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MODELING DISTORTIONARY TAXATION

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ABSTRACT

There has been a lot of discussion recently regarding the macroeconomic consequences of a distortionary taxation system. However the way this distortionary taxation scheme or instrument is modeled in macroeconomic analysis, as well as the ability of these models to capture the effects implied by this distortionary taxation system, is subject to criticism. This work provides a formal analysis in an attempt to build a methodological tool (i.e. a functional form), in order to capture the distortionary consequences of the tax system. This tool could be a useful instrument in economic analysis regarding the effects of a distortionary taxation system, and its relation to the other macroeconomic variables, like for example debt, deficit, and inflation.

Keywords: Distortionary Taxation; Income Tax; Tax Revenue; Tax evasion; Tax Compliance; Dynamic path of debt

JEL clasification: C00; H30; H20; H26; H24; H60; E62; B41

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1. Introduction

In the area of public economics, the notion of distortionary taxation is related, on one hand, to the distortions income tax brings to the labor market by affecting the behaviour of tax payers via *income* and *substitution effects* (Stiglitz (1988), Atkinson and Stiglitz (1980), Cowel (1981)), or by affecting labor mobility (Goodspead (2002)). On the other hand, the distortionary taxation system is linked to the notion of "*tax compliance*"[1] (Andreony et al (1998)), and "*tax evasion"* (Allingham and Sandmo (1972), Clotfelter (1983), Feinstain (1991)). This is because the higher the tax burden, the most likely tax evasion will occur. In the presence of tax evasion, unreported income escapes taxation and, as a result, the higher the required taxes, or equivalently the tax rates, the greater the distortionary effect of income taxation. This is also related to *tax incidence,* since for those who are burdened by taxation it is easier to evade and pay fewer taxes. Hence, the tax rate has a substitution effect encouraging tax evasion and an income effect discouraging it (Yitzaki (1974), Clotfelter (1983)).

From the above it is obvious that the choice of tax rate has a decisive role to play in the framework of distortionary taxation, and its relation to the notion of *tax compliance – tax evasion – tax collection*. It is also clear that the tax rate is related to the behaviour of tax payers; it determines their behaviour regarding the ways of tax burden avoidance. As a result, within a distortionary taxation system, tax revenues are lower, compared to a non distortionary system, for example *lump-sum taxation.*

Turning now to the majority of macroeconomic models that use taxation in their analysis, we can distinguish among two main ways taxation is introduced: The first way refers to *lump sum taxation*. This approach treats the tax instrument as an exogenous variable. Consequently, it ignores the relation between taxes and income as well as its growth rate, since under this assumption there exist no distortions. A second way taxation is treated in the related literature is the *proportional taxation* scheme, where tax revenue is written as a fraction of income, i.e. $T = \tau Y$, where τ and Y represent tax rate and income, respectively. This is often called a *distortionary taxation* system (in the sense that changes in the level of income, caused by the fiscal authority's tax policy, affect tax revenues).

[1] This is also related to the notion of tax equity, efficiency and incidence.

Furthermore, the above mentioned proportional taxation scheme is consistent with Ramsey's approach, usually used in macroeconomics in order to study optimal policy problems in a dynamic setting, where the deviation from lump sum taxation allows the benevolent government to avoid the first best solution and hence a distortionary framework ensues. However, this approach is subject to criticism. Golosov et. al (2006) argue that, under the Ramsey setting, the main goal for the government is to mimic lump sum taxes, while it is not clear why the tax instrument takes a particular form. As a result, the Ramsey approach does not provide a theoretical foundation for distortionary taxation. Distortions are simply assumed and their overall level is determined exogenously by the need of the benevolent government to finance its spending.

A closer look at the above taxation schemes can make it clear that the notion of a distortionary taxation system, in the sense we described it previously including tax *compliance – tax evasion – tax collection,* is not depicted either in the lump sum or in the proportional taxation scheme. In fact, including this in the analysis would complicate it, since it will introduce a non linear dynamic relation that relates the behaviour of tax payers to the path of tax revenue, the tax rate and its relation to income[2]. This dynamic relation results in a loss of tax revenue for the fiscal authority, with a deterministic role in equilibrium (if equilibrium occurs at all) of economic models that investigate the relation between the distortionary taxation and the path of debt, or/and its relation to inflation and the path of income, usually discussed in the recent literature (Woodford (2001), (2003) Leeper (1991), (2005), Chari and Kehoe (2007)).

To the best of our knowledge, the approach just described is not treated in the analysis of the relevant literature. Consequently, we believe that if one wants to include into the analysis the consequences of a distortionary taxation system on the economy, it is vital to deal with the above mentioned problem of non linearity.

The purpose of this paper is to provide a methodology for dealing with this problem by using formal analysis. For this we build and suggest a general functional form which relates the behaviour of tax payers to the tax rate and the path of tax revenue and we then study its properties as well as its economic rationale. We finally use an example, in order to show the way this functional form can be applied to economic models.

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[2] This would imply that in the proportional taxation scheme ($T= \tau \cdot Y$), we should at least consider income to be a function of the tax rate, i.e. $Y = Y(\tau)$, which would complicate further the analysis, since the effect of a change in tax rate on tax revenue would come through income as well.

2. Analysis

The idea is to start with the financial consequences of a distortionary tax system, by using the theory of public economics.

As the relevant literature notes, the imposition of income tax is consistent with the behaviour of tax payers. This is also evidenced by economic history, since in many countries, for example in UK[3] and USA[4], several years elapsed between the first appearance and the formal introduction of income tax. Brown and Levine (1979) as well as Breake (1970) provide statistical evidence reporting income and substitution effects, while Barro (1979) refers to the collection cost incurred by the imposition and collection of taxes, with welfare consequences as well as revenue losses for the fiscal authority. Goodspead (2002) shows that in a federal type of economy an increase in the tax rate results in less tax revenue due to the behaviour of tax payers who move to regions where tax rates are lower.

