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Diversification, integration and cryptocurrency market

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DIVERSIFICATION, INTEGRATION AND CRYPTOCURRENCY MARKET

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Abstract

We investigate the degree to which cryptocurrencies provide diversification benefits to an investor. We use a stochastic spanning methodology to construct optimal portfolios with and without cryptocurrencies, evaluating their comparative performance both in- and out-of-sample. Empirical analysis seems to indicate that the expanded investment universe with cryptocurrencies dominates the traditional one with stocks, bonds and cash, yielding potential diversification benefits and providing better investment opportunities for some risk averse investors. We further explain our results by documenting that cryptocurrency markets are segmented from the equity and bond markets.

JEL classification codes: C12, C14, D81, G11.

Keywords: Cryptocurrencies, Portfolio choice, Second Order Stochastic dominance, Stochastic Spanning, Diversification, Market Integration, Market Segmentation.

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1 Introduction

The emergence of cryptocurrencies has drawn significant investment capital in the recent years. The rates of increase in market capitalization, in volumes traded, as well as in the price of certain cryptocurrencies are exponential, indicating that changes in the cryptocurrency market can occur very rapidly. Bitcoin was the first cryptocurrency to appear in 2008 and currently has the highest market capitalization. Soon after its appearance several similar cryptocurrencies (collectively called “altcoins”) started to emerge: Litecoin was launched in 2011 and supports faster transaction confirmation. Additionally, other applications of the blockchain technology that incorporate a cryptocurrency have also emerged, the most dominant being Ethereum which was launched in 2015 and currently has the second highest market capitalization. Ethereum is an extended blockchain application that can record contracts and includes a built-in cryptocurrency called Ether, which is used to pay for contract execution. Another very interesting cryptocurrency is Ripple.

A cryptocurrency is produced by cryptographic algorithms. This is then transported across cyberspace using protocols such as peer-to peer networking. Most of the issues surrounding the successful adoption of cryptocurrencies are connected to the question of whether they are digital or virtual currencies, and as such, how their value is determined. Another compelling question is the issue of whether digital currencies should be considered to be currencies or digital assets. There is a new and emerging literature regarding cryptocurrencies, with most emphasis surrounding Bitcoin and much of the study has attempted to address the question of whether it is more analogous to a fiat versus commodity money. For example, Yermack (2015) claims that Bitcoin has no intrinsic value while lacks additional characteristics that are usually associated with currencies. Although the role of cryptocurrencies can still be disputed, they are certainly subject of rising awareness.

One possible explanation for investors’ interest in cryptocurrencies is their alleged diversification benefits. To have significant diversification benefits, the cryptocurrency returns need to have low-positive or negative correlations with the traditional asset classes like bonds and equities returns. In this paper we revisit this question while our main contribution is that we construct optimal portfolios and assess their performance in a non-parametric way. We manage to do so by employing a stochastic dominance (SD) approach.

The Mean-Variance (M-V) dominance criterion is questionable for portfolio selection, if investment returns are not normally distributed, or the utility functions are not quadratic. It is consistent with expected utility for elliptical distributions such as the normal distribution (Chamberlain, 1983; Owen and Rabinovitch, 1983; Berk, 1997) but has limited economic meaning when the probability distribution cannot be characterized completely by its location and scale. SD presents a further generalization that accounts for all moments of the return distributions without assuming a particular family of distributions. SD ranks investments based on general regularity conditions for decision making under risk (Quirk and Saposnik, 1962; Hadar and Russell, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970) and can be seen as a model-free alternative to M-V dominance. SD is a central theme in a wide variety of applications in economics, finance and statistics (for overviews and bibliographies see, Levy, 2015; Mosler and Scarsini, 2012) and the concept is used in numerous empirical studies in finance. Due to its non-parametric attractiveness, SD is particularly appealing for asset classes and investment strategies with asymmetric risk profiles, for example small-cap stocks, junk bonds, derivatives and momentum strategies.

SD is traditionally applied for comparing a pair of given prospects, for example, two income distributions or two medical treatments. Davidson and Duclos (2000), Barrett and Donald (2003) and Linton et al. (2005), among others, develop statistical tests for such pairwise comparisons. A more general, multivariate problem is that of testing whether a given prospect is stochastically efficient relative to all mixtures of a discrete set of alternatives (Bawa et al.,1985; Shalit and Yitzhaki, 1994; Post, 2003; Kuosmanen, 2004; Roman et al., 2006). This problem arises naturally in applications of portfolio theory and asset pricing theory, where the mixtures are portfolios of financial securities. Post and Versijp (2007), Scaillet and Topaloglou (2010), Linton et al. (2014) and Post and Poti (2017) address this problem using various statistical methods. Their stochastic efficiency tests can be seen as model-free alternatives to tests for M-V efficiency, such as the Shanken (1985, 1986) test (without a riskless asset) and the Gibbons et al. (1989) test (with a riskless asset).

