An affine factor model of the Greek term structure

Hiona Balfoussia
AN AFFINE FACTOR MODEL OF THE GREEK TERM STRUCTURE

Hiona Balfoussia
Bank of Greece

ABSTRACT
This paper aims to contribute to our understanding of the dynamics driving the Greek term structure of nominal interest rates and to explore their possible macroeconomic determinants. A canonical, Vasicek-type latent affine factor model of the Greek term structure is estimated on data spanning the period March 1999 to February 2007. This framework allows us to directly examine the impact of the extracted factors on the shape of the yield curve over time and on the associated price and amount of risk in the term structure. In line with the related literature, three latent factors, i.e. a "level" factor, a "slope" factor and a "curvature" factor, appear to capture most of the time variation in the Greek nominal term structure of interest rates and to drive its dynamics. The evolution of these factors over time is examined on the basis of business cycle theory and related to macroeconomic fundamentals of the Greek economy.

Keywords: term structure, affine factors, stochastic discount factor, Kalman filter
JEL classification: E43, E44, G1

Acknowledgements: Valuable discussions with Heather Gibson are gratefully acknowledged. The author has also benefited from comments by Stephen Hall, Daniela Marcelli, Eric Swanson, George Tavlas, Hercules Voridis, the Bank of Greece workshop series participants and the participants of the ECB 2007 Workshop on “The Analysis of money market: role, challenges and implications from the monetary policy perspective”. The views expressed in this paper are those of the author and do not necessarily reflect those of the Bank of Greece.

Correspondence:
Hiona Balfoussia,
Economic Research Department,
Bank of Greece, 21 E. Venizelos Ave.,
10250 Athens, Greece,
Tel. +30 210 3202646
e-mail: hbalfoussia@bankofgreece.gr
1 Introduction

The objective of this paper is to contribute to our understanding of the dynamics driving the Greek term structure of nominal interest rates and to explore their possible macroeconomic determinants. The framework employed is founded on the theoretical concept of a unique stochastic discount factor pricing all financial assets in the economy, as used in the context of affine term structure models. It allows us to directly examine the impact of the extracted factors on the shape of the yield curve over time and on the associated price and amount of risk in the term structure.

We estimate an unobservable (latent) factor model of the Greek term structure. While such a model does not use macroeconomic information, we can nonetheless draw tentative conclusions on the interdependence between the bond market and macroeconomic fundamentals from our findings. Alternative risk dependence structures within a no-arbitrage framework can also be tested against each other.

The structure of the paper is as follows. In Section 2 we briefly discuss the affine class of term structure models, outlining its advantages and underlying intuition. After briefly reviewing the related literature in Section 2.1, we present the theoretical framework of a canonical, Vasicek-type latent affine factor model of the term structure in Section 2.2. Section 3 outlines the econometric methodology employed. In Section 4.1 we review recent research which has focused on the Greek bond market and discuss the Greek data. We present and discuss our results in Section 4.2. These indicate that, in line with the related literature, three latent factors capture most of the time variation in the Greek nominal term structure of interest rates. The evolution of these factors over time is discussed and related to macroeconomic fundamentals of the Greek economy. Section 5 provides some concluding remarks.

2 Affine Factor Models of the Term Structure

2.1 A brief literature review

Affine term structure models allow us to model the entire term structure, including the risk-free rate, and to extract and examine the underlying factors which drive its evolution over time. This class of models has become very popular in recent years and the related literature, which stemmed from the seminal papers of Vasicek (1977) and Cox, Ingersoll and Ross (1985), is expanding rapidly. The specifications have become increasingly sophisticated, allowing much greater flexibility than the
early, single-factor models and often capturing yield dynamics with great accuracy. A complete characterisation of this class of term structure models is provided by Duffie and Kan (1996), while Piazzesi (2003) offers a comprehensive overview of this literature.

Affine term structure models, like all stochastic discount factor models, relate equilibrium bond prices to the stochastic discount factor or pricing kernel. Additionally, they also impose cross-equation restrictions on the yield equations in a way that ensures no arbitrage across the entire term structure. Despite their complexities, because yields are affine functions of the underlying state vector, the models remain tractable and have closed form solutions.

A key characteristic of affine term structure models is that all missing bond yields can be recovered from observations on a small set of yields, in a way that is consistent with no-arbitrage.\(^1\) Indeed, certain multifactor models fit yields that were not included in the estimation within a couple of basis points. From this feature stems a principal theoretical advantage of this framework over others employed in this literature.

In addition to enabling us to unravel the dependence of implied, expected future short-rate paths on the evolution of the underlying factors, affine term structure models also allow us to separate risk premia from expectations about future short rates. This renders them important tools in our effort to understand the extent to which bonds are considered a safe investment.

A further technical advantage of affine term structure models over other modelling frameworks is that they require the estimation of relatively few parameters, while still offering a remarkably good fit to the data. The reduction of estimated parameters increases the efficiency of the estimates, while Ang and Piazzesi (2003) show it also improves the out-of-sample forecasting of yields.

Empirical studies seem to indicate that 3 unobservable factors suffice to explain much of the nominal yield dynamics. Knez, Litterman and Scheinkman (1994) were among the first to establish, using factor decompositions of the variance-covariance matrix of yield changes, that over 97% of the variance is attributable to just three principal components. Nonetheless, on occasions, a fourth factor may prove to be significant, as shown by Dai and Phillipon (2005).