Stiglitz (1988) argues that changes in the tax rate are consistent with changes in income as well as in tax revenue, reflecting the "*income*" and "*substitution*" *effects.* Clotfelter (1983) reports that the loss in the non reported income in USA in 1976 is between 7% and 9% of the reported, indicating lower tax revenues due to distortions. It is hence clear, in a distortionary taxation system that tax revenues (*Z)* are less than tax revenues in a system without distortions (*T)*, i.e. $Z \leq T$. We make the reasonable assumption that the welfare loss due to the imposition of an income tax is realized by tax payers and gives them an incentive for tax evasion[5]. The magnitude of tax evasion will depend on characteristics like the "maturity" and the idiosyncrasy of the society in relation to tax burden, the "tax consciousness" and the socioeconomic characteristics of the society, as well as on the ability of the government to collect taxes.

We can therefore write the relation between the realized (*Z*) and the potential (*T*) tax revenue as

$$Z = T \cdot \varphi(\tau)$$

where $\varphi(\tau) \in (0,1]$, with $Z \leq T$ and for $\tau = \frac{T}{Y}$ standing for the tax rate.

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[3] Income tax first appears in the UK in 1799 in order to finance the Napoleonic Wars, and is established in 1880, while it is characterized as "*hostile to any sense of freedom revolting to the feelings of Englishmen*".

[4] For the first time income tax appears in USA in order to finance the civil war, and while later tax was introduced in 1894, it is established in 1913 with the 13th amendment. In the meantime it had been declared as unconstitutional.

[5] Substitution effect is considered to be a legal way for tax evading.

We are interested in investigating the properties of function $\varphi(\tau)$, which is the qualitative representation of a distortionary taxation system and reflects the behaviour of tax payers as well as the factors that determine this behaviour. Furthermore, this function is the quantification of the distortions of the tax system.

Based on our previous analysis we can write the following:

1. $\varphi(\tau) \in (0,1]$
2. φ exhibits an *infimum* (infφ), indicating that there exists a certain minimum level of willingness to pay (income) taxes. This reflects people's "tax consciousness", which is, of course, related to the tax payers' belief that the social state needs a minimum level of (tax) revenues in order to provide for its citizens.
3. When there are no tax distortions, function φ is a constant function with $\varphi(\tau)=1$.

2.1 Graphical Representation

φ is a function representing the behaviour of tax payers by expressing their willingness to pay (or equivalently to avoid paying) taxes. This willingness is formed depending on the relation between the income to be taxed and the tax burden.

When taxes are a small part of taxable income, the willingness of tax payers to avoid paying taxes is small, indicating that when they are weighing their cost for paying taxes against their benefit from the state's social policy the latter outweighs the former. As the tax rate increases, tax revenues diminish at a decreasing rate, as a result of an increase of the tax payers' willingness to avoid paying taxes. Thus, in the case where the tax burden is relatively low, the difference between Z and T is expected to be relatively small. On the other hand, when taxes reflect a relatively large part of tax payers' income, then their willingness to pay taxes is reduced (and the greater the part of their income required to pay taxes, the lower their willingness to pay taxes). As a result tax revenue is reduced at an increasing rate as the tax rate increases.

Furthermore, when the imposed taxes are a large part of income, then the tax payers' willingness to pay taxes is the minimum possible, and the tax revenues approach their minimum possible levels at an increasing rate as the tax rate increases.

The mathematical way of writing these properties is to say:

$$0 < \varphi(\tau) \leq 1$$

$$\frac{\partial \varphi}{\partial \tau} < 0$$

and

$$\frac{\partial^2 \varphi}{\partial \tau^2} = \begin{cases} < 0, \text{ for } 0 < \tau < \tau^* \\ > 0, \text{ for } \tau^* < \tau < 1 \end{cases}$$

We can therefore draw φ as in figure 1 (where $\tau = \tau^*$ corresponds to an inflection point).

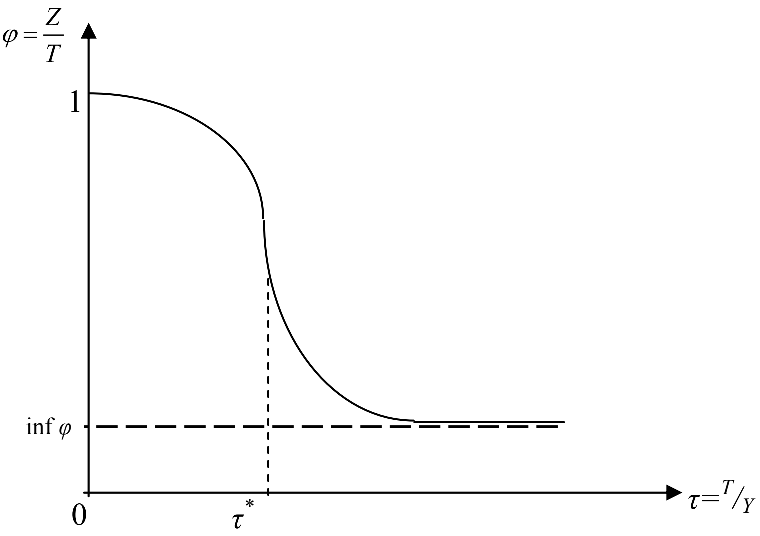

Figure 1

2.2 Properties of the function φ(τ)

In this section we conduct a formal analysis in order to investigate the properties of function $\varphi=\varphi(\tau)$, and to use them in order to prove the functional form that describes the dynamic behaviour of the realised tax revenues under a distortionary taxation system.