In a similar manner, the concept of stochastic spanning, introduced by Arvanitis et al. (2017), can be perceived as a model-free alternative to M-V spanning (Huberman and Kandel, 1987). Spanning occurs if introducing new securities or relaxing investment constraints does not improve the investment possibility set *uniformly* over a given class of investor preferences. Stochastic spanning, unlike M-V spanning, accounts for higher-order moment risk in addition to variance. Higher-order moment risk is arguably more relevant for analyzing spanning than for efficiency. Efficiency tests are generally applied to a given broad market index with limited skewness and kurtosis (at the typical monthly to annual return frequency), in which case the arguments of Levy and Markowitz (1979) for the M-V approximation are compelling. By contrast, a spanning test evaluates all feasible portfolios, including those concentrated in a small number of risky securities, for which the same arguments are unlikely to hold. Given this motivation, and for the second order stochastic dominance relation, Arvanitis et al. (2017) propose a theoretical measure for stochastic spanning, derive the null limit distribution for the associated empirical test statistic and propose a relevant testing procedure based on subsampling.

In an empirical application using actual market data we test whether a portfolio set originating from a traditional asset universe spans the same set augmented with cryptocurrencies. This could be of significant importance to the empirical analysis of financial markets. If the hypothesis of stochastic spanning is not true, then this implies that the introduction of the new securities (in our case cryptocurrencies) or the relaxation of investment restrictions is beneficial for some investors in the given class. Our focus is on the most common SD criterion of second-order stochastic dominance (SSD), which has a well-established economic interpretation in terms of expected utility theory and Yaari's (1987) dual theory of risk. We employ the S&P 500 Total Return Index, the Barclays U.S. Aggregate Bond Index and the one-month Libor rate to proxy the traditional asset universe, i.e., equities, bonds and the risk-free rate. We also use daily and weekly prices for four cryptocurrencies namely Bitcoin, Ethereum, Ripple and Litecoin. We address our research question both in- and out-of-sample.

In the in-sample analysis, using the Arvanitis et al. (2017) stochastic spanning test, we find that the portfolios based on the traditional investment opportunity set do not span the corresponding portfolio strategies that include cryptocurrencies. In the out-of-sample analysis, at any point in time we construct optimal portfolios based separately on an asset universe comprising traditional asset classes and on an asset universe augmented with cryptocurrencies, in a rolling window fashion. We compare the real performance of these portfolios using the Davidson and Duclos (2013) non-parametric stochastic dominance test, as well as parametric performance measures. We find that the expanded investment universe with cryptocurrencies empirically dominates the traditional investment universe with stocks, bonds and cash, making the investor better off, w.r.t. to all the aforementioned performance criteria.

Hence, given the above, the main contribution of this paper is the statistical finding that, both in-sample and out-of-sample, the augmented portfolio with cryptocurrencies could be a good option for some riks averse investors to help diversify their portfolio risks.

The results are not surprising. For most of the second half of 2016, Bitcoin had been on a steady march higher, driven by a number of factors such as the devaluation of the yuan, geopolitical uncertainty and an increase interest of the professional investors. In January 3, 2017, Bitcoin broke the $1000 mark for the first time in three years. The majority of Bitcoin trading is done in China and so the devaluation of the yuan and fears over capital controls had a significant impact on the price of the digital currency. But several other factors have also had a notable impact, for example the U.S. election results in November

2016. In the first three quarters of 2017 there were numerous broad market developments with material impacts on the digital asset space. For example, the denial of the U.S. Securities and Exchange Commission (SEC) of an exchange-traded fund (ETF), the Bitcoin fork, Initial Coin Offerings (ICOs) legality and China's stance on digital assets all contributed to a whirlwind move for the markets. Bitcoin and Ether are used to purchase tokens for ICOs so a big part of Bitcoin's and Ether's surge was the ICO craze.