\(^1\)It is necessary for the estimation of such models that the number of maturities available is greater than the number of factors in our model.
The main weakness of the latent affine class of term structure models is that they usually have little macroeconomic motivation. Consequently, they do not offer an economic interpretation of their factors, though they do offer a structural interpretation on the basis of their impact. Nonetheless, tentative conclusions on the main macroeconomic determinants of the term structure can often be drawn. Most papers with multiple latent factors try to make intuitive associations regarding their variables. Knez, Litterman and Scheinkman (1994) describe their three principal components as “level”, “slope” and “curvature” according to how shocks to these factors affect the yield curve and this terminology has been widely adopted. As Piazzesi (2003) points out, this interpretation of the driving forces of yields seems to be stable across model specifications, estimation samples and types of interest rates. Others name their factors after underlying fundamentals they can be related to. Among the first and most quoted such attempts is that by Pearson and Sun (1994). In general, macroeconomic intuition in the latent factor context usually stems from an examination of the time-series properties of the extracted factors, as well as from their comparison to observed data on macroeconomic fundamentals they may possibly represent. This is the approach we adopt in this paper.

2.2 Theoretical Framework

We briefly set out the canonical, affine Vasicek-type term structure model, in its latent multifactor form. As already mentioned, affine term structure models can be viewed in the context of the Stochastic Discount Factor theory of asset pricing, whereby the price of any asset at time $t$ is the expected discounted value of its price next period. Hence, $P_{n,t}$, the price of an $n$-period zero-coupon bond today is the expected discounted value of $P_{n-1,t+1}$, the price of the bond next period, when it has $n-1$ periods to maturity

$$P_{n,t} = E_t[M_{t+1}P_{n-1,t+1}]$$

where $M_{t+1}$ is the unique, positive stochastic discount factor.

Despite providing a comprehensive and intuitive theoretical framework for all asset pricing, the SDF theory does not direct us as to how $M_{t+1}$ should be modelled. We hence turn to the theory on affine term structure models to obtain a functional form for the stochastic discount factor. These models, though lacking macroeconomic intuition, propose a simple and general functional form for $M_{t+1}$, while at the same time eliminating arbitrage opportunities.
The simple multifactor affine Vasicek-type term structure model, one of the most popular specifications, contains three basic equations. The first is the transition equation for the underlying state vector $z_t$ relevant for bond pricing. This is specified as a Vasicek-type, autoregressive process

$$z_{t+1} = \left( I_{k\times k} - \Phi \right) \theta + \Phi z_t + \Omega_{k\times k}^{1/2} \epsilon_{t+1}$$

where

$$\epsilon_{t+1} \sim NID(0,I)$$

$k$ is the number of factors and $\Omega$ is a diagonal matrix with typical element $\sigma_i^2$.

The second equation is the process generating the pricing kernel which implicitly prices all assets in this system. This is also a function of the state vector

$$-\ln(M_{t+1}) = -m_{t+1} = \delta + \gamma' z_t + (\lambda + \beta z_t)' \Omega^{1/2} \epsilon_{t+1}$$

Finally, in any affine term structure model, the log-prices of all bonds are affine (linear) functions of the underlying state vector $z_t$

$$-\ln P_{n,t} = A_n + B_n' z_t$$

It follows that the yields are also affine in the state variables

$$R_{n,t} = \frac{A_n}{n} + \frac{B_n'}{n} z_t$$

where $R_{n,t}$ is the yield to maturity of an $n$-period zero-coupon bond at time $t$ and

$$R_{n,t} \simeq -\frac{1}{n} \ln P_{n,t}$$

By normalising $\delta = \frac{1}{2} (\lambda + \beta z_t)' \Omega (\lambda + \beta z_t)$ we set the short rate $s_t$ equal to a linear combination of the state variables, eliminating the constant term

$$R_{1,t} = s_t = \gamma' z_t$$

The price of risk associated with the shocks to the state vector $\epsilon_{t+1}$ is a linear
function of the underlying factors

$$\Lambda_t = (\lambda_{1 \times k} + \beta_{k \times k} z_t)$$  \hspace{1cm} (8)$$

Since in this framework the variance of the shocks, $\Omega$, is assumed to be constant over time, the price of risk defines the behavior of risk premia. If the price of risk is time-varying (i.e. if the matrix $\beta$ is non-zero) then risk premia will also be time-varying and vice versa.

The coefficients $A_n$ and $B_n$ depend on maturity and are non-linear functions of the underlying parameters. It can be shown that, in order to ensure no arbitrage in this system, the matrices $A_n$ and $B_n$ must be defined by the following recursions\(^2\)

$$A_n = A_{n-1} + B'_{n-1}(I - \Phi)\theta - \frac{1}{2}B_{n-1}'\Omega B_{n-1} - \lambda'\Omega B_{n-1}$$  \hspace{1cm} (9)

$$B'_{n} = \gamma' + B'_{n-1}\Phi - B'_{n-1}\Omega\beta$$  \hspace{1cm} (10)

where

$$A_0 = 0, \hspace{0.1cm} B_0' = 0_{1 \times k}$$  \hspace{1cm} (11)

and hence

$$A_1 = 0, \hspace{0.1cm} B_1' = \gamma'$$

Hence, although affine term structure models are linear in the state vector as set out in equation 4, $A_n$ and $B_n$ are nonlinear functions of the underlying parameters. The cross-equation restrictions imposed on this term structure system from equations 9 and 10 ensure arbitrage opportunities are eliminated not only for the bonds included in the estimation, but across the entire term structure.

### 3 Econometric Methodology: The Kalman Filter

The latent affine factor term structure model can be estimated using a Kalman filter and quasi-maximum likelihood. Once a dynamic system is expressed in a state-space representation, the Kalman filter provides an algorithm which sequentially updates a linear projection for the system. One of its many uses is to calculate linear least squares forecasts of the unobservable state vector on the basis of observed data.

We can view the affine term structure model as a state space system, with an

---

\(^2\)The derivation of the recursions 9 and 10 can be found in the Appendix.
observation equation linking observable yields to the state vector and a state equation describing the dynamics of the underlying factors. The state equation of our system is given by equation 2. This is the transition equation for the underlying state vector \( z_t \) which is modeled to follow a Vasicek-type, autoregressive process. The factors are taken to be latent (unobservable) and hence will be extracted from the estimation procedure.