The way the realized and potential tax revenues are related to each other in a distortionary taxation system is described by $Z = T \cdot \varphi(\tau)$, with $\varphi(\tau) \in (0,1]$, where $\tau = {}^T/_Y$ is the tax rate. So we can write

$$\frac{Z}{Y} \equiv z = \tau \cdot \varphi(\tau)\,.$$

When dealing with a distortionary taxation system, the related literature accepts that the loss in tax revenue is related to changes in tax rate. Here, we investigate how the tax revenue and the tax rate are related. This effect is depicted in the first order derivative of tax revenues with respect to the tax rate:

$$\frac{\partial z}{\partial \tau} = \varphi + \tau \cdot \varphi' = \psi(\tau),$$

where

$$\psi(\tau)\begin{cases} > 0, \text{ for } -\varphi' < \dfrac{\varphi}{\tau} \\ = 0, \text{ for } -\varphi' = \dfrac{\varphi}{\tau} \\ < 0, \text{ for } -\varphi' > \dfrac{\varphi}{\tau} \end{cases}$$

while its changes are described by the second order derivative

$$\frac{\partial^2 (z)}{\partial \tau^2} = 2 \cdot \varphi' + \tau \cdot \varphi'' = \psi'(\tau)$$

with

$$\psi'(\tau)\begin{cases} < 0, \text{ for } -\varphi' > \dfrac{\tau}{2}\varphi'' \\ = 0, \text{ for } -\varphi' = \dfrac{\tau}{2}\varphi'' \\ > 0, \text{ for } -\varphi' < \dfrac{\tau}{2}\varphi'' \end{cases}$$

As discussed earlier, $\varphi = \varphi(\tau)$ shows the tax payers' willingness to pay taxes. Hence its first order derivative $\varphi'(\tau) = \frac{\partial \varphi}{\partial \tau}$ shows the way this willingness changes as the tax rate changes. Equivalently, $-\varphi'(\tau)$ shows the tax payers' willingness to avoid paying taxes. We treat the ratio $\frac{\varphi(\tau)}{\tau}$ as a measure of the average willingness of tax payers to pay taxes.

The exact way tax revenues change due to changes of the tax rate, (i.e. the magnitude and the sign of $\psi(\tau) = \varphi + \tau \cdot \varphi'$) obviously depends on the magnitude of $\varphi' < 0$:

- When the absolute value of φ' is high enough it dominates the sign of ψ and so $\psi < 0$
- When the absolute value of φ' is small enough, then $\psi > 0$
- Obviously there exists an intermediate value of φ', for which $\psi = 0$

But φ' is the slope of function φ in figure 2. Hence, we can study φ in three areas: ***(i)*** its "left" (rather flat) part, where $\psi > 0$, ***(ii)*** its "right" (rather flat) part, where $\psi > 0$ and ***(iii)*** its "middle" part, where $\psi < 0$.

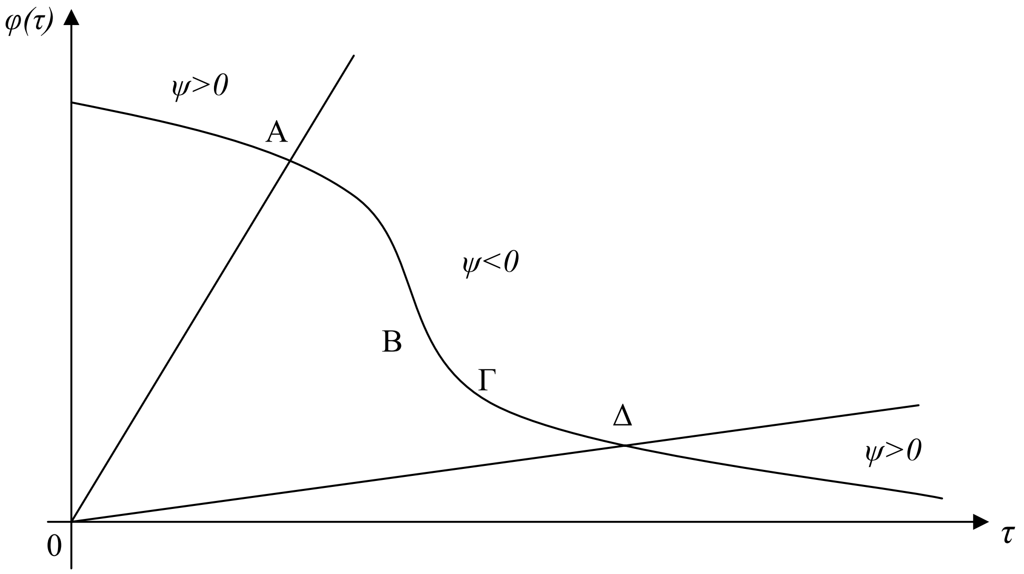

Figure 2

In what follows, we conduct a formal analysis in order to study the properties in each of the above mentioned parts/segments of function φ, as this is summarised in the following lemmas:

Lemma 1

When $\psi = \varphi + \tau.\varphi' > 0$ *and* $\psi' = 2\varphi' + \tau.\varphi'' > 0$, then: $-\varphi' \in \left(0, \min\left\{\frac{\varphi}{\tau}, \frac{\tau}{2}\varphi''\right\}\right)$.

