A note on the use of moving average trading rules to test for weak form efficiency in capital markets

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A NOTE ON THE USE OF MOVING AVERAGE TRADING RULES TO TEST FOR WEAK FORM EFFICIENCY IN CAPITAL MARKETS

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ABSTRACT
This work focuses on the sensitivity of the performance of the moving average (MA) trading rule of technical analysis to changes in the MA length employed. Empirical analysis of daily data from NYSE, the Vienna Stock Exchange (VSE) and the Athens Stock Exchange (ASE) reveal high variability of the performance of the MA trading rule as a function of the MA length for all these markets, a result that weakens the conclusions of previous works, regarding the validity of the hypothesis of weak form market efficiency. Further, the trading rule is found to have predictive power in ASE and VSE, but not in NYSE.

JEL classification: G14, G15, C22

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1. Introduction

The question of the predictability of future stock prices from past and current information, more formally the hypothesis of market efficiency, is one of the most fundamental ones in modern financial theory both for its theoretical merit and its implications for investing. According to Fama (1970) a market is efficient if “prices fully reflect all available information”.

Until the early 1990s the general conclusion of most empirical tests on market efficiency was that, with few exceptions, the hypothesis that capital markets are efficient was not rejected, at least in its weak and semi-strong form, i.e. when the information set used to predict future prices includes past prices, and all publicly available information respectively (see for example Fama (1991) and Elton and Gruber (1995) for comprehensive reviews). It should be noted at this point that market efficiency is directly linked with the mechanism of producing conditional expectations of returns, i.e. an asset pricing model and, thus, any conclusion regarding market efficiency is conditional upon the appropriateness of the asset pricing model used in conjunction with the hypothesis (joint hypothesis problem). However, in more recent research work, returns derived from trading rules were directly compared with the corresponding buy-and-hold returns. In that way the efficient market hypothesis becomes less dependent on a pricing model, as the only assumption that is made is that prices follow a submartingale process (i.e. $E(R_{t+1}/\Phi_t) \geq 0$ where $E$ is the expected value operator and $E(R_{t+1}/\Phi_t)$ is the expected return at time $t+1$ given the available information up to time $t$ ($\Phi_t$)). These newer results showed that buy or sell signals derived from such trading rules have predictive power, so market efficiency should be rejected even in its weak form.

The spark that ignited this new discussion on the subject with a large number of papers to follow was unquestionably the work of Brock et al. (1992) in which it was shown that trading rules have predictive power for the stocks of the New York Stock Exchange (NYSE). This conclusion was then generalized for both developed and emerging capital markets (see for example Hudson et al. (1996); Bessembinder and Chan (1995); Gençay (1996)). Among the trading rules used by researchers to test market efficiency the one employed most frequently is the so-called moving average (henceforth MA) rule. In contrast to other rules of technical analysis, the MA trading
rule is mathematically well defined (Nefci, 1991) and is used by most market analysts (Taylor and Allen, 1992). It is noted that although technical analysis stresses that a buy or a sell decision is best a composite one, based on as many conducive signals as possible (e.g. volume of trade, convergence-divergence indicators, etc. (Murphy, 1986)), quite often the MA rule, due to the precise signals it generates, is used as a stand-alone method, particularly in the automated trend following trading systems. In that way it becomes a purely “mechanical”, rather than technical, trading rule. Even in its “mechanical” use, however, the MA trading rule may have several versions (see for instance Pring (1991)). In one of its versions, two non-centered, moving averages with different length are initially created from the time series of stock prices:

\[
MAL_t = \left( \frac{1}{N} \sum_{i=0}^{N-1} \theta_i B^i P_t \right)
\]

\[
MAS_t = \left( \frac{1}{M} \sum_{i=0}^{M-1} \theta_i B^i P_t \right) \text{ with } N>M
\]

where \(MAL_t\) represents the relatively longer MA with length \(N\), calculated at time \(t\), \(MAS_t\) represents the relatively shorter MA with length \(M\), \(P_t\) is the stock price at time \(t\), \(\theta_i\) are non-time varying parameters, and \(B\) is the backward shift operator, i.e. \(B^i P_t = P_{t-i}\).

