On modeling banking risk

E. G. Tsionas
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Abstract
The paper develops new indices of financial stability based on an explicit model of expected utility maximization by financial institutions subject to the classical technology restrictions of neoclassical production theory. The model can be estimated using standard econometric techniques, like GMM for dynamic panel data and latent factor analysis for the estimation of covariance matrices. An explicit functional form for the utility function is not needed and we show how measures of risk aversion and prudence (downside risk aversion) can be derived and estimated from the model. The model is estimated using data for Eurozone countries and we focus particularly on (i) the use of the modeling approach as an “early warning mechanism”, (ii) the bank- and country-specific estimates of risk aversion and prudence (downside risk aversion), and (iii) the derivation of a generalized measure of risk that relies on loan-price uncertainty.

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1 Introduction

Recent problems with the banking sector have put issues of modeling risk at the forefront of research. Traditional risk measures like the z-score or the standard deviation of the returns on assets (ROA) are used widely but have no formal economic justification. In turn, questions related to financial stability which are intimately related to risk, remain unanswered. As one analyst remarks: “The problems of creating a solvency test for financials that operate in real-time or relative real-time is not easy. It’s not hard to do after the fact, but at any one time, no one – not even the directors of the company really know what is embedded in the books at a financial. Société Générale, Barings, Northern Rock, Bear Stearns – the list of surprise blowup go on and on.” (Hui, 2008)

One problem with the z-score is, of course, the volatility measure used in the denominator. For example, Lepetit and Strobel (2013) “compare the different existing approaches to the construction of time-varying z-score measures, plus an additional alternative one, using a panel of banks for the G20 group of countries covering the period 1992–2009.” Their main finding is that the mean/standard deviation of ROA for the full sample with the current capital-asset ratio is the preferred measure. The use of the z-score as an indicator of bank stability has a long history, see for example de Nicolo (2000), Cihak (2007) and Maechler et al (2007).

Boyd & Graham (1986) and Boyd et al. (1993) have pointed out that \( \frac{1}{z^2} \) is an upper bound of the probability of insolvency (that is the probability of a bank having a negative capital asset ratio plus ROA) from which it follows that the z-score can be used in the wider context of insolvency, prudence and stability of financial institutions. Strobel (2013) made an excellent point in arguing that, in fact, \( \frac{1}{1+z^2} \) provided a tighter bound on the probability of insolvency while \( \frac{1}{z^2} \) provides a good upper bound on the odds of insolvency. Of course the two concepts are closely related as they are functionally related through a simple mathematical transformation.

For any random variable, \( X \), it is of course true that \( z = \frac{X - E(X)}{\sqrt{var(X)}} \) is another well defined random variable (if the first two moments exist) which can be related to the odds of insolvency in the case of financial institutions. The problems with such a catch-all measure, despite of course its apparent simplicity are many: (i) It does not account for the degree of risk aversion implicit in the construction of portfolios of financial institutions. (ii) It does not provide by itself an estimate, when used in practice, of the proper measure of variance; which is why in many studies panel data is used (see the

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1The author refers to the Altman measure.
excellent study by Lepetit and Strobel, 2013). (iii) Although insolvency is, apparently of interest, there is a transition stage to insolvency characterized by increasing “risk”. This “risk” is difficult to measure when using the z-score. (iv) Consequently, it becomes increasingly apparent that it is difficult to find a proper measure of “risk”, such as \( \text{var}(\% ) \), making the calculation of z-scores problematic and the construction of “early warning signals” quite difficult. For example, in time series studies, the measurement of variance does not allow, so far at least, for time-varying measures while in panel studies, repeated cross sections have to be used: Since it is implicitly assumed that the variance remains constant over time for the same financial institution, this introduces significant biases in the measurement of insolvency or the assessment of financial stability. (vi) The development of the literature on financial stability tends to move away from the simple z-score and adopts a wider perspective on “risk”. Of course, there is the related literature on identification of banking crises and construction of early warning mechanisms, see, for example, Berg (1999), Disyatat (2001), Demirguc-Kunt and Detragiache (2011), Kaminsky and Reinhart (1996, 1999), Logan (2000), and Vila (2000).

As stated in a seminal paper by Aspachs, Goodhart, Segoviano, Tsomocos and Zicchino (2006):

“In the ECB Financial Stability Review (December, 2005, p. 131), it is stated bluntly that “there is no obvious framework for summarising developments in financial stability in a single quantitative manner.” This is, to say the least, a considerable disadvantage when attempting to analyse financial stability issues. As the same ECB Special Feature on ‘Measurement Challenges in Assessing Financial Stability’, (ibid) put it, ‘Financial stability assessment as currently practiced by central banks and international organisations probably compares with the way monetary policy assessment was practised by central banks three or four decades ago – before there was a widely accepted, rigorous framework.”.’

The authors proceed to argue as follows:

“The point to note here is not so much the details [...> but that the crucial aspects of the impact of shocks on the banking system are contained in two variables, bank profitability and bank repayment rate, which in turn is equivalent to its probability of default (PD) ...” (page 8).

This paper falls squarely within this literature. We focus on a concept of risk directly related to the performance of loans. Since this cannot be known in advance there is genuine uncertainty about the performance of financial
institutions. This uncertainty must be taken formally into account in modeling properly the risk of financial institutions. In turn, this requires modeling the financial technology and embedding the problem in solid economic theory, provided by expected utility maximization. The framework of expected utility maximization goes, in fact, a long way in terms of measurement and assessment of risk, performance and stability. In this framework, we can provide explicit measures of Arrow-Pratt risk aversion as well as measures of downside risk aversion. Moreover, risk premia measures can be provided on a bank-specific basis. Hanschel and Monnin (2004) and Illing and Liu (2003) are examples of two papers that search for a metric of financial stability, without relying on an explicit structural model and focusing on the separate cases of Canada and Switzerland.

