<td>-0.31</td>
<td>-0.38</td>
<td>-0.24</td>
<td>-0.34</td>
<td>0.74</td>
</tr>
</tbody>
</table>
RESULTS

Sectoral Models
Drivers of credit risk
Coefficients of inertia
Coefficients of contagion
Misspecification tests
### Table 5.1 Factor Models: Industry

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>D</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial production&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-6.76</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1.81</td>
</tr>
<tr>
<td>Share of electrical equipment in industrial production</td>
<td>-5.28</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0.73</td>
</tr>
</tbody>
</table>

<sup>a</sup> Logged and HP filtered.

### Table 5.2 Factor Model: Building

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction: gross output&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-31.45</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4.49</td>
</tr>
<tr>
<td>Exchange rate&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-9.87</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2.58</td>
</tr>
<tr>
<td>Consumption per head</td>
<td>-19.05</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>4.91</td>
</tr>
<tr>
<td>Public sector investment</td>
<td>-5.90</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Notes. <sup>b</sup> logs
### Table 5.3 Factor Model: Commerce

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate (USD)</td>
<td>3.44</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1.55</td>
</tr>
<tr>
<td>Gross investment</td>
<td>-1.84</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0.80</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.79</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0.22</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.16</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0.05</td>
</tr>
</tbody>
</table>

### Table 5.4 Factor Model: Hostelry

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign tourists&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-3.25</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.91</td>
</tr>
<tr>
<td>Domestic tourism&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-25.86</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>6.17</td>
</tr>
<tr>
<td>Deaths due to terror</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>Real wages&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-48.19</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>9.62</td>
</tr>
</tbody>
</table>

Notes: HP filtered.
### Table 5.5 Factor Model: Transport & Storage

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of diesel fuel</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0.10</td>
</tr>
<tr>
<td>Employment in sector</td>
<td>-0.21</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>Wages in sector</td>
<td>0.00</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>Gross output in sector</td>
<td>-0.04</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>YTM indexed bonds</td>
<td>0.54</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.13</td>
</tr>
<tr>
<td>Exports</td>
<td>-3.41</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0.79</td>
</tr>
</tbody>
</table>

### Table 5.6 Factor Model: Financial Services

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment in sector</td>
<td>-25.69</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5.82</td>
</tr>
<tr>
<td>TASE 100</td>
<td>-2.57</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.52</td>
</tr>
</tbody>
</table>

### Table 5.7 Factor Model: Persons

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate: Bank of Israel</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.02</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.04</td>
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<tr>
<td>Inflation</td>
<td>0.74</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Notes. The dependent variable is expressed as $\log\left[\frac{RPC}{1-RPC}\right]$. 


### Coefficients of Inertia

<table>
<thead>
<tr>
<th></th>
<th>Persons</th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport-Storage</th>
<th>Financial services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.22</td>
<td>0.51</td>
<td>0.88</td>
<td>0.79</td>
<td>0.72</td>
<td>0.83</td>
<td>0.74</td>
</tr>
<tr>
<td>Lag order</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes. P-value of coefficients of inertia < 0.025.
## Coefficients of Contagion

<table>
<thead>
<tr>
<th></th>
<th>Commerce</th>
<th>Industry</th>
<th>Transport</th>
<th>Building</th>
<th>Hostelry</th>
<th>Financial Services</th>
<th>Persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td></td>
<td></td>
<td>-1.17 Δ(2)</td>
<td>0.19 (1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Building</td>
<td></td>
<td>-0.68 Δ(3)</td>
<td>-0.45(4)</td>
<td></td>
<td>-0.32 Δ_2(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commerce</td>
<td></td>
<td></td>
<td></td>
<td>0.19 Δ(3)</td>
<td></td>
<td></td>
<td>0.31 Δ(1)</td>
</tr>
<tr>
<td>Hostelry</td>
<td></td>
<td></td>
<td></td>
<td>0.56 (2)</td>
<td></td>
<td></td>
<td>1.29 Δ_2(1)</td>
</tr>
<tr>
<td>Transport – Storage</td>
<td></td>
<td>-0.35 Δ(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.56 (1)</td>
</tr>
<tr>
<td>Financial Services</td>
<td></td>
<td>-0.11 (3)</td>
<td>-0.18 Δ(3)</td>
<td>0.07 (4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persons</td>
<td></td>
<td></td>
<td>-0.06 (1)</td>
<td>-0.13 Δ(2)</td>
<td></td>
<td>0.14 (2)</td>
<td></td>
</tr>
</tbody>
</table>