Buy signals are (sequentially) generated at the times \(\tau_j^B\), where:

\[
\tau_j^B \equiv \inf \left\{ t : t > \tau_j^B, MAL_t - MAS_t > DP_{t-1} \right\}
\]

Sell signals are (sequentially) generated at the times \(\tau_j^S\), where:

\[
\tau_j^S \equiv \inf \left\{ t : t > \tau_j^S, MAS_t - MAL_t > DP_{t-1} \right\}
\]

where the initial times \(\tau_0^B, \tau_0^S\) are set equal to zero and \(D\) is the so-called “band” (a pre-specified non-negative constant).

More recent research work on exchange rates has shown that other non-linear methods such as the nearest neighbour predictor perform better than the MA trading rule in its mechanical form (Fernandez-Rodriguez et al., 2003). However, the
performance of the MA trading rule is improved, if it is combined with other indicators, as technical analysis claims. Indeed, Gençay and Stengos (1998) have shown that including past information on the volume of trade improves the performance of the MA trading rule; Fang and Xu (2003) have shown that the performance of the MA trading rule is improved if MA trading rule signals are combined with conventional time series forecasts.

So far, in most of the research papers on the subject, the hypothesis of efficient markets in its weak form is tested by comparing the performance of the trading rule with that of the passive investment strategy (buy-and-hold). This is conducted using the mean return of “buy” trading periods (i.e. the trading periods, usually days, for which, according to the trading rule, the capital should remain invested in the market) to that of either the “sell” trading periods (i.e. the trading periods for which the capital should be liquidated, or sold short) or the mean return of the whole time span covered by the data. A t-test for these means cannot be legitimately applied, mainly due to the existence of autocorrelation, and, on many occasions, a bootstrap methodology is employed to test for significance (see Brock et al. (1992) for further details).

However, as far as the MA trading rule is concerned either as a stand-alone method or in conjunction with other indicators, in the largest part of the published research work, the selection of the MA lengths is rather arbitrary (e.g., Gençay (1996) and Gençay and Stengos (1998) use the MA trading rule with N=200 and M=1). Although several scholars examine cases with MAs with different lengths in order to weaken the dependence of the results on the chosen length of the MA (see for example Brock et al. (1992); Hudson et al. (1996); Mills (1997)), the sensitivity of the results to changes in the MA length has not been examined systematically, as the choice of the lengths of the MAs is based only upon the popularity that some specific combinations of MA lengths enjoy among market analysts (e.g. Brock et al., 1992; Bessembinder and Chan 1995; Fang and Xu, 2003).

In this work a more in-depth analysis of the sensitivity of the performance of the MA trading rule to changes in the MA length is presented. The aim is to answer questions including: (a) what is the functional dependence of the MA trading rule returns on MA length? (b) does this functional dependence imply stationarity? (i.e. do MA trading rule successive returns fluctuate around a certain level?) (c) are there any deterministic components? (d) can lengths for which the predictive performance of the
trading rule is maximised be identified (and in that way the opinion of many technical analysts that the choice of specific lengths leads to maximum returns be justified)?

In addition to the sensitivity analysis, a very preliminary and purely qualitative study of the predictive performance of the trading rule as compared to that of the buy-and-hold strategy will also be attempted in this article. Extensive quantitative work on the statistical significance of the differences of the MA trading rule predictive performance, as compared to the performance of a buy-and-hold strategy, may be found in most of the published papers (e.g., Brock et al., 1992; Hudson et al., 1996; Bessembinder and Chan, 1995).