It is true that Bitcoin's rally attracts interest in alternative cryptocurrencies like Ethereum. It is also bringing a broader investment base. But while Bitcoin's rise increases investors' interest for Ethereum and other altcoins, Ethereum's popularity depends at the same time on a number of other factors, especially in the business and financial communities. Corporates have focused on the Ethereum wishing to use the technology for smart contract applications, i.e., contracts that automatically execute according to a computer algorithm when contract terms are met. Several financial institutions and large companies have invested in Ethereum technology and as a result in March 2017, the Enterprise Ethereum Alliance (EEA) was formed with the involvement of some of the most valuable companies in the world, including Microsoft, JP Morgan and IBM. On the other hand, Ripple is at times overwhelmingly independent and at others somewhat dependent to Bitcoin and Ethereum. Its relative independence from Chinese markets and the lack of involvements in ICOs shielded Ripple and the events in the digital asset space over the last year largely did not involve Ripple directly. Litecoin, however, reacted quite dramatic to all news from China since it is really popular among Chinese investors. Generally, in the last year Litecoin's bullish run coincides with other altcoins' soaring gains including Ripple and Ethereum.

Finally, we explain the documented diversification benefits of cryptocurrencies by using the approach of Cambell and Hamao (1992) to establish that cryptocurrency markets are segmented from equity and bond markets. More specifically, under diversification benefits and hence market segmentation, assets of the different markets are not priced by the same discount factor (see, for example, Ferson et al., 1993; Bekaert and Urias, 1996; De Roon et al., 2003). To our knowledge, there is no relevant literature providing evidence about the integration/segmentation of cryptocurrency markets with equity and bond markets. We find that cryptocurrency markets are segmented from equity and bond markets and exhibit characteristics of a unique asset class. We obtain data on a set of variables documented to forecast returns in equity and bond markets: the dividend yield, the default spread (Bessembinder and Chan, 1992), the term spread (Fama and French, 1989), the money supply growth (Chen, 2007) and the growth in the Baltic Dry Index (Bakshi et al., 2011).

The paper is organized as follows. In Section 2 we describe the stochastic spanning methodology. In Section 3 we provide the empirical application. We present the results from the in-sample and out-of-sample analysis. Section 4 presents the evidence on markets' segmentation. The final Section concludes the paper. We discuss the computational strategy in the Appendix.

2 Stochastic spanning

2.1 Preliminaries and null hypothesis

Let the investment opportunity set $\Lambda := \left\{\lambda \in \mathbb{R}_{+}^{N} : 1_{N}^{\mathrm{T}}\lambda = 1\right\}$, i.e. the N-simplex, which represents the set of convex combinations of N base assets. Those are not necessarily restricted to be individual securities but are generally defined as the vertices of Λ, which could in turn emerge as feasible combinations of some individual securities. The analysis considers the random investment returns of the base assets represented by the random vector $X := (x_1, \dots, x_N)$, with support bounded by $\mathscr{X}^N := [\underline{x}, \overline{x}]^N$, $-\infty < \underline{x} < \overline{x} < +\infty$. $\mathscr{X}$ can be chosen arbitrarily as a closed superset of the maximal support of the base assets. As in Arvanitis et al. (2017), to be realistic we do not allow for unbounded investment opportunities, because of the risk of financial ruin and the associated negative spill-overs to counterparties. For any realistic investment problem, private contracts, law and regulation may limit the investment possibilities. These restrictions will, for example, prohibit any risk neutral investor from borrowing an infinite amount of money and assuming an infinite and concentrated position in a single high-risk security.

Let F denote the continuous joint c.d.f. of X and $F(y,\lambda) := \int 1(X^{\mathrm{T}}\lambda \leq y)dF(X)$ the marginal c.d.f. for portfolio $\lambda \in \Lambda$. In order to define stochastic spanning, we use the following integrated c.d.f.:

$$F^{(2)}(x,\lambda) := \int_{-\infty}^{x} F(y,\lambda)dy = \int_{-\infty}^{x} (x-y)dF(y,\lambda). \tag{1}$$

This measure corresponds to Bawa's (1975) first-order lower-partial moment, or expected shortfall, for return threshold $x \in \mathscr{X}$.

This study focuses on the effects of changing the set of base assets or investment constraints. For this purpose, we introduce a non-empty polyhedral subset $\mathrm{K} \subset \Lambda$. A polyhedral structure is analytically convenient and can for example emerge if we remove some of the extreme points of Λ and/or further restrict Λ with appropriate linear constraints. We denote generic elements of Λ by λ, κ etc. We assume that $\lambda \in \Lambda$ and $\kappa \in \mathrm{K}$. Given the above we will give the definition of stochastic spanning of Arvanitis et. al. (2017) and we will descibe their measure for stochastic spanning.