Notes. The table reads horizontally. P-value of contagion coefficients < 0.025. Δ_s indicates that contagion is in seasonal difference and (p) denotes that lag order of contagion.
Long-run Residuals

\[ u_{jt}^* = Y_{jt} - \alpha_j^* - \sum_{k=1}^{K} \beta_{kj}^* z_{kt} - \sum_{n=1}^{N} \gamma_{nj}^* x_{nt} - \sum_{h \neq j}^{J} \theta_{hj}^* Y_{ht-1} \]

\[ \beta_{kj}^* = \frac{\sum_{i=0}^{b} \beta_{kji}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \quad \gamma_{nj}^* = \frac{\sum_{i=0}^{d} \gamma_{jni}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \quad \theta_{hj}^* = \frac{\sum_{i=1}^{c} \theta_{hji}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \]
# Goodness of Fit & Specification Tests

<table>
<thead>
<tr>
<th></th>
<th>Adjusted $R^2$</th>
<th>Standard error</th>
<th>CV</th>
<th>LM1</th>
<th>LM2</th>
<th>Forecast test</th>
<th>ADF long run residuals</th>
<th>KPSS dynamic simulation residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.54</td>
<td>0.06</td>
<td>0.36</td>
<td>0.61</td>
<td>0.67</td>
<td>-6.75</td>
<td>0.11</td>
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<tr>
<td>2</td>
<td>0.95</td>
<td>0.76</td>
<td>0.05</td>
<td>0.25</td>
<td>0.08</td>
<td>0.96</td>
<td>-8.37</td>
<td>0.23</td>
</tr>
<tr>
<td>3</td>
<td>0.94</td>
<td>0.38</td>
<td>0.06</td>
<td>0.59</td>
<td>0.62</td>
<td>0.90</td>
<td>-7.58</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
<td>1.31</td>
<td>0.05</td>
<td>0.32</td>
<td>0.60</td>
<td>0.36</td>
<td>-8.91</td>
<td>0.23</td>
</tr>
<tr>
<td>age</td>
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<td>0.53</td>
<td>0.12</td>
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<td>0.07</td>
<td>0.06</td>
<td>-8.95</td>
<td>0.09</td>
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<tr>
<td>ces</td>
<td>0.92</td>
<td>0.48</td>
<td>0.14</td>
<td>0.30</td>
<td>0.22</td>
<td>0.40</td>
<td>-6.38</td>
<td>0.06</td>
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<tr>
<td>5</td>
<td>0.93</td>
<td>0.27</td>
<td>0.08</td>
<td>0.30</td>
<td>0.20</td>
<td>0.74</td>
<td>-9.99</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Innovation Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport-Storage</th>
<th>Financial services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>0.039</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Building</td>
<td>0.347</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Commerce</td>
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<td>0.030</td>
<td>0.092</td>
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<tr>
<td>Hostelry</td>
<td>0.174</td>
<td>0.184</td>
<td>0.036</td>
<td>0.187</td>
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<td></td>
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<tr>
<td>Transport-Storage</td>
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<td>0.079</td>
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<td>0.216</td>
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<td>Financial services</td>
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<td>-0.165</td>
<td>0.025</td>
<td>-0.043</td>
<td>-0.290</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Notes. Bartlett test of sphericity = 26.9 P-value for zero correlations df = 21 is 0.172.
Model Properties

Impulse responses
Counterfactual simulation
Variance decomposition
(Contagion induced correlation)
Technical Details