2. Data and Markets

The data set used in this work consists of daily closing prices of the Standard and Poor-500 Index (henceforth SP) of NYSE, the General Index (henceforth GEN) of the Athens Exchange (henceforth ASE) and the ATX index of the Vienna Stock Exchange (henceforth VSE) for the period 27 April 1993 to 27 April 2005.

NYSE comes first in the world in terms of capitalization value. Additionally, it is also the most extensively and thoroughly searched capital market in the world.

ASE and VSE were chosen for several reasons. Both markets until the late 80’s had low capitalization value, few listed companies, thin volume of daily trade and traders were almost exclusively local investors. Since then, the situation has gradually changed substantially in both markets in several perspectives. At first, important changes have occurred during the 90’s in the legislative and regulatory framework, in both VSE and ASE, towards harmonization with the standards of the more developed mature financial markets (for details see for instance Alexakis and Xanthakis, 1995; Laopodis, 2004; Kenourgios et al., 2008, for ASE and Huber, 1997 for VSE). These reforms resulted to a vast increase in the total capitalization value and in the average volume of transactions during the period under study in both markets. Indeed, total capitalization increased from 13.6 and 28.3 million USD at end 1993 to 145.1 and 126.3 million USD at end 2005 for ASE and VSE respectively (source: World Federation of Exchanges). Further, the historical, traditional and cultural links of both Greece and Austria with many of the ex-communist countries of Eastern Europe gave them a comparative advantage and the willingness to invest in
that region, undertaking the entailed risk, soon after the fall of communism. Foreign Direct Investments (henceforth FDIs) from resident enterprises of Greece and Austria in that region increased very rapidly. Indeed, for Austria outward FDIs in Eastern Europe from about 27% of total FDIs in 1993 reached about 44% in 2005 (source: Oesterreichische Nationalbank). The situation is similar for Greece (outward FDIs for that region increased from about 8% of total outward FDIs in year 2000 to about 23% in year 2006 (source: Bank of Greece)).

The brisk economic growth in the region of Eastern Europe could not leave global investment capital indifferent. International investors took a long position in shares of companies listed in VSE and ASE which are direct investors in Eastern Europe. In that way they tried on one hand to take advantage of the high growth rates of the economy of the region and on the other hand to reduce exposition to the entailed risk of investing directly to that region.

A direct reflection of the international capital inflows channelled toward VSE and ASE is in the foreign participation in the total capitalization value and the average volume of transactions. For ASE foreign investors owned less than 10% of the total capitalization in 1993 (approximate indirect assessment, as no exact official figure exists), but more than 37% of the total capitalization at end of April 2005, while their share in the average daily volume of transactions at April 2005 exceeded 54% (source: Central Securities Depository of ASE, Monthly Statistical Bulletin, April 2005). The situation in VSE is similar with foreign investors to account for nearly half of the exchange’s monthly turnover at end 2005.

Owing to these reforms and growth over the last decade, both markets have attracted much attention by international financial analysts, so it is of importance to study both markets in terms of the predictive performance of the MA trading rule and compare the results with those for NYSE for the same period.

3. Methodology

Due to the fact that reforms and globalization occurred gradually in both ASE and VSE the total time period was separated into three sub-periods each of four-years long (1993-1997, 1997-2001, 2001-2005). The beginning of the time period under study starts a month after the removal of the last short-term capital restrictions in
Greece\textsuperscript{1}. The last period begins right after the upgrading of ASE to a developed market status by Morgan Stanley Capital International. Previously to that date ASE belonged to the so-called European Emerging Markets.

The sensitivity of the total returns from the trading rule to changes in the MA length will be examined separately for each sub-period and market. In all cases the length of the short moving average will be kept constant and set equal to one; all $\theta_i$ parameters will be set equal to one; D will be set equal to zero; the length of the long moving average will vary from 5 to 100. The length of 100 was chosen as a reasonable upper limit considering the time period that the data span, as above that length the total number of transactions signalled by the trading rule was very small\textsuperscript{2}.