Definition 1. (Stochastic Spanning): *Portfolio set* Λ *is second-order stochastically spanned by subset* $\mathrm{K} \subset \Lambda$ *if all portfolios* $\lambda \in \Lambda$ *are weakly second-order stochastically dominated by some portfolios* $\kappa \in \mathrm{K}$*:*

$$\forall \lambda \in \Lambda, \exists \kappa \in \mathrm{K} : \forall x \in \mathscr{X} : G(x, \kappa, \lambda; F) \leq 0$$
$$G(x, \lambda, \tau; F) := F^{(2)}(x, \lambda) - F^{(2)}(x, \tau). \tag{2}$$

From Arvanitis et al. (2017) the following scalar-valued functional of the population c.d.f. serves as a measure for deviations from stochastic spanning:

$$\eta(F) := \sup_{\lambda \in \Lambda} \inf_{\kappa \in \mathrm{K}} \sup_{x \in \mathscr{X}} G(x, \kappa, \lambda; F) \geq 0. \tag{3}$$

If $\eta(F) = 0$, then there is no feasible portfolio $\lambda \in \Lambda$ that is not weakly dominated by a portfolio $\kappa \in \mathrm{K}$. If $\eta(F) > 0$, then stochastic spanning does not occur.

According to the above definition K stochastically spans Λ iff any arbitrary element of the latter set is weakly dominated by some element of the former set. Using the utility class interpretation, this is equivalent to that for an arbitrary element of Λ there exists an element of K, weakly prefered to the former, by every risk averse utility. *Hence, by Proposition 2 of Arvanitis et al. (2017) the stochasic spanning measure can be reformulated in terms of expected utility as:*

$$\eta(F) = \sup_{\lambda \in \Lambda; u \in \mathscr{U}_2} \inf_{\kappa \in \mathrm{K}} \mathbb{E}_F \left[u\left(X^{\mathrm{T}} \lambda\right) - u\left(X^{\mathrm{T}} \kappa\right) \right]; \tag{4}$$

$$\mathscr{U}_2 := \left\{ u \in \mathscr{C}^0 : u(y) = \int_{\underline{x}}^{\overline{x}} w(x) r(y; x) dx \; w \in \mathscr{W} \right\}; \tag{5}$$

$$r(y; x) := (y - x) 1(y \leq x), \; (x, y) \in \mathscr{X}^2. \tag{6}$$

In this formulation, $\mathscr{U}_2$ is a set of normalized, increasing and concave utility functions that are constructed as convex mixtures of elementary Russell and Seo (1989) ramp functions $r(y; x)$, $x \in \mathscr{X}$. Stochastic spanning ($\eta(F) = 0$) occurs if no risk averter ($u \in \mathscr{U}_2$) benefits from the enlargement ($\Lambda - \mathrm{K}$). The representation of spanning in 4 is quite useful for the numerical implementation of the associated testing procedure derived in Arvanitis et al. (2017) (see Appendix for a detailed description of the implementation).

2.2 Statistical test and critical values

In empirical applications, the c.d.f. F is latent and the analyst has access to a discrete time series of realized returns $(X_t)_{t=1}^{T}, X_t \in \mathscr{X}$, t = 1, ..., T. Let $F_T(x) := T^{-1}\sum_{t=1}^{T} 1(X_t \leq x)$ denote the associated empirical c.d.f.. Given the above, Arvanitis et al. (2017) use the following scaled empirical analogue as a test statistic for stochastic spanning:

$$\eta_T := \sqrt{T} \sup_{\lambda \in \Lambda} \inf_{\kappa \in \mathrm{K}} \sup_{x \in \mathscr{X}} G(x, \kappa, \lambda; F_T).$$

We will use the test statistic η_T to test the null hypothesis of stochastic spanning. Arvanitis et al. (2017) establish the asymptotic distribution of the test statistic under the null hypothesis. The basic decision rule to reject $\mathbf{H}_0$ against $\mathbf{H}_1$ if and only if $\eta_T > q(\eta_\infty, 1-\alpha)$ where $q(\eta_\infty, 1-\alpha)$ denotes the $(1-\alpha)$ quantile of the distribution of η_∞ for any significance level $\alpha \in]0,1[$ and η_∞ is the relevant weak limit. However this is infeasible due to the dependence of $q(\eta_\infty, 1-\alpha)$ on the latent c.d.f. F as well as on the temporal dependence of the returns process. However, feasible decision rules can be obtained by using a subsampling procedure to estimate $q(\eta_\infty, 1-\alpha)$ from the data.