In this paper we propose a structural model (although not a general equilibrium model as in Aspachs et al, 2006). The essential features of the model are, however, the same as we rely on the metrics of bank profitability and the probability of default explicitly through the financial institution’s optimization problem. The model can be estimated using standard econometric techniques, like GMM for dynamic panel data and latent factor analysis for the estimation of covariance matrices. The model relies on expected utility maximization for a financial institution under uncertain loan prices. An important feature of the model is, of course, that it relies on the neoclassical approach to optimization with a well-defined technology set provided by its representation via a distance function. An explicit functional form for the utility function is not needed and we show how measures of risk aversion and prudence (downside risk aversion) can be derived and estimated from the model. The model is estimated using data for Eurozone countries and we focus particularly on: (i) the use of the modeling approach as an “early warning mechanism”, (ii) the bank- and country-specific estimates of risk aversion and prudence (downside risk aversion); and (iii) the derivation of a generalized measure of risk that relies on the loan-price uncertainty.

The remainder of the paper is organized as follows. The model and duality are presented in section 2. The primal approach and its disadvantages are discussed in section 3. Data and estimation techniques are presented in section 4. The empirical results are discussed in section 5. The paper concludes with a summary of the approach and results.

2 The model and duality

Suppose \( x \in \mathbb{R}_+^K \) is a vector of inputs, \( w \in \mathbb{R}_+^K \) is the vector of input prices, \( y \in \mathbb{R}_+^M \) a vector of outputs and \( p \in \mathbb{R}_+^M \) is the vector of their prices. We think of outputs as various types of loans while inputs include capital, labor etc.

The technology set is defined by:
\[ \mathcal{T} = \{(x, y) \in \mathbb{R}^{K+M} : x \text{ can produce } y \} \]

Suppose \((x, y) \in \mathcal{T}\) if and only if: \(F(x, y) \geq 1\) for a general transformation function \(F(x, y)\). The banks face uncertain output prices as some loans might not perform or, alternatively, perform to an unknown extent. Suppose

\[ p = \mu + C' \varepsilon \]

where \(\varepsilon \sim N_M(0, I)\) and \(C'C = \Sigma\). The profit of the bank is: \(\Pi = p'y - w'x - \kappa\) where \(\kappa\) denotes fixed costs -the use of which has been realized for the first time by Appelbaum and Ullah (1997)\(^3\). The bank maximizes the expected utility of profits: \(\mathcal{E}u(\Pi)\), where the expectation is taken with respect to \(\varepsilon\). Profits can be written as:

\[ \Pi = (\mu + C' \varepsilon)'y - w'x - \kappa = \mu_{\Pi} + \varepsilon'C y \]

where expected profit is:

\[ \mu_{\Pi} = \mu'y - w'x - \kappa \]

The problem of the bank can be restated as:

\[ V(w, \kappa, \mu, C) = \max_{(x,y)\in\mathbb{R}^{K+M}^+} : \mathcal{E}u(\mu_{\Pi} + \varepsilon'C y) , \text{ s.t } F(x, y) \geq 1 \]  

Using the envelope theorem we can establish the following properties of the value function:

\[ - \nabla_w V(w, \kappa, \mu, C) = \mathcal{E} u'(\Pi) \cdot x(w, \kappa, \mu, C) \]  

\[ - \frac{\partial V(w, \kappa, \mu, C)}{\partial \kappa} = \mathcal{E} u'(\Pi) \]

\[ \nabla_{\mu} V(w, \kappa, \mu, C) = \mathcal{E} u'(\Pi) \cdot y(w, \kappa, \mu, C) \]

where \(x(w, \kappa, \mu, C)\) and \(y(w, \kappa, \mu, C)\) denote the vectors of optimal inputs and outputs and \(\nabla_x\) denotes the gradient with respect to \(x\). The role of fixed costs, \(\kappa\), is to establish the expected value of the first derivative of the utility function in (5) although this is not strictly necessary as we can establish:

\(^3\)See papers by Kumbhakar (2002a,b), Kumbhakar and Tsionas (2010) and Kumbhakar and Tveteras (2013). These papers do not deal with the (arguably more difficult) case of multiple outputs and thus many output prices which is the focus of the present paper.
\[- \frac{\partial V(w, \kappa, \mu, C)}{\partial w_k} = \frac{x_k(w, \kappa, \mu, C)}{y_m(w, \kappa, \mu, C)} \]

which provides the optimal input-output ratios \((k = 1, \ldots, K, m = 1, \ldots, M)\).

If fixed costs are available then we can obtain immediately the input and output functions as:

\[ x_k(w, \kappa, \mu, C) = - \frac{\partial V(w, \kappa, \mu, C)}{\partial w_k}, k = 1, \ldots, K \]

\[ y_m(w, \kappa, \mu, C) = \frac{\partial V(w, \kappa, \mu, C)}{\partial \mu_k}, m = 1, \ldots, M \]

From the first-order conditions of the expected utility maximization problem we have:

\[ \frac{w_k}{w_1} = - \frac{\partial F(x, y)}{\partial x_k} \frac{\partial F(x, y)}{\partial x_1}, k = 2, \ldots, K \]

\[ \frac{\mu_m + \sigma_m \Lambda_m}{\mu_1 + \sigma_1 \Lambda_1} = \frac{\partial F(x, y)}{\partial y_m} \frac{\partial F(x, y)}{\partial y_1}, m = 2, \ldots, M \]

where \(\sigma^2_m\) is the \(m\)th diagonal element of \(\Sigma\) and \(\mu = [\mu_m, m = 1, \ldots, M]\).