As short selling is not allowed in ASE and the derivatives market was not in operation for most part of the period under study a question arises regarding the treatment of “sell” periods investment-wise. The most obvious investment alternative during the “sell” periods under such circumstances is to put the liquidated capital in a current account. Both scenarios (i.e. that of simply being out of the market and that of investing the liquidated capital at the risk free rate) will be considered. The mean annual interest rate of a deposit account will be used as a proxy for the risk free interest rate (source: IMF, International Financial Statistics).

The information necessary to identify the form of the functional dependence of trading rule total returns on the MA length can be obtained from the econometric analysis of the time series of trading rule returns calculated for successive lengths of the longer MA. The first step towards this analysis is to examine the series of successive MA trading rule total returns for the possible existence of a unit root. If these series do not contain a unit root, then, in general, each can be described by a stationary ARMA model and a mean level, around which the series fluctuates, can be estimated. This mean level could then be used as a reference to test the predictive performance of the trading rule against the return generated by a buy-and-hold strategy. In fact the main implicit assumption needed to justify the methodological approach used in previous studies for the testing of weak form market efficiency

\textsuperscript{1}The date of the enactment of the Greek law adopting the EU directive for the abolishment of currency restriction was the 23\textsuperscript{rd} of March 1993.

\textsuperscript{2} For a length of the longer MA equal to 100 there are on average 0.87, 0.48 and 0.5 transactions per month for NYSE, ASE and VSE respectively.
based on the predictive power of the MA trading rule, is that the series of successive MA total returns is stationary. By contrast, if the series of successive MA returns is non-stationary, it can wander extensively without reference to any specific level, and its variance is not constant but tends to infinity as the series size tends to infinity. For such cases, the results themselves regarding the predictive performance of the MA trading rule based on the use of only specific MA lengths is not sufficient evidence for a decision for or against the existence of weak form market efficiency to be based upon.

The most commonly used test for the existence of a unit root is the so-called Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979). However, as is well known (see for instance Hamilton (1994); Enders (1995)), the critical values for ADF tests depend on the presence of any deterministic components. Inevitably, that makes it difficult to perform exploratory analysis when the researcher does not know the exact specification of the model as the tests for unit roots are conditional on the presence and character of any deterministic regressors and vice versa.

In this article, the guidelines suggested by Dolado et al. (1990) and Hamilton (1994) will be followed. More specifically, the existence of a unit root will be tested initially in a model with a constant and a linear trend ($a_2 L$):

$$\Delta R_L = \alpha_0 + \gamma R_{L-1} + \alpha_2 L + \sum \beta_i \Delta R_{L-i} + u_L$$

(1)

where $R_L$ is the trading rule total return for MA length $L$, $\alpha_0$, $\gamma$, $\alpha_2$, $\beta_i$ are the model parameters, $u_L$ is the stochastic disturbance and $\Delta$ is the difference operator.

After estimation of equation (1), if the hypothesis that $\gamma=0$ is not rejected and the existence of a time trend is rejected, then (1) is re-estimated without a time trend. If the hypothesis that $\gamma=0$ is not rejected but the constant is not found significant, (1) is again re-estimated without a constant. Critical values for the deterministic components at each stage are given by Dickey and Fuller (1979). If the hypothesis $\gamma=0$ is rejected at any stage, it is concluded that $R_L$ does not contain a unit root.