Proof:

- When $\varphi'' < 0$ and $\psi' = 2\varphi' + \tau.\varphi'' > 0$ then $0 < -\varphi' < \frac{\tau}{2}\varphi'' < 0$, which is an inconsistency.
- When $\varphi'' > 0$ and $\psi' = 2\varphi' + \tau.\varphi'' > 0$ then $0 < -\varphi' < \frac{\tau}{2}\varphi''$. Besides, $\psi = \varphi + \tau \cdot \varphi' > 0$ $\Rightarrow -\varphi' < \frac{\varphi}{\tau}$. So, we can write $-\varphi' \in \left(0, \min\left\{\frac{\varphi}{\tau}, \frac{\tau}{2}\varphi''\right\}\right)$.

This case corresponds to the convex $(\varphi'' > 0)$ rather flat (φ' relatively close to 0) part of the graph of φ (: to the right of point Δ' in figure 2).

Lemma 2

When $\psi = \varphi + \tau.\varphi' < 0$ *and* $\psi' = 2\varphi' + \tau.\varphi'' < 0$, then: $-\varphi' \in \left(\max\left\{ \frac{\varphi}{\tau}, \frac{\tau}{2}\varphi'' \right\}, +\infty \right)$.

Proof:

- When $\varphi'' < 0$ and $\psi' = 2\varphi' + \tau.\varphi'' < 0 \Rightarrow -\varphi' > 0 > \frac{\tau}{2}\varphi''$.

  Besides, $\psi = \varphi + \tau \cdot \varphi' \Rightarrow -\varphi' > \frac{\varphi}{\tau}$. So, we can write $-\varphi' > \max\left\{ \frac{\tau}{2}\varphi'', \frac{\varphi}{\tau} \right\}$.

- When $\varphi'' > 0$ and $\psi' = 2\varphi' + \tau \cdot \varphi'' \Rightarrow -\varphi' > \frac{\tau}{2}\varphi'' > 0$

  and $\psi = \varphi + \tau \cdot \varphi' \Rightarrow -\varphi' > \frac{\varphi}{\tau}$

This is the case of the "steep" parts $\left(\varphi' < \min\left\{ \frac{\varphi}{\tau}, \frac{\tau}{2}\varphi'' \right\} \right)$ of the graph of φ that lie around its inflection point B ($\varphi' < 0$ *and* $\varphi' > 0$).

Lemma 3

When $\psi = \varphi + \tau \cdot \varphi' > 0$ *and* $2\varphi' + \tau \cdot \varphi'' < 0$, *then it must be* $-\varphi' \in \left(\frac{\tau}{2}\varphi'', \frac{\varphi}{\tau} \right)$.

Proof:

- When $\varphi'' < 0$, it is $\frac{\tau}{2}\varphi'' < 0 < -\varphi' < \frac{\varphi}{\tau}$. This corresponds to the relatively flat $\left(-\frac{\varphi}{\tau} < \varphi' < 0 \right)$ concave part $(\varphi'' < 0)$ of function φ *and* more precisely to its part lying to the left of point *A* in figure 2.

- When $\varphi'' > 0$, it is $0 < \frac{\tau}{2}\varphi'' < -\varphi' < \frac{\varphi}{\tau}$ which corresponds to the relatively flat $\left(0 < \frac{\tau}{2} \cdot \varphi'' < -\varphi' < \frac{\varphi}{\tau} \right)$ convex $(\varphi'' > 0)$ part of φ and specifically to $\Delta\Delta'$ in figure 2.

Lemma 4

When $\varphi+\tau\cdot\varphi'<0$ *and* $2\varphi+\tau\cdot\varphi''>0$ *, then it must be* $-\varphi'\in\left(\frac{\varphi}{\tau},\frac{\tau}{2}\varphi''\right)$.

Proof:

- When $\varphi''<0$ it is $0<-\varphi'<\frac{\tau}{2}\varphi''<0\ddot{x}$, which leads to inconsistency.

- When $\varphi''>0$ it is $0<\frac{\varphi}{\tau}<-\varphi'<\frac{\tau}{2}\varphi''$, corresponding to the convex $(\varphi''>0)$, relatively flat $\left(-\frac{\tau}{2}\varphi''<\varphi'<-\frac{\varphi}{\tau}\right)$ part of φ *and* precisely to ΓΔ in figure 2.

Following our previous discussion, figure 3 shows the way tax revenues $z={}^{Z}/_{Y}$ change with tax rates:

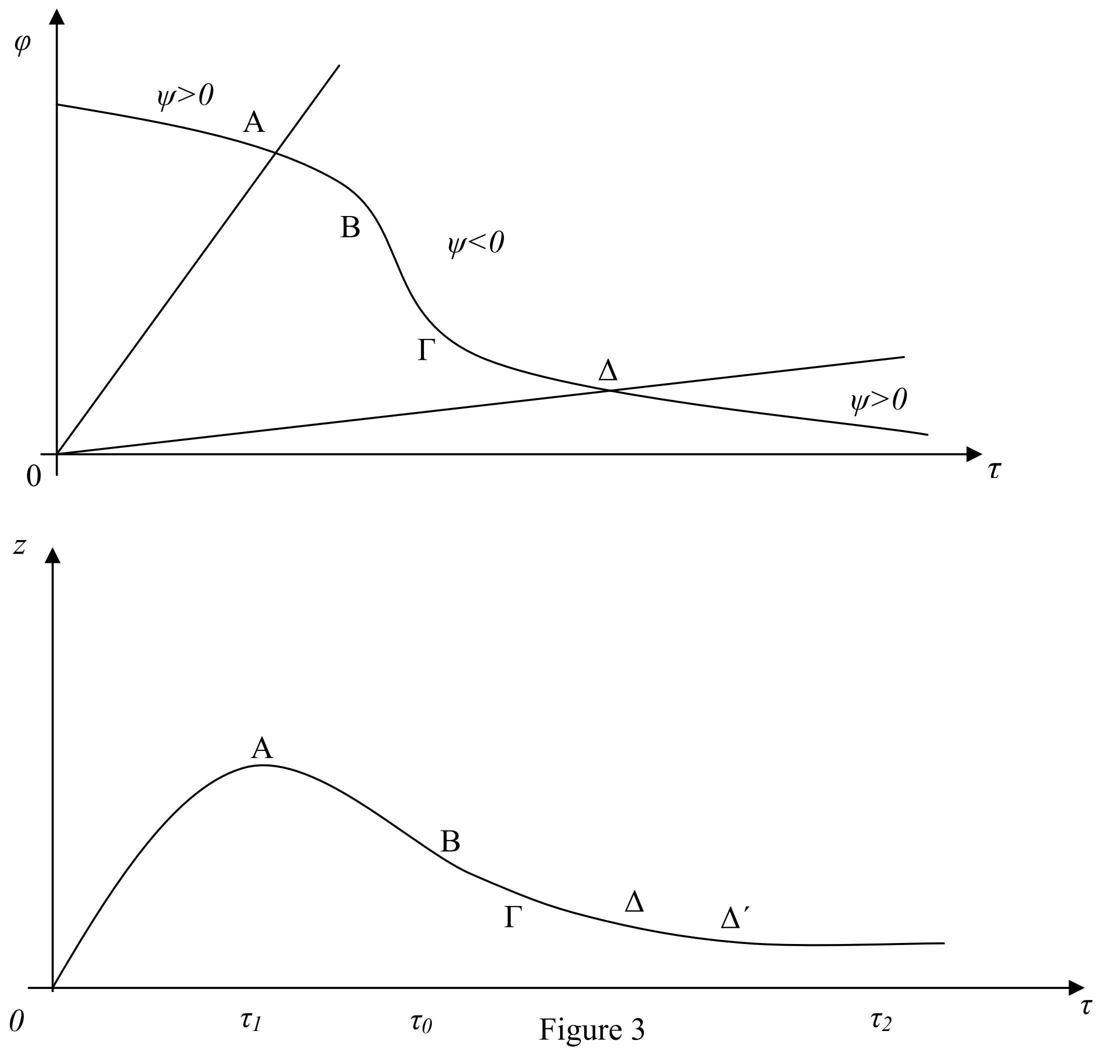

Figure 3

From the above analysis it is obvious that the formation of tax revenues depends, in a non linear way, on the tax rate. Based on our previous discussion, we can summarize the properties of the tax revenue function, expressed as a function of the tax rate, as follows:

$$z \equiv \frac{Z}{Y} = z(\tau)$$

with

$$z'(\tau)\begin{cases} > 0, & \text{for } \ 0 < \tau < \tau_1 \ \ or \ \ \tau > \tau_2 \\ < 0, & \text{for } \qquad \tau_1 < \tau < \tau_2 \end{cases}$$

and

$$z''(\tau)\begin{cases} < 0, & \text{for } \ 0 < \tau < \tau_0 \\ > 0, & \text{for } \quad \tau_0 < \tau \end{cases}$$

3. Theoretical rationale

In this section, we try to place economic rationale into our analysis. Following the previous formal analysis, as well as the literature, our attempt will be to justify the relation between the dynamic path of tax revenue and tax rate under a distortionary taxation regime, where the behaviour of tax payers is endogenized and plays an important role for the outcome. Thus, in each part of the functions $z(\tau)$ and $\varphi(\tau)$, we try to theoretically and logically justify the dynamic path of tax revenue. More precisely:

To the left of point A of figure 3 tax revenue (*z*) increases with a diminishing rate as tax rates increase. In this case, it is $\psi = \varphi + \tau \cdot \varphi' > 0 \Rightarrow -\varphi' < \frac{\varphi}{\tau}$ and $\psi' < 0$. That is, the tax payers' willingness to avoid paying taxes ($-\varphi'$) is less than the average willingness ($^{\varphi}/_{\tau}$). Hence, there is room for the government to raise tax rates. The low distortions in this case, are consistent with the income effect and a positive income elasticity with respect to tax rate[6], as well as with low levels of tax collection cost, tax evasion and tax compliance. Under these conditions the fiscal authority is efficient in increasing tax revenue by increasing tax rates.

Between points A and Δ of figure 3*,* tax revenues follow a diminishing path as the tax rate increases. In this area, increases in tax rates result in a relatively high tax burden and

[6] For the proof see Appendix.

hence, the economy is experiencing a high level of distortions. Here, it is $\psi = \varphi + \tau \cdot \varphi' < 0$, indicating that tax revenues fall, since the willingness to pay taxes as the tax rate increases, decreases at a rate greater than the average level $\left(-\varphi' > \frac{\varphi}{\tau}\right)$. In this case, the economy experiences a *substitution effect* (related to the high level of tax evasion etc), with negative income elasticity with respect to taxes[7] $\varepsilon_{Y,T} < 0$ (as the fiscal authority is willing to increase its tax revenue by increasing the tax rate). The rate of reduction of tax revenue depends on the way tax payers weigh the increase in taxes, and the effect this has on their income which determines their behaviour. The rate at which tax revenues fall is relatively low in ΑΓ, where the difference between *Z* and *T* is relatively small and the realized tax revenue reduces at a diminishing rate. On the other hand, the difference between Z and T is relatively high in ΓΔ, where tax revenue reduces at an increasing rate.