The observation equation for each of the four maturities is given by equation 5, augmented by a homoskedastic error \( \eta_t \)

\[
R_{n,t} = \frac{A_n}{n} + \frac{B_n'}{n} z_t + \eta_t^n
\]

where

\[
\eta_t^n \sim NID(0, H_n^2)
\]

Hence it is implied that all the yields are measured with a normally distributed error.\(^3\) \( H_n \) is estimated and constrained to be equal across maturities, to reduce the number of estimated parameters.

The iterations of the Kalman filter are started at the analytical unconditional mean and variance of the factors.

\[
E(z_t) = \Theta
\]

\[
\text{vec}(Var(z_t)) = [I_{k^2} - (\Phi \otimes \Phi)]^{-1}\text{vec}(\Omega)
\]

where \( \otimes \) denotes the Kronecker product.\(^4\)

In the model set out above \( \Phi, \Theta, \Omega, \gamma, \lambda, \beta \) and \( H_n \) are the parameters to be estimated. In order for the price of risk parameters to be identified, only \( \theta_k \) is unconstrained and estimated, while \( \theta_i \) for all \( k \neq i \) are set to zero. Further, since the factors are latent rather than observable, their weights in the stochastic discount factor cannot be separately identified. Hence \( \gamma \) is also normalised to a vector of ones.

\(^3\)The assumption that all yields are measured with error can be avoided by using the Chen and Scott (93) maximum likelihood technique, which allows as many yields as there are factors to be assumed without error. However this is a rather arbitrary assumption, as is the presumption that the econometrician can know which of the yields are measured with error. The assumption that all yields are measured with some error seems more plausible. Although this implies that equation 5 cannot be inverted to compute the state vector, the Kalman filtering technique allows us to obtain our estimates.

\(^4\)See chapter 11 in Hamilton (94) for further details.
The two-factor parameterisation\(^5\), (i.e. \(k = 2\)), is

\[
\Phi = \begin{bmatrix}
\phi_1 & 0 \\
0 & \phi_2
\end{bmatrix}, \quad \theta = \begin{bmatrix}
0 \\
\theta_2
\end{bmatrix}, \quad \Omega = \begin{bmatrix}
\sigma_1^2 & 0 \\
0 & \sigma_2^2
\end{bmatrix}, \quad \gamma = \begin{bmatrix}
1 \\
1
\end{bmatrix}, \quad \lambda = \begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix}, \quad \beta = \begin{bmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{bmatrix}
\]

Setting \(\beta_{i,j} = 0\) for any \(i, j\) constrains the price of risk (and risk premia) to be constant, i.e. \(\Lambda_t = \lambda\). Setting \(\lambda = 0\) as well, sets risk premia to zero, i.e. \(\Lambda_t = 0\). Estimating both \(\lambda\) and \(\beta_{i,j}\) allows risk premia to vary across the sample. Hence

\[
\Lambda_t = (\lambda + \beta z_t) = \begin{bmatrix}
\lambda_1 + \beta_{11} z_{1,t} + \beta_{12} z_{2,t} \\
\lambda_2 + \beta_{21} z_{1,t} + \beta_{22} z_{2,t}
\end{bmatrix}
\]

Constant price of risk models are nested within time-varying price of risk models with the same or higher number of factors. Furthermore, models with any number of factors nest models of the same risk specification with fewer factors. Hence, these specifications can be tested against each other using standard likelihood ratio tests.

4 The Greek nominal term structure

4.1 Data and recent research

Few studies have so far been undertaken on the Greek term structure of interest rates. The reason for this is twofold. Firstly, the sovereign bond market has long been hindered by various forms of institutional rigidities from operating freely, from allowing, that is, prices and yields to be solely determined by the forces of demand and supply. Historically, the most pronounced distortion was the obligation of financial institutions to invest a fixed share of their deposits in government debt, which essentially forced the financial sector to finance the government at rates which, far from being competitive, were often below inflation. This distortion was gradually relaxed and was abandoned in 1994, by which time most of the rigidities hampering the Greek financial sector had also been removed. Secondly, a fully functioning government securities market was essentially only really set in place with the foundation and operation of the Electronic Secondary Securities Market (HDAT) in 1998, the institution which now handles all trading of government securities. Thirdly, the maturity structure of Greek sovereign debt has not always been constant, or even complete. In the mid-nineties the weighted average maturity of newly-issued Greek

---

\(^5\)In the interest of economy of space we present the two-factor parameterisation rather than the three factor one.
sovereign debt was around 2 years, primarily as a consequence of the high interest rates of the period and a long history of high and variable inflation. As policy rates and inflation decreased and the overall circumstances and prospects of the Greek economy improved, the maturity spectrum increased substantially over our sample period, the average maturity of newly-issued debt standing at just above 10 years in 2006.\footnote{As interest rates rapidly declined, the circulation of bonds with a longer time to maturity substantially facilitated sovereign debt management.} \footnote{It should be noted that Greece’s obligation to fulfil the Maastricht criteria indirectly required the existence of long-term (i.e. 10-year) fixed-income securities, as long-term bond yields were expected to be within a 2% margin of the average corresponding yield of the three best-performing countries.} \footnote{An important landmark is 1997 when the first fixed-coupon, fixed-income security was issued.} However, this characteristic of the market during its early steps raises hurdles, both data-related and more fundamental ones, to the economist who wishes to study and model the Greek term structure.