This methodology, however, cannot, in general, distinguish a unit root process, the first differences of which in general follow a stationary ARMA model, from a random walk, the first difference of which follow specifically a white noise process.
(as is usual in finance, our use of statistical terms is somewhat loose; by random walk an ARIMA(0,1,0) process is actually meant). However, on this occasion, it is observed that in most cases where the unit root hypothesis is not rejected on the basis of the ADF test, the residuals of the first differences are not different from white noise. This implies that the process is not just non-stationary but specifically a random walk (an ARIMA(0,1,0)). For this reason, a second ad hoc testing procedure leading to a more direct identification of a random walk (in the sense described above) is suggested. This procedure is based on the nature of the sample autocorrelation function (ACF) of the original series, as well as that of the ACF of the first and second differences of the original series. If in the ACF of the original series, six or more of the first consecutive autocorrelations are statistically significant the null hypothesis is set that the original series has a unit root in its dynamics (for a justification see Appendix). The series is then differenced. If the first differences of the original series do not differ from white noise, the additional hypothesis that the original series is a random walk is made and the original series is differenced twice\(^3\).

With the assumption that the first differences of the original series are white noise, it can be easily shown that the overdifferenced series (i.e. the second differences of the original series) is a non-invertible moving average process of first order, and its ACF is theoretically expected to be equal to -1/2 at lag one and zero elsewhere (for a proof see Appendix). The statistical significance of the difference between the estimate of the correlation at lag one (ACF(1)) of the overdifferenced series and the theoretically expected value of -0.5 can be tested using the statistic:

\[
\text{t}_{\text{stat}} = \frac{-0.5 - \text{ACF}(1)}{\frac{1}{\sqrt{N}}}
\]

where N is the series size. For the standard error the typical approximation to Bartlett’s (1946) formula has been used. The possible existence of a drift in the random walk process can be tested by an examination of the statistical significance of the mean of the first differences of the original series. This ad-hoc methodology is

\(^3\) It is noted that in general the first differences of a unit root I(1) process follow a stationary ARMA process. Hence, if the residuals of the first differences of the original series are not white noise, an ARIMA(p,1,q) model is estimated. The residuals of this model are then differenced and the resulting ACF is examined in the way described in the text.
more illuminating, as compared to the ADF tests, in regard to the nature of a stochastic process as it gives the opportunity for a better inspection at each stage.

It must be noted that the proposed methodology is not a new test for random walks. Indeed, we have modified and synthesized material which already exists in disparate regions of the econometrics and time series literature and present it in a concise way as a procedure for random walk testing. A further advantage of this procedure is that it is easily programmable and we have created a routine in MATLAB environment for its automatic application, which we would be happy to supply to other scholars on request. It must also be stressed that it is necessary to use this method in conjunction with the ADF tests due to its bias towards non-rejection of stationarity (e.g. see Dickey and Fuller, 1979; Enders, 1995).

Both methodologies will be used for all markets and time periods to uncover the stochastic process that successive MA trading rule returns follow.

4. Results and discussion

Figures 1, 2 and 3 show the variation of trading rule total returns as a function of the length of the longer MA (the length of the short MA is equal to one for all cases) for SP, GEN and ATX respectively for all time periods with no investment during “sell” periods (left graphs) and investment in a deposit account during “sell” periods (right graphs). The horizontal dash line represents the (total) returns from the buy-and-hold strategy.

From these figures it is visually evident that total returns for many of the cases seem to wander extensively in their own right indicating the possible existence of a unit root.

Table 1 summarizes the results of the econometric analysis based on the ADF tests of successive trading rule returns for each trading period and market. Following the methodology described in the previous section, a stochastic model is suggested for each case (last column of Table 1). It is noted that these results refer to the cases where no investment of the liquidated capital is made during the periods for which an investor who follows the trading rule’s signals is out of the market. However, the results corresponding to the cases for which an investment in a deposit account is
made for the out-of-the-market periods are exactly the same as far as the stationarity test is concerned, therefore, are not reported.

In cases where the existence of a unit root is rejected, the following (stationary) model is estimated:

$$R_L = \alpha_0 + \gamma_1 R_{L-1} + \ldots + \gamma_p R_{L-p} + \alpha_1 L + u_L$$

Cases where the value of $\gamma_1 + \gamma_2 + \ldots + \gamma_p$ was found greater than 0.9 were loosely called “near unit root” processes. Such processes for $\alpha_1 = 0$, although they are stationary, exhibit long swings away from their mean level.