- \( x_{it} = a_i + b_i C_t + d_i x_{it-1} + e_i C_{t-1} + v_{it} \)
- Full dynamic simulation: 2000 Q1 – 2010 Q3
- Perturb C in 2000 Q1
- Impulse = \( Y_{nt}(\text{perturbed}) - Y_{nt} \) (FDS)
- Perturb \( u_n \) in 2000 Q1
- Fix \( x \)'s from 2000 Q1
- Fix C from 2000 Q1
- Use model estimates of \( u_{nt} \)
- Set contagion coefficients (\( \theta \)) = 0
Impulse Response: Shock to Credit Risk in Building
Impulse Response: Cyclical Shock
## Variance Decomposition of Credit Risk

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport</th>
<th>Financial Services</th>
<th>Persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>2.11</td>
<td>3.39</td>
<td>1.44</td>
<td>9.24</td>
<td>7.01</td>
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<tr>
<td>Cycle</td>
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<td>2.50</td>
<td>1.09</td>
<td>4.15</td>
<td>1.04</td>
<td>1.45</td>
<td>0.39</td>
</tr>
<tr>
<td>Factors</td>
<td>1.88</td>
<td>2.15</td>
<td>0.86</td>
<td>8.16</td>
<td>6.91</td>
<td>0.65</td>
<td>0.14</td>
</tr>
<tr>
<td>Residual</td>
<td>0.54</td>
<td>0.76</td>
<td>0.38</td>
<td>1.31</td>
<td>0.53</td>
<td>0.48</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Epilogue

• Why don’t regulators publish sectoral credit risk data for banks?
• Why aren’t banks required to publish these data?
• If they did, maybe banks would be more cautious
Epidemiology

- $N = x_t + y_t + z_t$
- $x$: susceptibles
- $y$: infectives
- $z$: removals (deaths, immunes)
- $\pi$: probability of contact with an infective
- $(1-\pi)y$: probability of no contact with infectives
- $1 - \beta$: probability of immunity
- $\alpha = \beta[1-(1-\pi)y]$ probability of infection
Bernoulli (1760)

\[ N_t = x_t + z_t \]
\[ \dot{x}_t = -(\alpha + \delta_t)x_t \]
\[ \dot{z}_t = -\delta_t z_t + (1 - \beta)\alpha x_t \]
\[ N_t = N_0 e^{-\delta t} \left(1 - \beta + \beta e^{-\alpha t}\right) \quad (\delta_t = \delta) \]
\[ \alpha = \beta = 0.125 \text{ (smallpox)} \]
Verhulst (1838)

\[ N = x_t + y_t \]
\[ \frac{\dot{y}_t}{y_t} = \alpha x_t \]

\[ y_t = \lim_{t \to \infty} \frac{Ny_0}{y_0 + (N - y_0)e^{-\alpha t}} \rightarrow N \]
Kermack-McKendrick (1927)

\[ N = x_t + y_t + z_t \]

\[ \dot{x}_t = -\alpha y_t \]

\[ x_t \]

\[ \dot{z} = \delta y_t \]

\[ x_t \xrightarrow{t \to \infty} < N \]
Reed-Frost Model
Abbey (1952)

\[ x_0 + y_0 = N + 1 \quad y_0 = 1 \]

\[ \alpha_t = \beta \left[ 1 - (1 - \pi)^{y_{t-1}} \right] \]

\[ f(\Delta y_1) = \frac{x_0!}{\Delta y_1!(x_0 - \Delta y_1)!} \alpha_1^{x_0} (1 - \alpha_1)^{x_0 - \Delta y_1} \quad \alpha_1 = \beta \pi \]

\[ f(\Delta y_t) = \frac{x_{t-1}!}{\Delta y_t!(x_{t-1} - \Delta y_t)!} \alpha_t^{x_{t-1}} (1 - \alpha_t)^{x_{t-1} - \Delta y_t} \]

\[ x_{t-1} = N + 1 - y_{t-1} \]