The area to the right of Δ of figure 3 is characterized by high tax rates and a very low willingness to pay taxes. The cost stemming from tax collection - tax evasion - tax compliance is also reaching its maximum levels. However, in this area as the tax rate approaches its highest level $\left(: \frac{T}{Y} \to 1\right)$, the willingness of paying taxes tends to be constant and very low, as *φ moves* asymptotically to its infimum. At these levels of tax rates, already a big part of the tax payers' income is paid for taxes, and hence any further increase of the tax rate would reduce their income even more. Thus, the *substitution effect* is limited and so when the tax rate is further increased, tax revenue increases. Practically, this implies a lower level of labor supply (which refers to the points "below" Δ in the labor supply curve[8], L^s), where there is no further substitution effect. Thus, *within* ΔΔ′, where there is small room for substitution, an increase in tax rates increases tax revenue with a diminishing rate.

In the area to the right of Δ′ *of figure 3* there is no substitution effect. This is because in this area substituting labor for leisure would mean an even lower income which might not be enough for covering the tax payers' living expenses. In practical terms, what happens in this case is that the tax payers have been adjusting their behaviour (regarding their labor supply –i.e. income and substitution effects– and their attitude towards paying taxes – i.e. tax evasion etc) all the way up until the very high taxes of this area and, as a result, there is not much they can do in order to further reduce their tax burden. As a result tax revenues increase with an increasing rate as the tax rate increases towards its maximum level.

[7] See Appendix
[8] See figure A.1 in Appendix.

4. Example: the budgetary consequences of tax distortions

So far we have made a formal analysis of the way income tax distortions work in the economy. We also show that, under the assumptions made, the realized tax revenue is not monotonically related to the change in tax rate. To the contrary, tax revenues exhibit a non linear property, with their dynamic path depending on the behavioural characteristics of the tax payers, composing this way the *income* and *substitution* effect.

In this section our intention is to show how the methodological tool we have theoretically and formally proved can be used in economic analysis, like for example the dynamic path of the debt level.

In a simple closed-economy version and abstracting from monetary considerations, the budget constraint for the economy can be written as

$$B_t = \left(1+r_t\right) B_{t-1} + \left(G_t - Z_t\right) \tag{1}$$

where B_t is the stock of government debt (i.e. bond outstanding) at period t, r_t is the interest rate, and $(G_t - Z_t)$ is the government's primary fiscal deficit with G_t and Z_t standing for public spending and the realized tax revenue, respectively.

Following our previous analysis we are interested in considering the effect of a distortionary taxation system on the dynamic path of government's revenue, as well as the government's debt. For this purpose, we write the realized (due to distortions) tax revenue as $Z_t = T_t \cdot \varphi(\tau)$, where $\varphi(\tau) \in (0,1]$. Dividing by income, Y_t, and after the appropriate rearrangements[9] we write the budget constraint as:

$$b_t = \frac{\left(1+r_t\right)}{\left(1+n_t\right)} b_{t-1} + \left(g_t - z_t\right), \tag{2}$$

where tax revenue is given by $z_t = \tau_t \cdot \varphi(\tau)_t$, and n_t stands for the rate of growth of output.

Solving equation (2) forward, gives the intertemporal budget constraint

$$b_M = \left(\prod_{j=1}^{M}\left(R_j\right)\right) \cdot b_0 + \sum_{k=1}^{M}\left(\frac{\prod_{j=1}^{M}\left(R_j\right)}{\prod_{j=1}^{k}\left(R_j\right)}\left(g_k - z_k\right)\right) \tag{3}$$

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[9] The small letters stand for the capital letters divided by Y_t.

with $R_j = \dfrac{(1+r_j)}{(1+n_j)}$.

We are interested in investigating the way the dynamic path of debt is affected by the government's fiscal policy (i.e. by increases of the tax rate). Differentiating (3) with respect to tax rate, gives

$$\frac{\partial b_M}{\partial \tau_k} = \sum_{k=1}^{M}\left(\frac{\prod_{j=1}^{M}(R_j)}{\prod_{j=1}^{k}(R_j)}\left(\frac{\partial(g_k - z_k)}{\partial \tau_k}\right)\right) \Rightarrow \frac{\partial b_M}{\partial \tau_k} = -\sum_{k=1}^{M}\left(\frac{\prod_{j=1}^{M}(R_j)}{\prod_{j=1}^{k}(R_j)}\left(\frac{\partial z_k}{\partial \tau_k}\right)\right) \Rightarrow$$

$$\frac{\partial b_M}{\partial \tau_k} = -\sum_{k=1}^{M}\left(\frac{\prod_{j=1}^{M}(R_j)}{\prod_{j=1}^{k}(R_j)}\left(\varphi(\tau)_k + \tau_k \cdot \varphi'(\tau)_k\right)\right) \gtrless 0 \tag{4}$$

Integrating (4), for $\tau_0, \tau_1 \in (0,1]$ gives:

$$\int_{\tau_0}^{\tau_1} \frac{\partial b_M}{\partial \tau_\kappa} d\tau_\kappa = -\sum_{\kappa=1}^{M}\left[\frac{\prod_{j=1}^{M}(R_j)}{\prod_{j=1}^{k}(R_j)} \int_{\tau_0}^{\tau_1} \left(\varphi(\tau_\kappa) + \tau_\kappa \cdot \varphi'(\tau_\kappa)\right) d\tau_\kappa\right] \gtrless 0 \Rightarrow$$

$$\Delta b_M \equiv b_M(\tau_1) - b_M(\tau_0) = -\sum_{\kappa=1}^{M}\left[\frac{\prod_{j=1}^{M}(R_j)}{\prod_{j=1}^{k}(R_j)} \int_{\tau_0}^{\tau_1} \left(\varphi(\tau_\kappa) + \tau_\kappa \cdot \varphi'(\tau_\kappa)\right) d\tau_\kappa\right] \gtrless 0 \tag{5}$$

Since we have shown that $\varphi + \tau \cdot \varphi' \gtrless 0$, expression (5) provides a description that the dynamic path of debt is not monotonically related to increases of the tax rate. So, as the government wants to increase its tax revenue in order to pay-off the debt, it is the distortions of the tax system, which depend on the tax payer's behaviour, that result in the path of debt exhibiting a non linear property.