The results from the second econometric approach are shown in Table 2. An example of how this methodology works is shown in Figure 4. The upper part of Figure 4 shows the ACF of the series of the trading rule total returns for GEN for the period 2001-2005. The horizontal lines on both sides of the zero-axis represent the 95% confidence interval. As is evident, there are more than 6 consecutive autocorrelations outside the confidence interval clearly indicating the presence of a unit root. In the middle part of Figure 4 the ACF of the first differences of the original series is shown. No correlation seems to be significant indicating a white noise process (hence, a random walk process for the original series) and the value of the so-called Ljung-Box statistic for whiteness (Ljung and Box, 1978) at lag 20 is 19.99 which is not statistically significant, confirming whiteness (this statistic follows the chi-square distribution with, in this case, 20 degrees of freedom). Finally the lower part of Figure 4 shows the ACF of the second differences of the original series. The value of autocorrelation at lag 1 is -0.43 which, using equation (2), is not found to be statistically different from the theoretically expected (for a random walk) value of -0.50. Further, a significance testing for the mean of the series of first differences shows that the mean is not significantly different from zero. On the basis of the above evidence the model finally selected is the random walk. The same model was also selected using the methodology based on ADF test.

From the results in Tables 1 and 2, it is observed that both methods are consistent in all cases in which successive returns from the trading rule were found to be clearly stationary with the first methodology (GEN 1997-2001, SP 1993-1997, ATX 1993-1997). However, with the second methodology the case, which using the first approach was characterized as near unit root process (ATX 2001-2005), is now
characterized as a random walk. This is not surprising, however, as the tendency of the alternative methodology towards non rejection of non-stationarity has already been discussed in the previous section.

Despite these differences, however, by and large, the results from both methodologies indicate a high variability of trading rule total returns as a function of the length of MA so that successive changes (first differences) in trading rule total returns are essentially random, or nearly so.

Although our primary purpose in this work is to compare the performance of the trading rule for various MA lengths and not to formally compare its performance with that of the passive investment strategy, with the aid of Figures 1, 2 and 3 it is not difficult to make such comparisons and draw interesting qualitative conclusions. At first it is evident that with the inclusion of the extra return from investing in the risk free asset (deposit account) during the out-of-the-market periods trading rule total returns for GEN clearly outperform that of the buy-and-hold for all time periods. The same is true for ATX with the exceptions of only a few MA lengths for the 2001-2005 period. In contrast, in NYSE the performance of the trading rule for SP in all time periods is clearly lower than that of buy-and-hold, with the exception of only some very short MA lengths indicating no predictive power for the trading rule in NYSE. It is noted that for ASE for the period 1997-2001 the trading rule outperforms the buy-and-hold strategy for all MA lengths even without the inclusion of the extra return and at the same time the series of successive trading rule total returns is clearly stationary, as reported in Table 1. This provides clear evidence of predictive power of the trading rule. Consequently weak form efficiency is clearly rejected. The same is true for ATX for the time period 1993-1997.

These results are qualitatively similar to those of previous studies for NYSE, VSE and ASE. Indeed, for NYSE, several authors have documented that the MA trading rule cannot beat the market in the most recent period (e.g. Kwon and Kish, 2002; Cai et al., 2005). In contrast, Kenourgios et al. (2008), for a time period quite similar to the one used in this work, using the FTSE-20 high capitalization index of ASE and following the established methodology regarding the use of the MA trading rule, find higher performance for the trading rule as compared to the buy-and-hold strategy, as is our conclusion for GEN. Likewise, for VSE Shmilovici et al. (2003) reject the hypothesis of weak form efficiency using daily closing prices of the ATX index.
5. Summary and Conclusions

One of the main purposes of this work was to attempt to discover the possible functional dependence of total returns using the moving average trading rule on the length of the moving average, hoping, amongst others, to identify length(s) for which the trading rule’s performance could be maximized. What we found (indeed quite surprisingly) is that such a functional relationship cannot be established due to the high variability of trading rule total returns. Indeed, trading rule total returns as a function of MA length for most cases evolve following a simple random walk process, or autoregressive processes with long periods without reversion to mean (near unit root processes).