1st order Binomial Markov Chain

\[ E(y_t) \xrightarrow{t \to \infty} < N + 1 \]
Criticisms

• $\pi$ exogenous: no precaution
• $\pi$ should vary inversely with $y$
• $\beta$ exogenous: no investment in resistance
• $\beta$ should vary inversely with $y$
• Infection rate ($\alpha$) varies inversely with $y$
• Mechanical not behavioral
• Heterogeneity in susceptibility $\sqrt{\phantom{a}}$
Verhulst Model with Endogenous Infection

\[ y_t + x_t = N \]

\[ \frac{\Delta y_t}{y_{t-1}} = \alpha_t x_{t-1} \]

\[ \alpha_t = \overline{\alpha} y_{t-1}^{-\theta} \]

\[ \Delta \ln y_t \approx \ln \overline{\alpha} - \theta \ln y_{t-1} + \ln(N - y_{t-1}) \]

\[ y_t \xrightarrow{t \to \infty} < N \]
Definitions

Population of N lives in square lattice
Individuals have n 1\textsuperscript{st} order neighbors
n = 4 in chessboard lattice
C: NxN sparse contact matrix
c_{ik} = 1 if i and k are neighbors
c_{ii} = 0
\alpha \text{ probability of contamination on contact}
Y_i = 1 if i is contaminated = contagious
q: number contaminated
Static Contagion

\[ y = \alpha C y + \varepsilon \quad N - \text{vector} \]

\[ y = A \varepsilon \quad A = [I_N - \alpha C]^{-1} \]

\[ \frac{\partial y_i}{\partial \varepsilon_k} = a_{ik} \]

\[ q_k = \sum_{i=1}^{N} a_{ik} \]

\[ \hat{\pi}_{ML} (SAR) \]

\[ \varepsilon_i = e + \omega_i \]
Impulse Responses

Spatial Multipliers
N=961

Effect vs Distance From Epicenter

- $\alpha=0.25$
- $\alpha=0.2$
- $\alpha=0.1$
Instrumental Variables

\[ y = \alpha Cy + \beta z + \varepsilon \]
\[ y = A(\beta z + \varepsilon) \]
\[ A = I + \pi C + \pi^2 C^2 + \ldots \]
\[ \tilde{z} = Cz, \quad \tilde{z} = C^2 z \]
\[ \hat{y} = \beta [z + \pi \tilde{z} + \pi^2 \tilde{z} + \ldots] \]
\[ y = \alpha C \hat{y} + \beta z + \varepsilon \]
Contagion v Correlation

- $Y_1$ and $Y_k$ are correlated
- Via $\alpha$: Contagion
- Via $e$: Correlated shocks
- Contagion is identified
Dynamic Contagion

\[ y_t = \alpha Cy_{t-1} + \varepsilon_t \]
\[ y_{t-1} = \alpha Cy_{t-2} + \varepsilon_{t-1} \]
\[ y_t = \alpha^2 C^2 y_{t-2} + \varepsilon_t + \alpha C \varepsilon_{t-1} = \varepsilon_t + \alpha C \varepsilon_{t-1} + \alpha^2 C^2 \varepsilon_{t-2} + ... \]
\[ \frac{\partial y_{it}}{\partial \varepsilon_{kt-p}} = \alpha^p c_{ik}^p \text{ (domino contagion) } \]
\[ \frac{dy_{it}}{d \varepsilon_{it-p}} = \alpha^p c_{ii}^p \text{ (boomerang contagion) } \]

\[ p \lim \hat{\alpha} = \alpha \text{ if } y_{t-1} \text{ weakly exogenous} \]
Diffusion

\[ q_0 = 1 \]
\[ E(q_1) = q_0 + \alpha n \]
\[ E(q_2) = q_1 + \frac{1}{2} \alpha (n - 1) + \frac{1}{2} \alpha (n - 2) \]
\[ E_{t-1}(q_t) = q_{t-1} + \alpha n - \frac{3}{2} \alpha \]
\[ E(q) \Rightarrow N \]
Endogenous Contact

- $U_{ik} = F[c_{ik}, H(c_{ik}/Y_k = 1)] \quad c_{ik} = 1$
- $U_{0ik} : c_{ik} = 0$
- $c_{ik} = 1: d_{ik} = U_{ik} - U_{0ik} > 0$
- $P(c_{ik} = 1) = 1/[1+\exp(-d_{ik})]$
- $f(q) = 1/q \quad f' < 0$
- Endogenous resistance too
- Exposure avoidance
- Investment in resistance
Contagion with Endogenous Exposure

\[ y_t = \frac{\alpha C}{q_{t-1}} y_{t-1} + \epsilon_t \]

\[ E(q) \Rightarrow < N \]