In most of the studies concerning sustainability issues of fiscal debt, the distortionary element of the taxation system is absent, (see for example Chalk and Hemming (2000)). Thus a general rule emerging from most of the analyses is that the debt follows a diminishing path if the government generates surplus. However, the qualitative element of our analysis contributes to the endogenous way distortions are introduced. In the presence of distortions, the path of debt depends on the ability of the fiscal authority to generate a surplus or to reduce the deficit and, hence to finance its debt by using its tax instrument. Higher surpluses, monotonically correspond to diminishing debt over time, when the taxation scheme refers to lump sum or to proportional taxation, as referred to in section 1, where increasing the tax rate causes the tax revenue to increase as well, so that to appropriately reduce the government's debt, without loss of revenue, a distortionary taxation system would actually imply. It is worth mentioning that when distortions are modelled explicitly, there is a dynamic process where changes in the tax rate affect (via the behaviour of tax payers) income (i.e. the tax base), affecting tax revenue, which, in our example, affects the dynamic path of debt.

Besides, the path of debt depends on the sign of $\varphi(\tau)_k + \tau_k \cdot \varphi'(\tau)_k \gtrless 0$ which, following the non linear property of the distortionary tax system, depends on the part of function φ or z on which the economy lies, or, alternatively, it depends on the magnitude of tax rate.

Figure 4 below, shows the dynamic path of debt related to a distortionary taxation scheme. As can be seen from this figure, the steady states of this dynamic path occur for $\tau = \tau_A$ and $\tau = \tau_\Delta$.

We have therefore shown that, distortions can be introduced in macroeconomic analysis when distortionary taxation is treated in the way we discussed in the previous sections. This way, one can overcome the restrictions imposed by the adoption of lump-sum or proportional taxation (also followed by the Ramsey approach) discussed in section 1, which also constitute the criticism of Golosov et al. (2006).

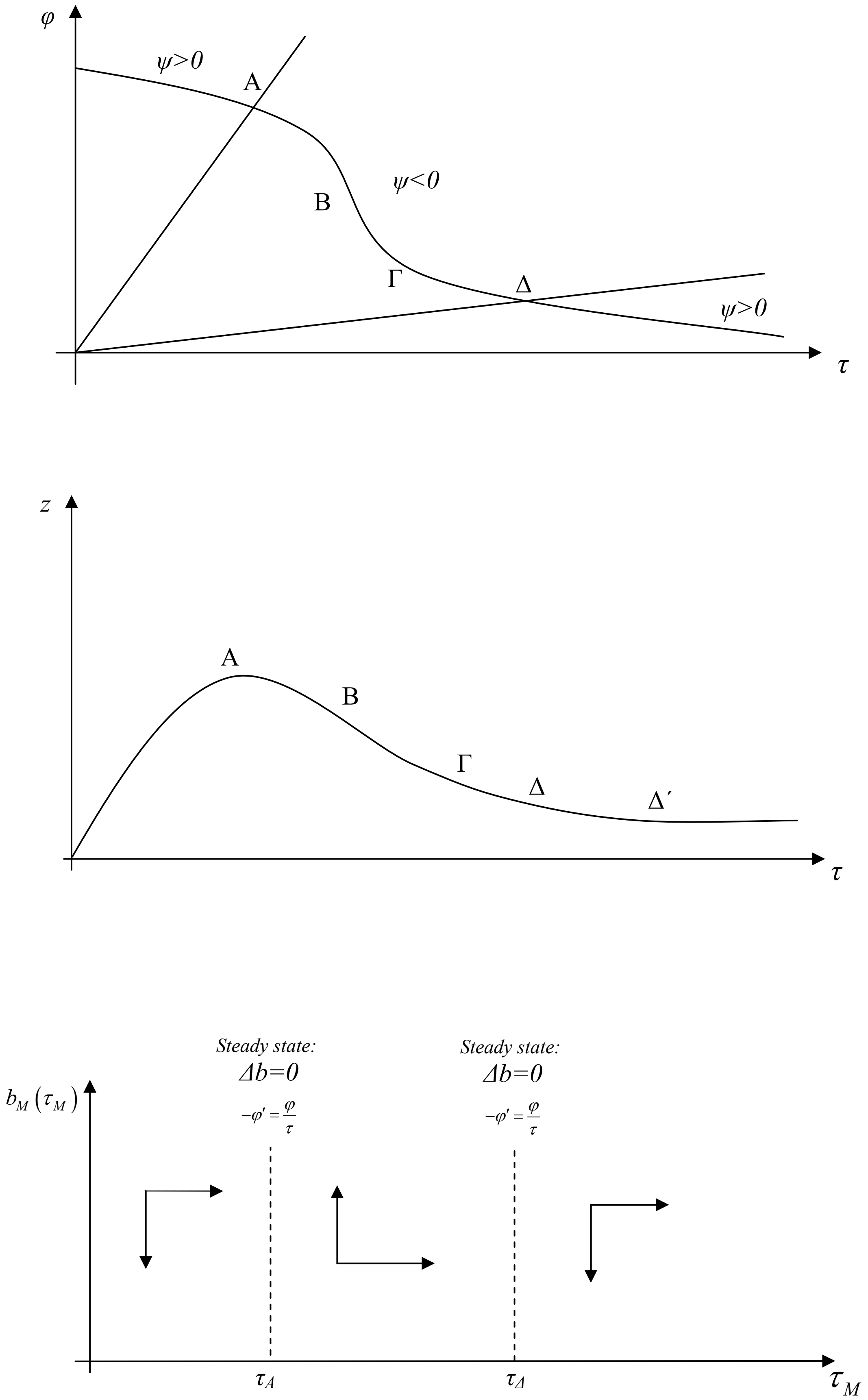
φ
ψ>0
A
B
ψ<0
Γ
Δ
ψ>0
τ
z
A
B
Γ
Δ
Δ′
τ
Steady state:
Δb=0
−φ′ = φ/τ
Steady state:
Δb=0
−φ′ = φ/τ
b_M(τ_M)
τ_A
τ_Δ
τ_M