As a consequence of the above limitations, research so far undertaken on the Greek bond market is limited and recent, and typically uses small samples. The most notable paper is that of Manousopoulos and Michalopoulos (2005) which is, to our knowledge, the only study to extract theoretical prices from observed ones. The study examines a single year, 2004, and aims to compare the performance of a battery of interpolation methods. However, the focus is on goodness of fit and statistical properties rather than on the underlying dynamics and their possible interpretation in terms of macroeconomic fundamentals. Other work includes that of Mylonidis and Nikolaidou (2003), Varelas (2005) and Artikis (2003). However, these focus on specific maturities only and aim, usually, to examine the validity of the expectations hypothesis rather than to formally model and study term structure dynamics. In contrast, this paper presents an estimation of an affine latent factor model of the entire term structure of Greek nominal interest rates, with an emphasis on its macroeconomic underpinnings.

Based on the related empirical literature on affine latent factor term structure models, one can expect to find three factors driving a nominal term structure using a latent affine factor model. Our aim is to examine whether this is indeed the case and to relate the factors extracted to macroeconomic fundamentals of the Greek economy. We expect to find a “level” factor, associated with long-term real and nominal expectations for the economy and capturing the long-run attractor of the yield curve. A second factor is likely to be a “slope” factor which may be capturing cyclical fluctuations. A third one is likely to be a curvature factor, affecting the shape of the yield curve mostly at the very short end.
Data availability confines the period covered to between March 1999 and February 2007. This gives us eight years of data at a weekly frequency. We use yields on the 3-year, 5-year and 10-year benchmark bonds provided by HDAT, while for the short end we use the one-year ATHIBOR which is replaced by the EURIBOR in 2001.

4.2 Discussion of Results

4.2.1 Estimated coefficients

We have estimated a three factor model with time-varying and constant prices of risk. Table 1 presents parameter estimates of the observation and risk equations. The corresponding t-statistics are in the second row, in italics. We see that the persistence parameter \( \phi \) of all factors is highly significant and very close to unity, as is common in this literature. The corresponding estimates of the factors’ standard deviations are also highly significant, as expected. Hence, this specification seems to be capturing three significant and highly persistent time-varying factors driving the yield curve. The lamdas on two of the three factors are highly significant. We will see later on that these two factors correspond to short and medium-run fluctuations. Long-run sources of risk seem not to be assigned a statistically significant price.

The betas estimated in this model are however insignificant, implying, implausible though it may seem, that the risk premium in the Greek nominal term structure, as modelled here to stem from our three latent factors, is not significantly time-varying over our sample.\(^9\) Moreover, on the basis of a likelihood ratio test, the constant price of risk specification appears significantly superior to the one with time-varying prices of risk. One reason for this finding may be that the short end of the yield curve is poorly represented in our dataset, since we only use the 1-year rate. The short end of the yield curve is generally more complex in its curvature. The absence of more information might be making it more difficult for the model to pick up time variation in the risk premium. Additionally, the fact that we are using an interbank rate rather than a T-bill at the one year maturity may be introducing an additional distortion at this end of the spectrum. Another alternative interpretation may be the lack of heteroskedasticity in the risk structure of the affine term structure model. However, in the light of the highly time-varying risk premia captured in most empirical research on various term structures, which find the risk premium to have significant time variation, this finding requires further investigation.

\(^9\)We report the estimates of the specification with constant prices of risk. All results available upon request.
Table 1
Estimated parameters: Three factor model with constant prices of risk

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \phi )</th>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>0.996</td>
<td>0.987</td>
<td>0.944</td>
<td>-1002.8</td>
</tr>
<tr>
<td>0.1</td>
<td>884.6</td>
<td>396.6</td>
<td>97.4</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>0.233</td>
<td>0.508</td>
<td>0.754</td>
<td>0.372</td>
</tr>
<tr>
<td></td>
<td>457.2</td>
<td>4.2</td>
<td>4.2</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>77.1</td>
<td></td>
<td>11.6</td>
</tr>
</tbody>
</table>

Note: \( t \)-statistics in second row, in italics.

Figure 1 plots fitted against actual yields and Table 3 presents the corresponding \( R^2 \). The observed yields seem to be fitted very well throughout the sample. For all maturities the goodness of fit is above 99%. In view of this, one may conclude that, although rather arbitrary, the constraint of equal measurement error variance across maturities may be inconsequential when compared to the gain in terms of reduction in parameter space.

Table 2
Adjusted \( R^2 \)

<table>
<thead>
<tr>
<th>Maturity</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 months</td>
<td>99.70%</td>
</tr>
<tr>
<td>3 years</td>
<td>99.74%</td>
</tr>
<tr>
<td>5 years</td>
<td>99.88%</td>
</tr>
<tr>
<td>10 years</td>
<td>99.93%</td>
</tr>
</tbody>
</table>

As already mentioned, having estimated an affine term structure model one can calculate the implied yield for any maturity, whether used in the original dataset or not. It is interesting to calculate the 1-month yield implied by our model. This can be compared to data available on the 1-month short rate in order to assess the model’s goodness of fit at the very short end of the maturity spectrum. Figure 2 plots these two variables against each other. We see that, despite the very good fit to yields within our dataset (and indeed to yields of maturities which fall between the ones included) the fit to this very short rate which is outside the range of our sample is not equally good, especially over the first few years of our sample. The very short end of the yield curve was particularly volatile during much of 1999 as the markets are thought to have experienced several “scares” that the drachma’s central rate in the ERM and hence the subsequent final conversion rate from the drachma to the euro may have had to have been revised. While these fears did not materialize, the restlessness of the period is evident in our data and even more so in data on shorter maturities, rendering the very short end of the yield curve hard to capture during the first part of our sample. This illustrates that the choice of yields to include is
important and that the more complex curvature of the short end of the yield curve is harder to fit in the absence of precise information. It may also help explain why no time-varying risk premium component has been captured. Risk premia on yields of short maturities, while smaller, would be expected to be more volatile than those on yields of longer maturities, the latter being relatively smoother as averages of expected one-period risk premia over a much longer horizon. Since we only use the segment of the yield curve from one year onwards, we may be missing some of this high time variation.