The subsampling procedure, begins by generating $(T - b_T + 1)$ maximally overlapping subsamples of $(X_s)_{s=t}^{t+b_T-1}$, $t = 1, \cdots, T - b_T + 1$, and then evaluates the test statistic on each subsample value thereby obtaining $\eta_{b_T;T,t}$ for $t = 1, \cdots, T - b_T + 1$. The empirical distribution of subsample test scores can be described by the following c.d.f. and quantile function:

$$s_{T,b_T}(y) := \frac{1}{T - b_T + 1} \sum_{t=1}^{T-b_T+1} 1(\eta_{b_T;T,t} \leq y); \tag{7}$$

$$q_{T,b_T}(1-\alpha) := \inf_y \left\{ s_{T,b_T}(y) \geq 1-\alpha \right\}. \tag{8}$$

We reject $\mathbf{H}_0$ if and only if $\eta_T > q_{T,b_T}(1-\alpha)$. This subsampling routine is asymptotically exact and consistent under reasonable assumptions on the subsample length and significance level (see Online Appendix B in Arvanitis et al., 2017).

Although the test has asymptotically correct size the quantile estimates $q_{T,b_T}(1-\alpha)$ may be biased and very sensitive to the choice of subsample size b_T in finite samples of realistic dimensions (N and T). Arvatitis et al. (2017) propose a correction procedure via a regression-based method. For a given significance level α, the quantiles $q_{T,b_T}(1-\alpha)$ are evaluated for a range of the subsample size b_T. Next, the intercept and slope of the following regression line using OLS regression analysis are estimated:

$$q_{T,b_T}(1-\alpha) = \gamma_{0;T,1-\alpha} + \gamma_{1;T,1-\alpha}(b_T)^{-1} + \nu_{T;1-\alpha,b_T}. \tag{9}$$

Finally, the bias-corrected $(1-\alpha)$-quantile is evaluated as the OLS predicted value for $b_T = T$:

$$q_T^{BC}(1-\alpha) := \hat{\gamma}_{0;T,1-\alpha} + \hat{\gamma}_{1;T,1-\alpha}(T)^{-1}. \tag{10}$$

Arvanitis et al. (2017) argue that the asymptotic properties are not affected, while computational experiments show that the bias-corrected method is more efficient and more powerful in small samples.

3 Empirical application

In the empirical application we test whether the inclusion of cryptocurrencies in the asset universe could make some risk averse investors better off compared to the case where the asset universe consists of only traditional asset classes (stocks, bonds and cash). We address our research question both in- and out-of-sample.

We use data on daily closing prices of a number of indices obtained from Bloomberg. We employ the *S&P* 500 Total Return Index, the Barclays U.S. Aggregate Bond Index and the one-month Libor rate to proxy the traditional asset universe, i.e., the equity market, the bond market and the risk-free rate, respectively. To access the cryptocurrencies asset class, we use daily data on Bitcoin, Ethereum, Ripple and Litecoin US dollar closing prices extracted from the Bitfinex exchange market through the CoinMarketCap. Ether was first publiclly traded in July 2015 and data availability for cryptocurrencies before that date was an issue. So, the dataset spans the period from August 7, 2015 to December 29, 2017, a total of 604 daily return observations.

Table 1 reports summary statistics regarding the performance of the employed assets over this period. We can see that the daily average return of cryptocurrencies is higher than that of stocks and bonds. The Sharpe ratio of cryptocurrencies is also considerably higher. Moreover, cryptocurrencies exhibit considerable standard deviation, compared to the mean. Skewness and kurtosis are extremely high, indicating large deviations from normality. Although the one-month Libor rate is not constant, its daily return is calculated and is considered constant, therefore the standard deviation is zero, and we do not calculate skewness and kurtosis.

3.1 In-sample analysis

In this section we test in-sample the null hypothesis that the traditional asset class spans the augmented with cryptocurrencies asset universe. We get the subsampling distribution of the test statistic for subsample size $b_T \in [120, 240, 360, 480]$. Using OLS regression on the empirical quantiles $q_{T,b_T}(1-\alpha)$ and for significance level $\alpha = 0.05$, we get the estimate q_T^{BC} for the critical value.

We find that the regression estimate $q_T^{BC} = 0.3408$ is lower than the value of the test statistic 0.34834. Thus, we reject the hypothesis that the traditional asset class spans the augmented asset class with cryptocurrencies.

In order to validate our results, we carry out an additional test. Particularly, we test whether the results on the outperformance of cryptocurrencies are robust to the choice of traditional asset universe. To this end, we include both the S&P 500 and dynamic trading strategy (i.e. SMB or HML) into the traditional asset universe. Moreover, we include the Russell 2000 equity index, the Vanguard value and the Vanguard small-cap index funds.