These findings have important practical as well as the theoretical implications. From the practical perspective they do not justify the opinion of many technical analysts that the choice of specific lengths leads to maximum returns. On the other hand, caution is need on how these results are interpreted in terms of their implications for weak form market efficiency testing. We stress that our findings do not cast any doubt on the validity of the results themselves regarding the predictive performance of the MA trading rule for specific combinations of MA lengths, as presented in previous published papers. What we do worry about, however, is on the way that these results are interpreted in terms of the hypothesis of weak form market efficiency. Indeed, given the high variability of the performance of the MA trading rule as a function of the length of the longer MA, as documented in this work, by just finding out that trading rules with some specific combinations of MA lengths can or cannot beat the market is not enough evidence for or against weak form market efficiency.

Further, in accordance to the results of previous works regarding the predictive power of the MA trading rule, our results indicate that the MA trading rule can beat the market in ASE and VSE but not in NYSE. The approach we used to come to this conclusion, after further improvements, could be proved superior to the existing one, as, in contrast to our approach, the later is based on only specific combinations of MA lengths, as already discussed.
The important implications of these results for the predictability of stock returns suggest that it is worth pursuing further research using much longer data sets and different markets to establish the derived results on a more solid basis.
References


Appendix

a) The choice of six consecutive statistically significant autocorrelations at the ACF of successive MA trading rule returns, as a criterion of possible non-stationarity is justified as follows: Consider a representative near random walk process, say the first order autoregressive process \( R_L = \varphi L R_{L-1} + u_L \), where \( u_L \) is a white noise process, with finite length and value of the autoregressive parameter greater than roughly 0.9. The autocorrelation coefficient at lag 5 is approximately 0.59 (i.e. \( \varphi^5 \)). The 95% confidence interval for a series with length of about 96 (as is the length of the series in the manuscript), can be calculated using the so-called Bartlett’s approximation formula (Bartlett, 1946) for the estimation of variances:

\[
\text{VAR}(r_k) = \frac{1}{N} \left( 1 + 2 \sum_{i=1}^{m} r_i^2 \right), \quad k > m
\]

where \( N \) is the series length and \( r_k \) is the autocorrelation coefficient at lag \( k \). Then for \( k = 5 \) the 95% confidence interval becomes about +/- 0.49. Hence, the autocorrelation coefficient is clearly outside the confidence interval. At lag 6, the value of the autocorrelation coefficient is approximately 0.53 while the confidence interval becomes also approximately +/- 0.52. Hence, for a particular time series of such length, the situation regarding the statistical significance of the value of the autocorrelation coefficient at lag 6 is unclear, while for lag 7 and above the value of the autocorrelation coefficient is below the confidence interval. This justifies our choice for at least 6 consecutive significant autocorrelations as a necessary condition for non-stationarity.

b) Let \( X_t \) be a white noise process, and \( Z_t = X_t - X_{t-1} \) its first differences. Then \( Z_t \) may be seen as a special case of a first order moving average process (MA(1)) of the general form:

\[
Y_t = U_t - \theta U_{t-1},
\]

where \( U_t \) is a white noise process and \( \theta \) the parameter of the model. For invertibility \( \theta \) must lie between -1 and 1 (see for example Hamilton, 1994). Therefore, \( Z_t \) may be seen as an MA(1) process with \( \theta = 1 \). The process will be stationary, as it is a moving average process, but not invertible. As is well known (Hamilton, 1994) the ACF of an MA(1) has only one non-zero coefficient at lag one and its value is be given by:
\[ r_i = -\frac{\theta}{1+\theta^2}. \] Substitution for \( \theta=1 \) gives: \( r_1 = -\frac{1}{2} \) q.e.d.
Figure 1