4.2.2 The extracted factors - evolution and relation to the macroeconomy

We now turn to the extracted latent factors themselves, arguably the most interesting and intuitive part of our estimates. Figure 3 plots the three extracted factors against time, while Figure 4 presents the corresponding impulse responses of the spot yield curve to a positive shock of one standard deviation to each of the latent factors. Knez, Litterman and Scheinkman (1994) were the first to describe their three principal components as “level”, “slope” and “curvature”, according to how shocks to these factors affect the yield curve. Indeed, the impulse response curve corresponding to the first factor is completely flat. This is characteristic of a level factor, a shock to which will affect the level of the entire yield curve, causing a parallel shift. The slope factor is strongly negatively correlated with all spreads, as visualised in Figure 5, but mostly with those at the medium and long end of the yield curve. The strongest correlation is with the 5 to 10 year spread, the correlation coefficient being -0.98, followed closely by the 3 to 10 year and the 3 to 5 year spreads. The impulse response curve of factor two is a negatively sloping straight line, indicative of a slope factor, that is one which affects the short end of the yield curve much more than the long end. Finally, the impulse response curve of the third factor is downward sloping but decreasingly so. This appears to be a typical “curvature factor”, i.e. one affecting the shape of the yield curve across maturity but very much more so at the short end and less than any other at the long end of the spectrum. It is a factor which is most highly correlated with the slope at the short end of the yield curve. As Piazzesi (2003) points out, the level, slope and curvature interpretation of the driving forces of yields seems to be stable across model specifications, estimation samples and types of interest rates. This observation appears to be confirmed by the dynamics of the Greek bond market.

Having established the effect of each of the extracted factors on the yield curve,
the natural question to ask is whether we can associate these factors with some underlying fundamentals which may be driving the yield curve. Most papers with multiple latent factors try to draw such parallels, in an effort to render the extracted factors more intuitive, often naming their factors after underlying fundamentals they can be related to. Pearson and Sun (1994) are among the first and most quoted such attempts, followed by many others. Despite this trend, a direct comparison of the extracted latent factors to raw data on the corresponding macroeconomic variables is not often attempted, perhaps due to the inherent difficulty in drawing clear one-to-one associations between theoretical factors and macroeconomic timeseries. Nonetheless, this is the approach we follow here.

The level factor of the term structure is typically associated with some measure of long-run expectations for the economy. It is particularly difficult to compare this factor with actual data, as information on long-run expectations is not available for Greece. The longest horizon for which survey data on market expectations is collected is the 2-year horizon, provided by Consensus Forecasts. Hence, we use these, bearing in mind their limitations.

Figure 6 plots the level factor against an annual survey measure of expected real GDP growth two years ahead, used as a proxy for the expected long-run real return on capital, which should according to economic theory be reflected in the yield curve. One may hope that, depending on the period under examination, medium-term expectations for real growth and for real variables in general would not necessarily differ very substantially from longer-term expectations, at least regarding the direction of the expectation which is of concern here, given that usually little precise additional information is available for the longer horizons. It would appear that this series roughly follows the same trend as the factor, though their frequencies are very different. Expected growth of real GDP was on the increase until 2003. This period includes the two years immediately prior to Greece’s entry into EMU, during which the ongoing efforts to meet the Maastricht criteria, the flow of EU structural funds channeled towards investments in infrastructure and the markets’ anticipation of the country’s admittance to the monetary union largely drove investors’ confidence. The subsequent euphoria and realisation of the substantial real implications of the country’s acceptance into the eurozone continued to dominate the economic climate, while the real improvement in key macroeconomic fundamentals, including the still-ongoing sharp decline in interest rates, continued to boost expectations for growth. This positive sentiment was further prolonged in anticipation of the 2004
Athens Olympics, as evidenced by the high expected growth rates in the years immediately prior to the hosting of the games. Expected GDP growth subsequently declined, as these one-off events went by and the international environment deteriorated. Geopolitical instability in the Middle East sparked fears of imminent and prolonged oil-price increases and a possible dampening of world economic growth. While most of the pessimistic projections of the time were not subsequently realised, the markets’ concerns at the time were real and as such should be reflected in the bond market’s fluctuations. Overall, the extent to which developments over our sample period are reflected in the level factor seems intuitive. Indeed, our factor captures the positive sentiment and expectations for strong growth associated with the anticipation of admittance to EMU, an indication that this success was perceived by market participants as signaling improved chances of real and long-lasting economic progress and stability for Greece. In contrast, the temporary peak in real growth expectations related to the hosting of the Olympic Games is not present in the level factor which had already started on a mild downward trend, implying that this event was considered unlikely to impact on long-run real growth, hence not affecting expectations at the long end of the yield curve.

However, while the expected long-run rate of return on capital must be inter alia driving the level of the yield curve, long-run expectations of the level of inflation are usually thought to dominate. Once again we face a lack of data on long-run expectations. Unlike major economies, surveys of the market’s expectations of yields on long bonds and 10-year ahead inflation are not conducted, the two-year horizon being once again our best proxy. However, admittedly expectations of long-run inflation may differ substantially from medium-horizon expectations. Indeed, not all turns in two-year ahead expectations would be mapped in expectations for inflation at say a 5 or 10-year horizon. Nonetheless, Figure 7 plots the level factor against the available survey measure of expected inflation. The similarity with the level factor is still present, though less clear. The main difference between the two series is the increase in expected 2-year ahead inflation in 2002 and 2003, in contrast to the level factor’s decline. The increase in the former could be a result of anticipated inflationary pressures accompanying the expected increased economic activity related to the Olympics Games. However, it is more likely that these expectations reflect primarily short and medium-term inflation expectations, as formed in the light of the international geopolitical unrest of the period and the resulting sharp increase in the international price of crude oil. Hence two-year ahead inflation expectations at the time could reflect the impact of expected high oil prices over the short to
medium-term. In contrast, our level factor clearly declines in 2004 and 2005, these short and medium term inflationary prospects apparently not filtering through to long-run expectations.