We find that the regression estimate $q_T^{BC} = 0.33870$ is again lower than the value of the test statistic 0.34834.

We can see that in any case the optimal portfolios based on the investment opportunity set that includes cryptocurrencies are not spanned by the corresponding optimal portfolio strategies based on the traditional investment opportunity set.

The results of this in-sample-analysis indicate that the performance of traditional portfolios, consisting of stocks, bonds and cash, can be improved by including cryptocurrencies. Thus, some risk averse investors could benefit from the augmentation.

3.2 Out-of-sample analysis

In this section, we examine whether cryptocurrencies could provide diversification benefits out-of-sample. Although in the in-sample tests we reject the null hypothesis of stochastic spanning, it is not known a priori whether the augmented with cryptocurrencies portfolios will outperform the traditional ones in an out-of-sample setting. This is because by construction these portfolios are formed at time t based on the information prevailing at time t, while the portfolio returns are reaped over $[t, t+1]$ (next day). The out-of-sample test is a real-time exercise mimicking the way that a real-time investor acts.

We form optimal portfolios separately for two asset universes: one that includes traditional asset classes, i.e., equities, bonds and risk-free asset and an augmented one with cryptocurrencies. We resort to backtesting experiments on a rolling horizon basis. The rolling horizon simulations cover the 604 working day period from 08/07/2015 to 12/29/2017. At each day, we use the data from the previous year (269 daily observations) to calibrate the procedure. We solve the resulting optimization problem for the stochastic spanning test and record the optimal portfolio of the traditional assets as well as the optimal portfolio of the traditional assets and the cryptocurrencies. The clock is advanced and the realized returns of the optimal portfolios are determined from the actual returns of the various assets. The same procedure is then repeated for the next time period and the ex post realized returns over the period from 09/01/2016 to 29/12/2017 (334 working days) are computed for both portfolios.

Figure 1 illustrates the cumulative performance of the traditional optimal portfolio as well as the augmented with cryptocurrencies optimal portfolio for the sample period from 09/01/2016 to 12/29/2017. We observe that the augmented optimal portfolio has more than 307 times higher value at the end of the holding period compared to the beginning, while the traditional portfolio has only 1.265 times higher value. Not surprisingly, the relevant performance of portfolios with cryptocurrencies is 242 times higher compared to the equity market, the bond index and the Libor 1-month. The optimal augmented portfolio includes Ethereum and Ripple, and small amount in Litecoin, but none of the traditional assets.

We repeat the backtesting experiment extending the traditional asset class with the SMB and HML indices, as well as the Russell 2000 equity index, the Vanguard value and the Vanguard small-cap index funds, in the same way as we did in the in-sample analysis. Figure 2 exhibits the cumulative performance of both the traditional optimal portfolio and the optimal augmented portfolio for the sample period from 09/01/2016 to 29/12/2017. The same observations hold here, i.e., the value of the augmented optimal portfolio is more than 307 times higher at the end of the holding period while the traditional portfolio has only 1.348 times higher value. Again, the relevant performance of the portfolio with cryptocurrencies is 228 times higher than the performance of the equity indices, the bond index and the Libor 1-month. The optimal augmented portfolio is the same as before.

3.3 Out-of-sample performance assessment

In this section, we compare the out-of-sample performance of the two optimal portfolios formed by the respective two asset universes by using both non-parametric and parametric tests.

3.3.1 *Non-parametric tests*

There is a number of pairwise stochastic dominance tests presented in the literature; see, for example Barret and Donald (2003), Davidson and Duclos (2000), Linton et al. (2005) and Davidson and Duclos (2013). Here we prefer to use the Davidson and Duclos

(2013) stochastic dominance test, mainly for two reasons. First, the test allows for correlated samples. Second, the Davidson and Duclos (2013) test has as null hypothesis that one portfolio does not stochastically dominate another, i.e., the nondominance. The majority of stochastic dominance tests posit the null of dominance. But rejecting dominance of one portfolio does not necessarily imply that the other one is stochastically dominant. However, under the Davidson and Duclos (2013) test, rejecting the nondominance of one portfolio, leaves us with the only remaining alternative, i.e., dominance. Adding up to this idea, we test the null of nondominance of the optimal portfolio based on stocks, bonds, cash and cryptocurrencies over the traditional asset universe. The alternative hypothesis is that the augmented asset universe based optimal portfolio stochastically dominates the traditional asset based optimal portfolio. We test the null hypothesis using second order stochastic dominance criteria[1].