Equation (10) shows that expected utility maximization requires cost minimization, as expected. In (11) we have defined the following expressions which will become evidently quite important in our discussion:

\[ \Lambda_m = \frac{\partial}{\partial u'(\Pi)} \left\{ u'(\Pi \varepsilon_m) \right\}, m = 1, \ldots, M \]

From (11) it is clear that expected utility maximization is consistent with ordinary profit maximization at relative output (or shadow) prices which are given by:

\[ \bar{p}_m = \frac{\mu_m + \sigma_m \Lambda_m}{\mu_1 + \sigma_1 \Lambda_1}, m = 2, \ldots, M \]

If, in fact, the transformation function is an output distance function, then exploiting its linear homogeneity with respect to outputs, we can establish (11) in a somewhat different form:

\[ \frac{\mu_m + \sigma_m \Lambda_m}{\sum_{m'=1}^{M} (\mu_{m'} + \sigma_{m'} \Lambda_{m'})} = \frac{\partial \log F(x, y)}{\partial \log y_m}, m = 1, \ldots, M \]

We denote \(\alpha = -\frac{u''(\mu \Pi)}{u'(\mu \Pi)}\) as the Arrow-Pratt measure of risk aversion and \(\delta = \frac{u''(\mu \Pi)}{u'(\mu \Pi)}\) as the measure of downside risk aversion or “prudence”. We have
\( \alpha > 0 \) and, normally, we would expect that \( \delta \geq 0 \). The left-hand-side of (14) would provide the virtual relative prices, \( \tilde{p}_m \) that would be consistent with classical profit maximization - but in this case prices are normalized so that they lie on the boundary of the unit simplex in \( \mathbb{R}^M \). Besides the first two moments of prices, these virtual prices depend on \( \Lambda_m \)'s which are related to the underlying utility function as in (12). Of course, we wish to avoid expressing the utility function in a specific functional form since that would make the analysis specific to the particular functional form. In that way, we see that what we have assumed so far is enough to deliver measures of Arrow-Pratt measures of risk aversion as well as downside risk aversion.

As we show in Appendix A, we have:

\[
\Lambda = -\frac{\alpha}{1 + \delta \text{tr}(yy^\prime \Sigma)} \cdot Cy
\]

which is an \( M \times 1 \) vector whose elements are the \( \Lambda_m \)'s. Therefore, the \( \Lambda_m \)'s can be related to two fundamental characteristics of risk, namely the Arrow-Pratt measures \( \alpha \) and \( \delta \). It is important to emphasize that these measures depend on underlying bank profitability (since the banks maximize expected utility of profit) as well “probability of default” in the sense that loan price uncertainty is explicitly taken into account.

If \( \delta = 0 \) or it can be ignored approximately, then knowing the \( \Lambda_m \)'s would identify \( \alpha \) from the above equation (15). If such an assumption cannot be made then, up to a factor of proportionality, the \( \Lambda_m \)'s can be identified either directly from the technology as in (14) or from (13) under the assumption that observed and virtual output prices coincide. The factor of proportionality induces a restriction which is either \( \tilde{p}_1 = \mu_1 + \sigma_1 \Lambda_1 = 1 \) in the first case, or \( \sum_{m=1}^M (\mu_{m'} + \sigma_{m'} \Lambda_{m'}) = 1 \) in the second. Both restrictions imply a relationship between \( \alpha \) and \( \delta \) from (15). In turn, (14) or (13) can be used to recover \( \Lambda_m \) and solve (15) for one of \( \alpha \) or \( \delta \) while the other can be recovered from the restriction. If we use the restriction to solve for \( \delta \) as a function of \( \alpha \) conditional on \( y \) and \( \Sigma \) then from (15) we would have a (very simple) system of \( M - 1 \) equations in the single unknown \( \alpha \). In Appendix A we show that the Envelope Theorem can be used with respect to the elements of Cholesky factorization of \( \Sigma \) to obtain separate information on \( E \{ u' (\Pi) \varepsilon_m \} \). If fixed costs are available, \( E u' (\Pi) \) can be obtained from (5) so, in principle, \( \alpha \) and \( \delta \) can be separately identified.

3 The primal approach

In this section we explore the implications of the primal or direct approach to the problem. The primal approach to the problem relies on using equations (10) and (11). From (11) we have:
\[ \frac{\tilde{p}_my_m}{\tilde{p}_1y_1} = \frac{\partial \log F(x, y)}{\partial \log y_m} / \frac{\partial \log F(x, y)}{\partial \log y_1}, m = 2, ..., M \] (16)

where \( \tilde{p}_m = \mu_m + \sigma_m \Lambda_m \). These equations can be written in the following form:

\[ \log \frac{\tilde{p}_m}{\tilde{p}_1} + \log y_m - \log y_1 = \log \left\{ \frac{\partial \log F(x, y)}{\partial \log y_m} / \frac{\partial \log F(x, y)}{\partial \log y_1} \right\} \] (17)

Suppose actual log relative prices satisfy:

\[ \log \frac{p_m}{p_1} = \log \frac{\tilde{p}_m}{\tilde{p}_1} + \zeta_m \] (18)

where \( \zeta_m \) reflects measurement error. Then, equation (17) can be reformulated as:

\[ \log \frac{p_m}{p_1} + \log y_m - \log y_1 = \log \left\{ \frac{\partial \log F(x, y)}{\partial \log y_m} / \frac{\partial \log F(x, y)}{\partial \log y_1} \right\} + \zeta_m \] (19)