One point of interest may be that the level factor becomes markedly less volatile from 2001 onwards. It is highly likely that, following the acceptance of Greece as a full member of EMU, whose main goal is price stability (defined as year-on-year inflation near but below 2%), long-run inflation expectations were stabilised at or close to the declared inflation target of the ECB, now applicable to the Greek economy. The ECB has arguably provided a long-run inflation expectations anchor more credible and stable than any declared monetary policy target could previously have been, even despite the best efforts of the country’s monetary policy authority. Moreover, the substantial improvement in the country’s economic circumstances and environment would undoubtedly be reflected in the yield curve, affecting its long-run nominal attractor in particular. Hence, inflation expectations having become less volatile in this new era, one may also wonder whether long-run expectations of the real underlying macroeconomic fundamentals such as expected real growth may now be more easily detectable in movements at the long end of the yield curve and more defining of the curve’s general level, than had previously been the case. If so, the recent decrease in the expected long-run real growth rate may have been largely responsible for the decline of the level factor towards the end of our sample.

Also notable in this context is the fact that, over the period examined, the average maturity of outstanding tradable debt has increased substantially. Given that in the mid-nineties the average maturity of newly-issued debt had been as low as 2 years, there was a relatively thin market for long bonds during the first years of our sample period, perhaps causing some of the wide fluctuations of the level factor during the first few years. Moreover, the cautious post-Olympics investment climate and the international geopolitical uncertainties may have led to a higher demand for long-run yields, i.e. a flight to quality, which, particularly given the increased liquidity in global financial markets, could well have led to an increase in bond prices and a decline in the rate of return on medium to long maturity sovereign bonds.

We turn now to the slope factor, which we have seen is strongly negatively correlated with changes in the slope of the yield curve across the maturity spectrum. This factor does not have a unique interpretation in the literature. In general however, it is taken to be associated with measures of business cycle fluctuations. Monetary policy shocks are usually found to have a significant slope effect, monetary policy

Figure 8 plots our factor against the evolution of the policy rate over the sample period. It would seem that the general trend is mapped closely. Until 2001 the slope factor declines, tracing the policy rate cuts of the Bank of Greece in the context of the convergence of short-term interest rates to the euro area level. Indeed, its decline from 11.6% in January 1999 to 3.75% in January 2001, the level of the ECB’s deposit facility rate at the time, led to a sharp increase in the slope of the yield curve. However, as further short rate cuts were virtually a foregone conclusion, the slope of the yield curve remained negative, albeit decreasingly so, for some time, reflecting this certainty. The slope eventually reversed to a positive one from the end of 2001 onwards and, as the ECB’s policy rate declined even further and real short rates became negative in Greece, the yield curve’s slope became clearly positive at all horizons and remained so until mid-2005. Throughout this period, the slope factor was negative and declining, picking up this increase in the slope at different maturities.\textsuperscript{10} As the ECB began its latest series of interest rate rises, bringing the short rate once again to levels above those of Greek inflation, the slope of the yield curve declined gradually and the factor increased. In summary, on the basis of a visual inspection of the two series, the slope factor does indeed appear to be related to monetary policy shocks and their impact on the slope of the yield curve, thus indicating that our first and most straightforward hypothesis on the slope factor seems to have some validity.

Shocks to output and real activity are, however, often found to also have a clear slope effect (Ang and Piazzesi 2003, Dai and Phillipon 2005, Lildholdt, Peacock and Panigirtzoglou 2007). Low current economic activity is usually associated with relatively low contemporaneous short rates, since the policy maker will routinely attempt to use monetary policy to boost demand. At the same time, agents would form expectations of recovery as the economy would be anticipated to move through the cycle. According to mainstream business cycle theory, real rates would be expected to increase, as would the rate of inflation, these expectations manifesting themselves in the bond market via a positively sloping term structure. Hence the

\textsuperscript{10}It is important to note that there are occasional points of jumps in our data, where one bond was replaced by another as benchmark. These are, in the majority of cases, very minor changes and become much sparser over time as the market becomes deeper at longer horizons. The only notable exception is in early November 2001, when returns on both the 3-year and the 5-year benchmark bonds recorded a contemporaneous increase, resulting in a corresponding sharp increase in the slope factor. This latter increase is however viewed as data-related and not as a reflection of economic fundamentals.
cyclicality of economic activity should be reflected in the yield curve and especially in its slope.\textsuperscript{11} To informally examine the relevance of such an argument we compare the extracted slope factor to different measures of capacity utilisation. Given its negative correlation with the slope itself one would expect the slope factor to have a positive correlation with such measures.

Figure 9 plots the slope factor against the ratio of potential to actual real GDP.\textsuperscript{12} This measure declined from 1999 to 2004, with the brief exception of 2002, implying that the Greek economy was getting closer to full capacity. As already mentioned, these were years of consistently high growth. They were associated with yield curves which were negatively sloping until 2001, reflecting the markets’ near-certainty that Greek short rates would decline to converge with European ones, and with increasingly positively sloping yield curves from then onwards. The slope factor begins to increase again from around 2005 onwards, as the slope of the yield curve begins decreasing and potential to actual real GDP is again on the increase.