We define the empirical versions of the dominance functions D^2_{Tr} and D^2_{Aug} (see Davidson and Duclos (2000) for the definition of dominance functions) as,

$$\hat{D}^2_{Tr} = \frac{1}{T}\sum_{t=1}^{T}(\max(z-y_t,0)), \tag{11}$$

where T is the number of observations in the distribution sample of traditional portfolio returns, y_t is the t -th observation, and z is the threshold of interest. Analogously, we define the dominance function for the augmented portfolio (Aug).

Then the augmented portfolio dominates at second order the traditional portfolio iff $\hat{D}^2_{Tr} > \hat{D}^2_{Aug}$ for all y in the joint support. The null hypothesis of nondominance is not rejected unless there is dominance in the sample.

For each threshold level z, let the standardized difference of the two dominance functions,

$$t(z) = \frac{\hat{D}^2_{Tr} - \hat{D}^2_{Aug}}{(Var(\hat{D}^2_{Tr}) + Var(\hat{D}^2_{Aug}) - 2Cov(\hat{D}^2_{Tr}, \hat{D}^2_{Aug}))^{1/2}}, \tag{12}$$

and the resulting test statistic,

$$t^* = \min_z t(z). \tag{13}$$

In order to simulate the p-values we use the bootstrap methodology described in Davidson and Duclos (2013). Notice that the relevant limiting results of Davidson and Duclos (2013) could be justified in the framework of rebalanced optimal portfolios via results such as Theorem 4.4.2 of Politis et al. (1999). The results entail $T-1$, i.e., 233 overlapping periods for the in-sample fitting of the two portfolios with corresponding out-

[1]Hodder et al. (2015) use the same stochastic dominance test to test the out-of-sample performance of alternative portfolios compared to a benchmark portfolio.

of-sample comparisons. The 233 p-values are considered from September 1, 2016 to December 29, 2017, using overlapping periods of 100 daily returns. Since it is impossible to aggregate the $T-1$ values of the Davidson and Duclos (2013) test statistic to get a unique measure of comparison, we compute quartile p-values from the distribution of the $T-1$ test statistics.

Table 2 reports the quartile p-values from the distribution of daily portfolio returns, under the null hypothesis that the augmented portfolio does not dominate the traditional one. Panel A considers the case when the traditional set includes the S&P 500 Equity Index, the Barclays US Aggregate Bond Index and the 1-month Libor, while Panel B considers the case when the traditional set includes additionally the SMB and HML indices, as well as the Russell 2000 equity index, the Vanguard value and the Vanguard small-cap index funds. We observe that for the 25% and 50% quartile p-values, we tend to reject the null hypothesis that the augmented optimal portfolio does not dominate the traditional one in all cases, with respect to the second order stochastic dominance criterion.

The results indicate that the null hypothesis of nondominance over the traditional portfolio can be rejected even for 50% of daily tests for both cases.

3.3.2 *Parametric tests*

We compute a number of commonly used parametric performance measures: the Sharpe ratio, the downside Sharpe ratio of Ziemba (2005), the upside potential and downside risk (UP) ratio of Sortino and van den Meer (1991), the opportunity cost, the portfolio turnover and a measure of the portfolio risk-adjusted returns net of transaction costs. The downside Sharpe and UP ratios are considerd to be more appropriate measures of performance than the typical Sharpe ratio given the asymmetric return distribution of cryptocurrencies.

For the downside Sharpe ratio, first we need to calculate the downside variance (or more precisely the downside risk),

$$\sigma_{P\_}^2 = \frac{\sum_{t=1}^{T}(x_t - \bar{x})_{-}^2}{T-1}, \tag{14}$$

where the benchmark $\bar{x}$ is zero, and the x_t taken are those returns of portfolio P at day t below $\bar{x}$, i.e. those t of the T days with losses. To get the total variance, we use twice the downside variance namely $2\sigma_{P\_}^2$ so that the dowside Sharpe ratio is,

$$S_P = \frac{\bar{R}_p - \bar{R}_f}{\sqrt{2}\sigma_{P\_}}, \tag{15}$$

where $\bar{R}_p$ is the average period return of portfolio P and $\bar{R}_f$ is the average risk free rate. The UP ratio compares the upside potential to the shortfall risk over a specific target (benchmark) and is computed as follows. Let R_t be the realized daily return of portfolio P for $t = 1,...,T$ of the backtesting period, where $T = 252$ is the number of experiments performed and let ρ_t be respecitvely the return of the benchmark (T-bills riskless asset) for the same period. Then, we have,

$$\text{UP ratio} = \frac{\frac{1}{K}\sum_{t=1}^{K}\max[0, R_t - \rho_t]}{\sqrt{\frac{1}{K}\sum_{t=1}^{K}(\max[0, \rho_t - R_t])^2}}. \tag{16}$$

It is obvious that the numerator of the above ratio is the average excess return over the benchmark and so reflects upside potential. In the same way, the denominator measures downside risk, i.e. shortfall risk over the benchmark.