This system of equation is not very useful since it requires output prices to estimate features of the technology whereas, in fact, the technology could, in principle, be estimated without assumptions about preferences and information about prices. However, since virtual prices are given by: \( \tilde{p}_m = \mu_m + \sigma_m \Lambda_m \) we have:

\[ p_m = \mu_m + \sigma_m \Lambda_m + \xi_m \] (20)

where \( \xi_m \) denotes possible deviations. From (15) we know that \( \Lambda_m \)'s depend on optimal decisions about inputs and outputs and thus on \( w, \kappa, \mu, \Sigma \) and also that \( \Lambda_m \leq 0 \). Although seemingly this gives rise to treat (20) as a system of equations with composite error terms, viz. the two-sided errors \( \xi_m \) and the one-sided components \( \Lambda_m \), matters are more complicated because from (15) the ratio of \( \Lambda_m \)'s has a particular form. For example, with two outputs and a lower triangular Cholesky factor \( C = \begin{bmatrix} c_1 & 0 \\ c_2 & c_3 \end{bmatrix} \) it is evident that:

\[ \frac{\Lambda_1}{\Lambda_2} = \frac{c_1 y_1 (w, \kappa, \mu, C)}{c_2 y_1 (w, \kappa, \mu, C) + c_3 y_2 (w, \kappa, \mu, C)} \] (21)

and we know it does not contain information about \( \alpha \) or \( \delta \). Moreover,

\[ \Lambda_1 = -\frac{\alpha}{1 + \delta \cdot \text{tr}(yy) \cdot \Sigma} \cdot c_1 y_1 \] (22)
so identification of both $\alpha$ and $\delta$ is not possible although $\alpha$ can be identified under the approximate (third-order) assumption that there is no downside risk aversion ($\delta = 0$). In this case we have:

$$\Lambda_1 = -\alpha \cdot c_1 y_1 (w, \kappa, \mu, C)$$

$$\Lambda_2 = -\alpha \cdot \{c_2 y_1 (w, \kappa, \mu, C) + c_3 y_2 (w, \kappa, \mu, C)\}$$

Using the first order conditions in share-equations form (see Appendix A) we have:

$$r_m \equiv \frac{\hat{p}_m y_m}{R} = \frac{RTS}{\sum_{k=1}^{K} \frac{\partial \log F(x, y)}{\partial \log x_k} \cdot \frac{\partial \log F(x, y)}{\partial \log y_m}}$$

which can be expressed in the form:

$$\log \hat{p}_m + \log y_m - \log TC = f_m (x, y; \theta) + v_m, \ m = 1, \ldots, M$$

where

$$f_m (x, y; \theta) \equiv \log \left( \frac{\partial \log F(x, y)}{\partial \log y_m} \right) - \log \sum_{k=1}^{K} \frac{\partial \log F(x, y)}{\partial \log x_k}$$

and $\theta \in \Theta \subset \mathbb{R}^p$ denotes the vector of unknown technology parameters. Moreover, $\hat{R} = \frac{TC}{RTS}$, $TC$ is total cost and RTS denotes and Panzar and Willig (1977) measure of returns to scale for multi-output production. Alternatively, we have:

$$\log \hat{p}_m + \log y_m - \log \hat{R} = \log r_m + v_m, \ m = 1, \ldots, M$$

where $v_m$s are standard (statistical) error terms. Identification of $\alpha$ and $\delta$ in this case relies on joint estimation of the system (10)-(11) where the left-hand-side shadow prices depend on $\hat{p}_m$ and thus on these two parameters.

### 4 Estimation techniques and data

For the $M \times 1$ vector of output prices $p$ we have observations for a given country ($c = 1, \ldots, C$) and a given bank within a country ($b = 1, \ldots, B_c$). So, in practice, the vector $p_{ct} = [p_{cb,t}]$ is possibly very high-dimensional for a given time period ($t = 1, \ldots, T$). This is especially the case because we want to combine countries (for example, peripheral countries or PIIGS versus non-periphery.) Moreover, in order to estimate the time-varying conditional covariance matrix of prices, $\Sigma_t$, we must account for the fact that they do
not have a constant conditional mean. Since we do not typically have large \( T \) (but we do have large \( N \)) we assume the following process:

\[ p_{cb,t} = a_b + A_c p_{cb,t-1} + u_{cb,t} \]  

(The vector \( p_{cb,t} \) is \( M \times 1 \) which is low-dimensional since in the leading case we have two or three outputs. Notation \( A_c \) means that estimation is performed using data for all banks in a given country \( c \). IN denotes “independent normal”, \( a_b \) is \( M \times 1 \) and \( A \) is matrix \( M \times M \). The process is a vector autoregression with a time-varying covariance matrix. The notation \( a_b \) means that we include bank-specific effects (dummy variables.) The model in (26) is estimated using data for a given country and all banks and time periods available for that country (or group of countries.) The model can be estimated using standard GMM techniques for dynamic panel data. We use moments of the type proposed in Arellano and Bover (1995) and Blundell and Bond (1999) [also Handbook of Econometrics, chapter 53].

The modeling of a dynamic covariance matrix in large dimensions is well known to be an exceedingly difficult matter and standard extensions of the GARCH model do not work well, see for example Engle and Kroner (1995) and Silvennoinen and Teräsvirta (2009).

Given the residuals \( \hat{u}_{cb,t} \) we follow Connor and Korajczyk (1986, 1988), Stock and Watson (2002), Bai and Ng (2002, 2011) and Bai (2003) to model the time-varying covariance matrix using a latent factor model with the principal components simplification, which has been shown to work well in practice (see in addition Bai and Li (2010), Forni et al (2000) and Lehmann and Modest (1988)). Suppose \( X_t = [\hat{u}_{c1,t}, ..., \hat{u}_{cB,t}]' \) for simplicity in notation. The latent factor model in vector form is

\[ X_{it} = \mu_i + \lambda_i' f_t + \varepsilon_{it} \]  

where the factors \( f_t \) (\( r \times 1 \)) and factor loadings \( \lambda_i \) (\( r \times 1 \)) are unobserved. Here, the index \( i \) corresponds to \((c,b)\). In matrix notation we have:

\[ X_t = \mu + \Lambda f_t + \varepsilon_t \]  

where \( \Lambda = [\lambda_1, ..., \lambda_N]' \) and \( \mu, \varepsilon_t \) are defined in conformable manner. The principal components estimator makes use of the decomposition