From a business cycle perspective this is the opposite of what we would expect. As aforementioned, one would expect periods of relatively lower growth to be associated with expansionary monetary policy and with upward sloping yield curves reflecting market participants’ expectations of recovery and vice versa. What we see here for most of our sample is, conversely, positively sloping yield curves during periods of strong growth and relatively high inflation. This should not however be taken as an indication of a mismatch between the extracted factor and the data, but rather as a visualization of the fact that the ECB’s monetary policy responds to the macroeconomic developments in the euro area as a whole and not in Greece alone. Indeed, during this particular period, Greece was experiencing strong growth and inflationary pressures while most of the eurozone countries were experiencing a relative slowdown. Therefore, the recent period of low interest rates coincided with a period of overheating and inflationary pressures in Greece. This asynchronous cycle implied \textit{de facto} that the short end of the yield curve does not reflect the business-cycle position of the Greek economy, as the latter was at odds with that of the euro area as a whole. Consequently the slope factor and indeed the slope of the yield curve exhibit a relationship inverse to the one implied by economic theory, reflecting monetary policy at the time.

\begin{footnotesize}
\textsuperscript{11}It is important to note that Greece has not actually experienced a full business cycle over our sample period. This is in fact an extended period of relatively strong GDP growth (mostly above EU average) and of inflation rates which, while clearly lower than Greece’s not-so-distant historical highs, nonetheless remained above the ECB’s declared price stability target. This limitation notwithstanding, we attempt to analyze the evolution of our factor on the basis of business cycle theory.

\textsuperscript{12}Source: OECD
\end{footnotesize}
Figure 10 plots the slope factor against an annual measure of change in private consumption.\textsuperscript{13} We note a decline in this growth rate up to 2000, reflecting perhaps the contractionary monetary and fiscal policy of the period preceding the country’s admission to the EMU. Private consumption markedly picked up henceforth, its growth approaching 5% in 2001 and remaining close to 4% until 2004. It decelerated from 2005 onwards, remaining however above 3%. Once again the business cycle implications of this graph are not reflected in the yield curve.

Perhaps most illuminating is Figure 11 which plots the slope factor against a quarterly survey judgement of capacity utilization in manufacturing.\textsuperscript{14} The two series seem to be following mirror paths until 2005. Henceforth, following the post-Olympics drop in business and consumer confidence, capacity utilization and the slope factor seem to have moved back into the relationship implied by traditional business cycle theory. That is, the slope of the yield curve increases as capacity utilization increases, reflecting expectations that the end of the cycle and its real and nominal implications is in sight. Indeed, from 2006 onwards the slope at all points of the yield curve has dropped, recently turning negative at levels of capacity utilisation previously associated with steeply positively sloping yield curves in our sample. This could be seen to imply that the recent series of increases in the policy rate has been appropriate for the economic circumstances of the overheated Greek economy, the markets evidently no longer expecting further major rises. The current rate may be perceived by investors as appropriate for the Greek economy, the yield curve perhaps revealing expectations of a curb in inflation as we move out of the boom period, as evidenced from the level and slope of the yield curve.

Finally, we turn to the last factor, identified as a curvature factor, the interpretation of which is not uniform in the literature. Figure 11 indicates that it could perhaps be seen as related to a short-run survey measure of inflation expectations from 2001 onwards. Indeed, we recall that during the first years of our sample the short end of the yield curve was excessively volatile, primarily reflecting speculative pressures on the drachma rather than domestic macroeconomic circumstances. It must however be noted that, while statistically significant, this factor captures shocks primarily related to maturities of less than a year which do not fall within our sample. In view of the fact that the goodness of fit at this out-of-sample end of the yield curve is not good, the estimated evolution of this factor over time is arguably less reliable that those of the other two factors, and hence the above interpretation.

\textsuperscript{13}Source: Economist Intelligence Unit
\textsuperscript{14}Source: OECD Main Economic Indicators
is tentative in comparison to the ones put forth for the level and slope factors.

5 Conclusion

This paper presents estimates of an affine latent factor model on Greek nominal bond returns. The model indicates that three latent factors adequately describe the dynamics of the Greek nominal term structure of interest rates, the identified "level", "slope" and "curvature" effects falling in line with related findings in the literature. Moreover, the evolution of the extracted factors over time seems largely intuitive when seen in relation to developments in macroeconomic fundamentals of the Greek economy over our sample period.
References

Ang A., Piazzesi M., 2003, “A no-arbitrage vector autoregression of term structure dy-
namics with macroeconomic and latent variables”, Journal of Monetary Economics, 50, 4, 745-787
Artikis P. G., 2003, “Measuring risk in the Greek bond market - An alternative ap-
proach”, Managerial Finance, 29, 9, 9-20
Econometrica, 53, 385-408
Chen R.-R., Scott L., 1993, “Maximum likelihood estimation for a multifactor equi-
librium model of the term structure of interest rates”, Journal of Fixed Income, 3, 14-31
structural VAR approach”, Working Paper, NYU
rates and the macroeconomy”, forthcoming in the Journal of Applied Econometrics
money market returns”, Journal of Finance, 49, 5, 1861-1882
Lildholdt, P., Panigirtzoglou , N., Peacock, C., 2007 “An affine macro-factor model of
the UK yield curve”, Bank of England Working Paper No. 322
methods: the Greek case”, Journal of Financial Decision Making, 1, 1, 33-46
Mylonidis N., Nikolaidou E., 2003, “The interest rate term structure in the Greek money
market”, European Review of Economics and Finance, 2, 23-38
Pearson N., Sun T. S., 1994, “Exploiting the conditional density in estimating the term
structure: an application to the Cox, Ingersoll and Ross model” Journal of Finance, 54, 1279-1304
policy and the economy”, FRBSF Working Paper, 17
Figure 1
Fitted and actual yields

Figure 2
Implied short rate
Figure 5
Slope factor and implied spreads

Figure 6
Level factor and 24-month ahead GDP growth rate expectations
Figure 7
Level factor and 24-month ahead inflation expectations