Variation of the MA trading rule total returns as a function of the MA length for SP with no interest paid (left graph) and interest paid (right graph). The dash line represents the total return from the buy-and-hold strategy.
Figure 2
Variation of the MA trading rule total returns as a function of the MA length for GEN with no interest paid (left graph) and interest paid (right graph). The dash line represents the total return from the buy-and-hold strategy.
**Figure 3**

Variation of the MA trading rule total returns as a function of the MA length for ATX with no interest paid (left graph) and interest paid (right graph). The dash line represents the total return from the buy-and-hold strategy.
Figure 4
Plot of the ACFs for the MA trading rule returns, their first and second differences for GEN (2001-2005). Dash horizontal lines represent the 95% confidence interval.
**TABLE 1**

Results on stationarity testing for the series of MA trading rule successive total returns based on ADF tests.

<table>
<thead>
<tr>
<th>INDEX</th>
<th>Time Period</th>
<th>Result of Stationarity Test</th>
<th>Model Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>1993-1997</td>
<td>trend stationary series</td>
<td>$R_L=14.62+0.09L+0.80R_{L-1}-0.22R_{L-4}+\varepsilon_L$</td>
</tr>
<tr>
<td></td>
<td>1997-2001</td>
<td>stationary series</td>
<td>$R_L=-1.14+0.80R_{L-1}+\varepsilon_L$</td>
</tr>
<tr>
<td></td>
<td>2001-2005</td>
<td>unit root</td>
<td>$\Delta R_L=0.443\varepsilon_{L-7}+\varepsilon_L$</td>
</tr>
<tr>
<td>GEN</td>
<td>1993-1997</td>
<td>unit root</td>
<td>random walk</td>
</tr>
<tr>
<td></td>
<td>1997-2001</td>
<td>stationary series</td>
<td>$R_L=37.95+0.85R_{L-1}-0.34R_{L-2}+0.26R_{L-3}+\varepsilon_L$</td>
</tr>
<tr>
<td></td>
<td>2001-2005</td>
<td>unit root</td>
<td>random walk</td>
</tr>
<tr>
<td>ATX</td>
<td>1993-1997</td>
<td>stationary series</td>
<td>random walk</td>
</tr>
<tr>
<td></td>
<td>1997-2001</td>
<td>unit root</td>
<td>random walk</td>
</tr>
<tr>
<td></td>
<td>2001-2005</td>
<td>stationary series(near unit root)</td>
<td>$R_L=11.80+0.92R_{L-1}+\varepsilon_L$</td>
</tr>
</tbody>
</table>

**TABLE 2**

Results on stationarity and random walk testing for the series of MA trading rule successive total returns based on the alternative methodology.

<table>
<thead>
<tr>
<th>Index</th>
<th>Time Period</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>1993-1997</td>
<td>unit root (ARIMA(4,1,0))</td>
</tr>
<tr>
<td></td>
<td>1997-2001</td>
<td>stationary series</td>
</tr>
<tr>
<td></td>
<td>2001-2005</td>
<td>unit root (ARIMA(0,1,7))</td>
</tr>
<tr>
<td>GEN</td>
<td>1993-1997</td>
<td>random walk</td>
</tr>
<tr>
<td></td>
<td>1997-2001</td>
<td>stationary series</td>
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<td>stationary series</td>
</tr>
<tr>
<td></td>
<td>1997-2001</td>
<td>random walk</td>
</tr>
<tr>
<td></td>
<td>2001-2005</td>
<td>random walk</td>
</tr>
</tbody>
</table>


80. Apostolides, A., “How Similar to South-Eastern Europe were the Islands of Cyprus and Malta in terms of Agricultural Output and Credit? Evidence during the Interwar Period”, July 2008.