\(^4\text{See also Bai and Shi (2011) for the wider issues in high-dimensional settings, along with Bai and Ng (2008), Bai and Li (2010), Bai (2010), Chamberlain and Rothschild (1983), and Jones (2001).}

\(^5\text{Suppose } \hat{u}_t = [\hat{u}_{c1,t}, ..., \hat{u}_{cB,t}]'. \text{ It is to be noted that since } \hat{u}_t = u_t + (\hat{u}_t - u_t) = u_t + \varepsilon_t \text{ and } \varepsilon_t = o_p(1) \text{ the analysis carries through in the sense that the principal component analysis is asymptotically valid.}

\(^6\text{\( \Lambda \) is not to be confused with \( \Lambda_m \) s.} \)
\[
S = \sum_{i=1}^{N} b_i^2 h_i h'_i
\]

where \(S\) is the sample covariance matrix, \(b_i^2\) is the \(i\)th largest eigenvalue and \(h_i\) denotes the corresponding eigenvector (Bai and Shi, 2011). The principal components estimator for \(\Lambda\) is then given by

\[
\hat{\Lambda} = [b_1 h_1, \ldots, b_r h_r]
\]

which gives

\[
\hat{\Sigma} = \hat{\Lambda}\hat{\Lambda}' + \hat{\Omega}_\varepsilon
\]

where \(\hat{\Omega}_\varepsilon = \text{diag} \left( S - \hat{\Lambda}\hat{\Lambda}' \right)\) is the estimator for the diagonal covariance of the error terms \(\varepsilon_t\). The advantage of the estimator despite, of course, its simplicity is that it can be applied for a given time period, treating different banks as variables within a given country and a given time period, yielding consistent estimators for \(\hat{\Sigma}_t\) (Bai, 2003, 2004). There are a number of procedures to select the number of factors, \(r\), see Bai and Ng (2012) who proposed information criteria.

With values of \(\hat{\mu}_t\) and \(\hat{\Sigma}_t\), taken as given, next we specify a translog form for the output distance function \(F(x, y)\) which is estimated allowing for the endogeneity of \(y_s\) and \(x_s\) using GMM with instruments being log-relative prices and time dummies. The instruments are motivated by the first-order conditions of profit maximization. Our implementation of GMM is the so-called CUE (continuously-updated-estimator) which has been shown to have better finite sample properties. Joint estimation of the first-order conditions in (10)-(11) is quite difficult because of the dependence on \(\Lambda_m\)s which are highly nonlinear functions of \(\alpha\) and \(\delta\).

Next, we use the normalizing restriction \(\sum_{m=1}^{M} (\mu_m + \sigma_m \Lambda_m) = 1\) that output prices lie on the boundary of the unit simplex in \(\mathbb{R}^M\). As we mentioned before, the restriction implies a relationship between \(\alpha\) and \(\delta\) from (15). In turn, (14) or (13) can be used to recover \(\Lambda_m\) and solve (15) for one of \(\alpha\) or \(\delta\) while the other can be recovered from the restriction.

The data set includes commercial, cooperative, savings, investment and real-estate banks in Eurozone countries that are listed in the IBCA-Bankscope database over the period 2001–2011. After reviewing the data for reporting errors and other inconsistencies we obtain an unbalanced panel dataset of

\footnote{Another difficulty is that the first order conditions must be estimated in log form and we need to take account of the Jacobian of transformation due to endogeneity of the variables involved. Using GMM this is automatically taken into account.}

\footnote{The author is grateful to Ms. Natasha Koutsomanoli-Filippaki who generously provided the data set and its description.}
29,023 observations, which includes a total of 4,065 different banks. For the
definition of bank inputs and outputs, we follow the vast majority of the liter-
ature and employ the financial intermediation approach\textsuperscript{9} proposed by Sealey
and Lindley (1977), which assumes that the bank collects funds, using la-
bor and physical capital, and transforms them into loans and other earning
assets. In particular, we specify three inputs, labor, physical capital and
financial capital, and two outputs, loans and other earning assets (which in-
clude government securities, bonds, equity investments, CDs, T-bills, equity
investment etc.). With respect to input prices, the price of financial capi-
tal is computed by dividing total interest expenses by total interest bearing
borrowed funds, while the price of labor is defined as the ratio of personnel
expenses to total assets. Moreover, the price of physical capital is defined
as the ratio of other administrative expenses to fixed assets. Regarding the
calculation of output prices, the price of loans is defined as the ratio of in-
terest income to total loans, while the price of other earning assets is defined
as total non-interest income to total other earning assets\textsuperscript{10}.

The number of banks by year included in our sample is: 2001: 2,214;
2008: 2,990; 2009: 2,951; 2010: 2,949 and 2011: 2,818. The additions to the
sample are not necessarily new market entrants, but rather successful banks
that are added to the database over time. Exits from the sample are due
either to bank failures or to mergers with other banks or are a consequence
of changes in the coverage of the Bankscope database. Our sample covers
the largest credit institutions in each country, as defined by their balance
sheet aggregates. Due to the specific features of the German banking system
(large number of relatively small banks), our sample is dominated by German
banks.

Breaking the sample into periphery versus non-periphery as well as France
and Germany separately, we can apply the factor model in (28) for separate
samples whose dimensionality is less than the dimensionality of the full sam-
ple in terms of the number of banks. In turn we can apply the principal-
components estimator in (30) and (31). The principal-components estimator
has also been applied to each country separately following preliminary es-
timation of (28). For the factor models, the use of Schwarz information
criterion (Bai and Ng, 2012) which turned out to give one factor for the vast
majority of cases, including the groups of periphery, non-periphery as well
as France and Germany.

The output distance function is estimated separately for the groups of

\textsuperscript{9} For a review of the various approaches that have been proposed in the literature for
the definition of bank inputs and outputs see Berger and Humphrey (1992).