Figure 4

5. Conclusion

The focus of our analysis was to build a methodological tool in order to deal with the problems arising when one is interested to include a distortionary taxation system in the analysis. As we argued, the consideration of a lump sum or a proportional taxation scheme as one that reflects a distortionary taxation system is subject to serious criticism and might lead to misleading results when analysing “optimal” economic policies in a macroeconomic framework. Our methodology provides a general functional form as a tool for capturing the distortionary consequences of the tax system. As a result, our analysis points out that when one considers a distortionary taxation system, the increase in the tax revenue due to an increase in tax rate, is not straightforward and is related to a non linear behaviour of the tax system. The behaviour of the tax payers (an important element in the distortionary taxation system) is a crucial ingredient of the path of tax revenue. This is also related to the dynamic path of government’s debt, creating conditions regarding its sustainability, since it is not independent of the dynamic path of tax revenue that is used in order to finance it. The non linear path of government’s debt is also rationalised.

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Appendix: income – substitution effects and labor supply

Proof

Consider the, commonly used in the literature[10], labor supply function of diagram A.1 bellow, that underlies a distortionary taxation system.

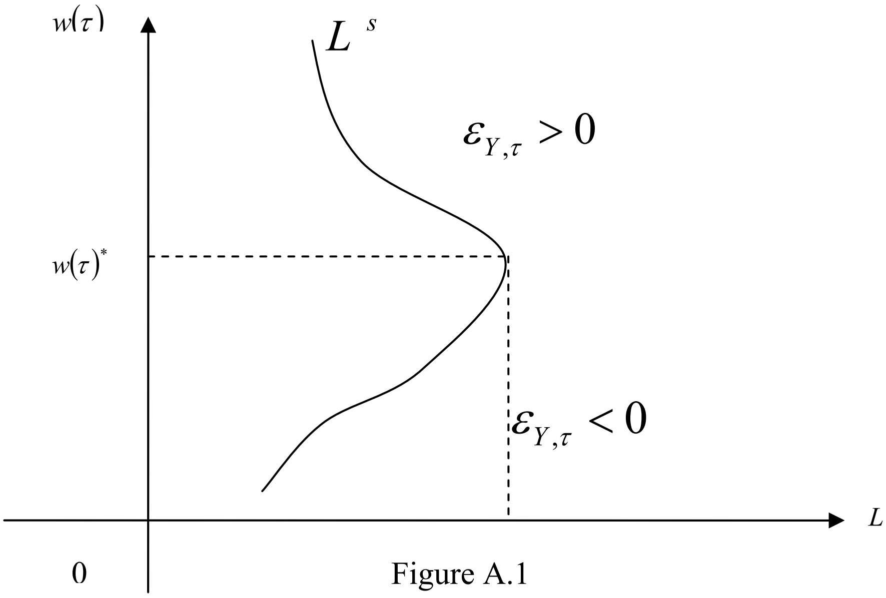

Figure A.1

Let $Y = \xi\left(L\left(W(\tau)\right)\right)$ be the production function where *W*, stands for wages, τ for the tax rate and *L* for the hours of work.

The relation between tax rate and output is given by

$$\frac{\partial Y}{\partial \tau} = \frac{\partial \xi}{\partial L} \cdot \frac{\partial L}{\partial W(\tau)} \cdot \frac{\partial W(\tau)}{\partial \tau} \Rightarrow$$
$$\frac{\partial Y}{\partial \tau} = \xi_L' \cdot \frac{\partial L}{\partial W(\tau)} \cdot \frac{\partial W(\tau)}{\partial \tau}$$

where $\xi_L' = w$ is the marginal productivity of labor. Multiplying both sides by $\frac{\tau}{Y}$, and after the appropriate rearrangements we get the elasticity of income with respect to taxes:

[10] See for example Stiglitz (1988) etc.

$$\varepsilon_{Y,\tau} = \tau \cdot \frac{W}{Y} \cdot \frac{\partial L}{\partial W(\tau)} \cdot \frac{\partial W(\tau)}{\partial \tau}.$$

Since it is non controversial that increases in taxes reduce wages, it follows that $\frac{\partial W(\tau)}{\partial \tau} < 0$. Consequently, when $\frac{\partial L}{\partial W(\tau)} < 0$, $\varepsilon_{Y,\tau}$ is greater than 0, indicating an "*income effect"*, consistent with wages greater than *w** and it $\varepsilon_{Y,\tau}$ is smaller than 0, when $\frac{\partial L}{\partial W(\tau)} > 0$ indicating a "*substitution effect"* for wages lower than *w**.

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