Figure 8
Slope factor and policy rate
Figure 9
Slope factor and ratio of potential over actual GDP

Figure 10
Slope factor and growth rate of private consumption
Figure 11
Slope factor and capacity utilisation in manufacturing

Figure 12
Curvature factor and consumer short run price expectations
Appendix

Recall that from the Stochastic Discount Factor theory

$$P_{n,t} = E_t[M_{t+1}P_{n-1,t+1}]$$

and, from our affine term structure model, that the log-stochastic discount factor and the log prices of all bonds are affine (linear) functions of the underlying state vector $z_t$

$$-\ln(M_{t+1}) = -m_{t+1} = \delta + \gamma' z_t + (\lambda + \beta z_t)' \Omega^{1/2} \epsilon_{t+1}$$

$$-\ln P_{n,t} = A_n + B'_n z_t$$

Taking logarithms

$$\ln P_{n,t} = \ln E_t[M_{t+1}P_{n-1,t+1}]$$

$$p_{n,t} = E_t[m_{t+1} + p_{n-1,t+1} - \frac{1}{2} \text{Var}_t[m_{t+1} + p_{n-1,t+1}]$$

where $p_{n,t} = \ln P_{n,t}$. Substituting for $m_{t+1}$ and $p_{n-1,t+1}$

$$-p_{n,t} = -\{E_t[m_{t+1} + p_{n-1,t+1} - \frac{1}{2} \text{Var}_t[m_{t+1} + p_{n-1,t+1}]}\}$$

$$= E_t[\delta + \gamma' z_t + (\lambda + \beta z_t)' \Omega^{1/2} \epsilon_{t+1} + A_{n-1} + B'_{n-1} z_{t+1}]$$

$$-\frac{1}{2} \text{Var}_t[\delta + \gamma' z_t + (\lambda + \beta z_t)' \Omega^{1/2} \epsilon_{t+1} + A_{n-1} + B'_{n-1} z_{t+1}]$$

The state vector follows the Vasicek-type process

$$z_{t+1} = (I - \Phi) \theta + \Phi z_t + \Omega^{1/2} \epsilon_{t+1}$$
Substituting we obtain

\[-p_{n,t} = E_t[\delta + \gamma' z_t + (\lambda + \beta z_t)\Omega^{1/2}e_{t+1} + A_{n-1} + B'_{n-1}((I - \Phi)\theta + \Phi z_t + \Omega^{1/2}e_{t+1})]
- \frac{1}{2}Var_t[\delta + \gamma' z_t + (\lambda + \beta z_t)'\Omega^{1/2}e_{t+1} + A_{n-1} + B'_{n-1}((I - \Phi)\theta + \Phi z_t + \Omega^{1/2}e_{t+1})]
= (\delta + \gamma' z_t) + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t]
- \frac{1}{2}Var_t[(\lambda + \beta z_t)'\Omega^{1/2}e_{t+1} + B'_{n-1}\Omega^{1/2}e_{t+1}]
= (\delta + \gamma' z_t) + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t]
- \frac{1}{2}Var_t[(\lambda + \beta z_t)'\Omega^{1/2} + B'_{n-1}\Omega^{1/2}]e_{t+1}]
= (\delta + \gamma' z_t) + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t]
- \frac{1}{2}[(\lambda + \beta z_t)'\Omega^{1/2} + B'_{n-1}\Omega^{1/2}]'(\lambda + \beta z_t)'\Omega^{1/2} + B'_{n-1}\Omega^{1/2}]
= (\delta + \gamma' z_t) + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t]
- \frac{1}{2}[(\lambda + \beta z_t)'\Omega^{1/2} + B'_{n-1}\Omega^{1/2}]\Omega^{1/2}(\lambda + \beta z_t) + \Omega^{1/2}B_{n-1}]
= (\delta + \gamma' z_t) + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t]
- \frac{1}{2}[(\lambda + \beta z_t)'\Omega\lambda + \beta z_t) + B'_{n-1}\Omega(\lambda + \beta z_t) + B'_{n-1}\Omega B_{n-1}]
= (\delta + \gamma' z_t) + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t]
- \frac{1}{2}[(\lambda + \beta z_t)'\Omega(\lambda + \beta z_t) + 2B'_{n-1}\Omega(\lambda + \beta z_t) + B'_{n-1}\Omega B_{n-1}]
\]

Setting \( \delta = \frac{1}{2}(\lambda + \beta z_t)'\Omega(\lambda + \beta z_t) \) we simplify to

\[-p_{n,t} = \gamma' z_t + A_{n-1} + B'_{n-1}[(I - \Phi)\theta + \Phi z_t] - B'_{n-1}\Omega(\lambda + \beta z_t) - \frac{1}{2}B'_{n-1}\Omega B_{n-1}]
= A_{n-1} + B'_{n-1}[(I - \Phi)\theta - B'_{n-1}\Omega\lambda - \frac{1}{2}B'_{n-1}\Omega B_{n-1} + (\gamma' + B'_{n-1}\Phi - B'_{n-1}\Omega\beta)z_t]
\]

Equating coefficients with our initial equation for \( \ln P_{n,t} \) we see that indeed

\[ A_n = A_{n-1} + B'_{n-1}((I - \Phi)\theta - \lambda'\Omega B_{n-1} - \frac{1}{2}B'_{n-1}\Omega B_{n-1}) \]

and

\[ B_n = \gamma' + B'_{n-1}\Phi - B'_{n-1}\Omega\beta \]

We have verified the recursions 9 and 10.

42. Christl, J., “Regional Currency Arrangements: Insights from Europe”, including comments by Lars Jonung and the concluding remarks and main findings of the workshop by Eduard Hochreiter and George Tavlas, June 2006.


56. Sideris, D. A., “Foreign Exchange Intervention and Equilibrium Real Exchange