\textsuperscript{10} The Bankscope database reports published financial statements from banks worldwide,
homogenized into a global format, which are comparable across countries and therefore
suitable for a cross-country study. Nevertheless, it should be noted that all countries suffer
from the same survival bias.
peripheral countries or periphery, non-periphery as well as France and Germany, which are the focus of our attention in this paper. Estimation of the output distance function includes bank- and time-specific fixed effects (dummy variables) and is performed using the CUE\textsuperscript{11} version of GMM starting from a random selection of initial conditions for the parameters, centered around the OLS estimator. Random initial conditions are generated as \( \hat{\theta} + h\hat{V} \) where \( \hat{\theta} \) is the OLS estimator, \( \hat{V} \) its covariance matrix and \( h \) is a constant which, in practice, we set equal to 10. In total, for each case, we generate 1,000 initial conditions and we finally choose the estimates corresponding to the lowest value of the GMM criterion. Given an \( R \times 1 \) set of moment conditions \( \mathbb{E} g(\theta, \mathcal{Y}_i) = O_{(R \times 1)} \) where \( \theta \in \Theta \subseteq \mathbb{R}^k \), \( \mathcal{Y}_i \) denotes the data, the empirical analogue is \( ^{12} \):

\[
\min_{\theta \in \Theta} f(\theta) = \left[ N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) \right]' W \left[ N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) g(\theta, \mathcal{Y}_i) \right]^{-1} \left[ N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) \right] \tag{32}
\]

for some weighting matrix \( W \), where \( N \) denotes the number of available observations. In the first stage we set \( W = I \). Since the optimal weighting matrix is \( W^{-1} \propto \mathbb{E} g(\theta, \mathcal{Y}) g(\theta, \mathcal{Y})' \) the problem is re-solved using \( W^{-1} = N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) g(\theta, \mathcal{Y}_i) \). Therefore, the optimization problem for CUE-GMM is the following:

\[
\min_{\theta \in \Theta} f(\theta) = \left[ N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) \right]' \left[ N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) g(\theta, \mathcal{Y}_i) \right]^{-1} \left[ N^{-1} \sum_{i=1}^{N} g(\theta, \mathcal{Y}_i) \right]
\]

Given \( G = N^{-1} \sum_{i=1}^{N} \frac{\partial g(\theta, \mathcal{Y}_i)}{\partial \theta} \), the well-known asymptotic result can be used to obtain the asymptotic covariance matrix of the estimator:

\[
N^{1/2} \left( \hat{\theta}_{GMM} - \theta \right) \rightarrow \mathcal{N} \left( 0, [G'\Omega G]^{-1} \right)
\]

Moreover, Hansen’s \( J \)-statistic \( J = Nf(\theta) \rightarrow \chi^2_{R-k} \).

\textsuperscript{11}Continuously updated estimation.

\textsuperscript{12}We rely on moment conditions using as instruments the inputs and time as well as their squares and interactions plus country-specific dummy variables. Hansen’s \( J \) test has a \( p \)-value of 0.31 failing to reject the null hypothesis of orthogonality between output distance function errors and the specific instruments. Exclusion of country-specific dummy variables produces a Hansen test whose \( p \)-value is 0.001. The minimization problem is solved using a standard conjugate-gradients algorithm. More computational details are available on request.
5 Empirical results

The expected utility framework provides a wealth of information regarding risk in financial intermediation. First of all, we report results related to Arrow-Pratt measures of risk aversion ($\alpha$) and downside risk aversion ($\delta$). These measures are bank-specific as well as time-varying due to the solution of equation in (15). Second the use of a latent factor model as in (28) provides a generalized measure of risk which is given by the determinant $\det(\Sigma_t)$ or its log. This measure of risk relies on all output prices collectively. Although it reflects the ‘risk’ of the banking system as a whole it should be accompanied by considerations of risk aversion, formalized by estimates of $\alpha$ and $\delta$. This is important as there cannot be a single measure of “risk” without a reference framework provided by a behavioral assumption which, in this paper, is expected utility maximization. In that way, financial stability depends not only on underlying, statistical or econometric measures of risk like the z-score or even the generalized variance $\det(\Sigma_t)$ but rather on the propensity of the system to increase risk aversion and prudence (downside risk aversion) during periods of crises. Conversely, an increase of risk accompanied by an increase in risk aversion and prudence can show that a crisis is developing and can, at least in principle, provide us with an early warning mechanism.

There remains the question of whether measures of risk aversion and prudence are rather retrospective in nature since, for example, risk aversion can rise in response to a crisis. From the perspective of expected utility maximization and almost every other behavioral assumption based on optimization (excluding cost minimization) this cannot be the case. Since the bank has more information about its assets, capital and loan performance, it will react to a deterioration of its financial position by taking the appropriate measures, before a generalized crisis has taken place. The bank will not react to the generalized crisis but rather to its own deterioration of financial indices based on its own optimizations using its own information. Of course, distress signals from the entire banking sector will contribute to an increase of risk aversion and prudence, but this is the retrospective, not the prospective element lying at the heart of an increase in measures of prudence and risk aversion.

In Figures 1 and 2 we provide histograms of these key parameters across all financial institutions and years.

These distributions, which combine evidence from all countries and all time periods are clearly multimodal indicating at least that there is considerable heterogeneity either over time or across banking systems in different countries.

In Figure 3, estimates of risk aversion are reported for the European

\footnote{To avoid cluttering the paper, we present results in graphical form. All estimation results are available on request.}
banking system excluding the periphery (thick line), France, Germany and the periphery’s banking system. It is impressive that risk aversion in the system as a whole (excluding peripheral countries) started increasing before the sub-prime crisis. Yet, for peripheral countries this happened after the crisis had been developed. It remains important, however, that an early warning signal is indeed at work here as overall risk aversion for the system started increasing at least one year before the sub-prime crisis.

In Figure 4, measures of prudence (downside risk aversion) are reported. For the system as a whole (excluding the periphery) as well as for France and Germany prudence increases steadily throughout 2001-2011. For the periphery the 2000s start with negative downside risk aversion which increases to values around zero and ends up with values close to 1.5.

In Figure 5, the generalized risk (variance) measures are reported. These measures are increasing throughout the 2001-2011 period indicating the accumulation of risk resulting from the expansion of credit. Possibly as the result of adopting policies of fiscal restraint, the generalized risk especially for the periphery seems to decrease during 2010-2011 for the system as a whole (except the periphery) as well as France and Germany but notably not for the peripheral countries themselves.

In Figure 6, risk aversion coefficient (α) sample distributions are reported for the peripheral countries over time. These distributions result from bank- and of course country-specific measures. These distributions are evidently non-normal and show the evolution of risk aversion over time from, basically, lower to higher values. It is not until 2008 that the financial system in the peripheral countries starts to develop increasing aversion towards risk, that is in the middle or even after the sub-prime crisis. The distributions clearly shift to the right after 2007-2008 showing that the banking system adjusted with a considerable lag, contrary to the non-periphery whose (aspects of) sample distributions are summarily reported in Figure 3. For non-peripheral countries the distributions of risk aversion started shifting to the right as early as 2006 providing an “early warning mechanism” with regard to the following sub-prime crisis: We consider this an important aspect of the model in terms of modeling and forecasting financial stability.

In terms of interpreting our results, we feel that the following point is important: In the core countries it was the toxic assets that were the issue. The crisis stemmed from the banks. This is true for some peripheral countries as well, e.g. Spain. In peripheral countries such as Greece, for example, the crisis moves from the sovereign to the banks. This distinct aspect of the crisis, with its dichotomy between periphery versus non-periphery, provides an explanation for the findings in Figure 3 and onwards. It is, for example, well known that the financial turmoil in late 2007 resulting from the collapse of the mortgage market was due to the unprecedented issuance volume of credit default swaps (CDS) from 1998 to 2007 (Stanton and Wallace, 2011). Buch, Eckmeier, and Prieto (2014) analyzed a macroeconomic
vector autoregression for the United States with a set of factors summarizing conditions in about 1,500 commercial banks. Their main findings are as follows: “Backward-looking risk of a representative bank declines, and bank lending increases following expansionary shocks. Forward-looking risk increases following an expansionary monetary policy shock. There is, however, substantial heterogeneity in the transmission of macroeconomic shocks, which is due to bank size, capitalization, liquidity, risk, and the exposure to real estate and consumer loans”. Another channel through which substantial heterogeneity in the transmission of macroeconomic shocks can arise, is precisely because of, possibly substantial, heterogeneity in the degree of risk aversion which, in this paper, is the main focus of our modeling and estimation. We find that risk aversion as well as downside risk aversion vary over time. This leaves open the possibility that it is the risk attitudes of banks that are responsible for the heterogeneity in their responses instead of “forward-looking” or “backward-looking” risk. At any rate, through the application of simple estimation techniques, we are able to deliver bank-specific and time-varying estimates of important aspects of risk along with a generalized risk measure of the banking sector with a solid foundation in economic theory.

Concluding remarks

This paper has developed a new model of expected utility of profit maximization for financial institutions, subject to the neoclassical production possibility restrictions. The essential feature of the model is loan-price uncertainty in a multivariate context, an issue that has not been considered so far in the literature. The model can be estimated using standard econometric techniques: GMM for dynamic panel data along with latent factor analysis for the estimation of covariance matrices. An explicit functional form for the utility function is not needed and we show how measures of risk aversion and prudence (downside risk aversion) can be derived and estimated from the model. The model is estimated using data for Eurozone countries and we focus particularly on: (i) the use of the modeling approach as an “early warning mechanism”; (ii) the bank- and country-specific estimates of risk aversion and prudence (downside risk aversion); and (iii) the derivation of a generalized measure of risk that relies on loan-price uncertainty. The empirical results show that prudential behavior and risk aversion differ substantially among the periphery and the rest of the Eurozone, as well as compared to French and German banks. Risk aversion in the French and German began to increase well in advance of the sub-prime crisis. The same is true for the Eurozone excluding the periphery. For the periphery, risk aversion followed the sub-prime crisis and started increasing only after 2008. Our generalized measure of risk shows that risk has been building up in the Eurozone since
the early 2000s and is still at high levels although it has already begun to
decrease for the large financial sectors of France and Germany.

Appendix A.

Proof of equation (12).

Our purpose is, first of all, to derive expressions for 

\[ E_u'(\Pi) \]

as well as

\[ E\left\{ u'(\Pi) \varepsilon \right\} \]

where

\[ \Pi = \mu \Pi + \varepsilon' Cy \].

Using a Taylor approximation around \( \varepsilon = 0 \)

\( \text{M} \times 1 \) we have:

\[ u'(\Pi) = u' (\mu \Pi) + u'' (\mu \Pi) \varepsilon' Cy + u''' (\mu \Pi) \varepsilon' Cy' \varepsilon \]

Taking expected values we obtain:

\[ E_u'(\Pi) = u' (\mu \Pi) + u''' (\mu \Pi) E_{tr}(yy' C) = u' (\mu \Pi) + u''' (\mu \Pi) Tr(yy' \Sigma) \]

Moreover,

\[ E\left\{ u'(\Pi) \varepsilon \right\} = u'' (\mu \Pi) \varepsilon' Cy + u''' (\mu \Pi) E\left\{ [\varepsilon' Cy' C'] \varepsilon \right\} \]

By the symmetry assumption the last term is zero and therefore:

\[ E\left\{ u'(\Pi) \varepsilon \right\} = u'' (\mu \Pi) Cy \]

Using the definitions \( \alpha = -\frac{u''(\mu)}{u'(\mu)} \) and \( \delta = \frac{u'''(\mu)}{u''(\mu)} \), we obtain:

\[ \Lambda \equiv \frac{E\left\{ u'(\Pi) \varepsilon \right\}}{E\left\{ u'(\Pi) \right\}} = -\frac{\alpha}{1 + \delta Tr(yy' \Sigma)} \cdot Cy \]

Virtual Revenue

Given virtual prices \( \tilde{p}_m \) define virtual revenue as

\[ \tilde{R} = \sum_{m=1}^{M} \tilde{p}_m y_m (w, \kappa, \mu, C) \].

Consider the first order conditions from expected utility maximization:

\[ -\varepsilon u'(\Pi) \cdot w_k = \lambda \frac{\partial F(x, y)}{\partial x_k}, k = 1, ..., K \]

\[ \varepsilon \left\{ u'(\Pi) \cdot (\mu_m + \sigma_m \varepsilon_m) \right\} = \lambda \frac{\partial F(x, y)}{\partial y_m}, m = 1, ..., M \]

where \( \lambda \) is the Lagrange multiplier of the problem. Multiplying these
equations by the (optimal) \( x_k \) and \( y_m \) we have:

\[ -\varepsilon u'(\Pi) \cdot w_k = \lambda \frac{\partial F(x, y)}{\partial x_k}, k = 1, ..., K \]
\( \partial u' (\Pi) \cdot (\mu_m + \sigma_m \Lambda_m) y_m = \lambda \frac{\partial F (x, y)}{\partial y_m}, m = 1, \ldots, M \)

Summation yields:

\( -\partial u' (\Pi) \cdot TC = \lambda \sum_{k=1}^{K} \frac{\partial \log F (x, y)}{\partial \log x_k} \)

\( \partial u' (\Pi) \cdot \tilde{R} = \lambda \frac{\partial \log F (x, y)}{\partial \log y_m}, \quad m = 1, \ldots, M \)

where \( TC \) is total (variable) cost. Following Panzar and Willig (1977) we define returns to scale as: \( RTS = -\frac{\sum_{k=1}^{K} \partial \log F (x, y)/\partial \log x_k}{\sum_{m=1}^{M} \partial \log F (x, y)/\partial \log y_m} \). Using this definition we have:

\( \tilde{R} = \frac{TC}{RTS} \)

Since virtual prices are \( \mu_m + \sigma_m \Lambda_m = \tilde{p}_m \), and \( \frac{\lambda}{\partial u' (\Pi)} = -\frac{\sum_{k=1}^{K} \partial \log F (x, y)/\partial \log x_k}{\sum_{m=1}^{M} \partial \log F (x, y)/\partial \log y_m} \) (from the equations corresponding to cost minimization) the first order conditions with respect to outputs can be written as:

\( r_m = \frac{\tilde{p}_m y_m}{\tilde{R}} = -\frac{RTS}{\sum_{k=1}^{K} \partial \log F (x, y)/\partial \log x_k} \cdot \frac{\partial \log F (x, y)}{\partial \log y_m} \)

where \( r_m \) denotes the observed revenue share of the \( m \) th output provided virtual revenue is defined as above, which requires only total variable costs and RTS. The \( \Lambda_m \)'s can be obtained directly from revenue shares, \( r_m \).

**Identification of \( \alpha \) and \( \delta \) in the dual approach**

The question we take up here is whether \( \alpha \) and \( \delta \) can be separately identified from the \( \Lambda_m \)'s. Taking the derivative of the value function with respect to all elements of the Cholesky factorization of \( \Sigma \) and using standard matrix-differential calculus we have:

\[
\frac{\partial V (w, \kappa, \mu, C)}{\partial \text{vec} (C)} = \mathbb{E} \left\{ u' (\Pi) \cdot (\varepsilon \otimes y) \right\}
\]

For example, in the case of two outputs, assuming \( C \) is lower triangular, we would have:

\[
\frac{\partial V (w, \kappa, \mu, C)}{\partial \text{vec} (C)} = \mathbb{E} \left\{ u' (\Pi) \cdot \begin{bmatrix} \varepsilon_1 y_1 \\ \varepsilon_2 y_1 \\ 0 \\ \varepsilon_2 y_2 \end{bmatrix} \right\}
\]

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From this equation it is clear that we can obtain all expressions of the form:

\[ E \{ u'(\Pi) \varepsilon_m \} \cdot y_{m'}(w, \kappa, \mu, C), m, m' = 1, \ldots, M \]

Since

\[ \partial \{ u'(\Pi) \varepsilon \} = -\alpha u'(\mu_{\Pi}) Cy \]

it is evident that these equations are informative for \( \alpha \) but they depend on \( u'(\mu_{\Pi}) \). Moreover, the equations

\[ \partial u'(\Pi) = u'(\mu_{\Pi}) \{ 1 + \delta \text{tr} (yy') \} \]

are informative for \( \delta \) and, again, they depend on \( u'(\mu_{\Pi}) \).

Since utility is ordinal we can normalize: \( u'(\mu_{\Pi}) = 1 \) in which case \( \alpha \) and \( \delta \) can be obtained from the two equations above. The elements \( \partial \{ u'(\Pi) \varepsilon \} \) and \( \partial u'(\Pi) \) can be obtained from \( \partial V(w, \kappa, \mu, C) \) for \( \partial u'(\Pi) \) if we have fixed costs, \( \kappa \), and then \( \partial \{ u'(\Pi) \varepsilon \} \) can be obtained from \( \partial V(w, \kappa, \mu, C) \) as shown above. If all costs are variable, application of duality with respect to \( C \) can still be used to obtain \( \alpha \) and \( \delta \) can be obtained through (15).
References


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Figure 1. Sample distributions of measures of risk aversion
Figure 2. Sample distributions of measures of downside risk aversion
Figure 3. Risk aversion measures over time
Figure 4. Downside risk aversion measures over time
Figure 5. Generalized risk measures over time
Figure 6. Risk aversion sample distributions over time (Periphery)


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