Next, we use the concept of opportunity cost presented in Simaan (2013) to analyse the economic significance of the performance difference of the two optimal portfolios. Let R_{Aug} and R_{Tr} be the realized returns of the optimal portfolio for the augmented and the traditional asset class respectively. Then, the opportunity cost θ is defined as the return that needs to be added to (or subtracted from) the traditional portfolio return R_{Tr}, so that the investor is indifferent (in utility terms) between the strategies imposed by the two different investment opportunity sets, i.e.,

$$E[U(1 + R_{Tr} + \theta)] = E[U(1 + R_{Aug})]. \tag{17}$$

A positive (negative) opportunity cost implies that the investor is better (worse) off if the investment opportunity set allows for cryptocurrency investing. Notice that the opportunity cost takes into account the entire probability density function of asset returns and hence it is suitable to evaluate strategies even when the assets' return distribution is not normal. For the calculation of the opportunity cost, we use exponential and power utility functions alternatively, consistent with second degree stochastic dominance. We also employ alternative values for the risk aversion parameter.

Next, we compute the portfolio turnover (PT) to get a feeling of the degree of rebalancing required to implement each one of the two strategies. For any portfolio strategy P, the portfolio turnover is defined as the average of the absolute change of weights over the T rebalancing points in time and across the N available assets, i.e.,

$$PT = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}(|w_{P,i,t+1} - w_{P,i,t}|), \tag{18}$$

where $w_{P,i,t+1}$ and $w_{P,i,t}$ are the optimal weights of asset i under strategy P (traditional or augmented) at time t and $t+1$, respectively.

Finally, we evaluate the performance of the two portfolios under the risk-adjusted (net of transaction costs) returns measure, proposed by DeMiguel et al. (2009) which indicates the way that the proportional transaction cost, generated by the portfolio turnover, affects the portfolio returns. Let trc be the proportional transaction cost, and $R_{P,t+1}$ the realized return of portfolio P at time $t+1$. The change in the net of transaction cost wealth NW_P of portfolio P through time is,

$$NW_{P,t+1} = NW_{P,t}(1+R_{P,t+1})[1-trc\times\sum_{i=1}^{N}(|w_{P,i,t+1}-w_{P,i,t}|). \tag{19}$$

The portfolio return, net of transaction costs is defined as

$$RTC_{P,t+1} = \frac{NW_{P,t+1}}{NW_{P,t}} - 1. \tag{20}$$

Let μ_{Tr} and μ_{Aug} be the out-of-sample mean of monthly RTC with the restricted and the expanded opportunity set, respectively, and σ_{Tr} and σ_{Aug} be the corresponding standard deviations. Then, the return-loss measure is,

$$R_{Loss} = \frac{\mu_{Aug}}{\sigma_{Aug}}\times\sigma_{Tr} - \mu_{Tr}, \tag{21}$$

i.e., the additional return needed so that the traditional portfolio performs equally well with the augmented portfolio. We follow the literature and use 35 bps for the transaction cost of stocks and bonds.

In the cryptocurrency market, transaction costs usually occur in two ways, namely trading fees and the bid-ask spread. Bid-ask spread is a type of risk premium to compensate market dealer for providing liquidity. To execute a transaction, investor should pay additional premium to the exchange. And usually, exchange will ask for a high premium to reduce loss by providing liquidity to the informed traders. But overall, the bid-ask spread is minor issue compared to the normal price deviation. In contrast, other fees create more frictions for investors to make arbitrage. For example, BTC-E charges a 0.2 to 0.5 percent fee per transaction along with fees to deposit or withdraw traditional currency. According to CryptoCoins News, there is currently a $20 fee for a wire deposit. Bitstamp and Bitfinex also charge trading fees and deposit/withdrawal/fees. In order to confirm the out-of-sample superiority of the augmented portfolio, we decided to use higher transaction costs for cryptocurrencies than for stocks and bonds, i.e., 50 bps.

Table 3 reports the parametric performance measures for the traditional and the augmented portfolios (Panels A and B respectively). These performance measures, although parametric, will supplement the evidence obtained from the previously discussed non-parametric stochastic dominace measure. The higher the value of each one of these measures, the greater the investment opportunities for cryptocurrencies. From the results, we can see that the inclusion of cryprocurrencies into the opportunity set increases both the Sharpe ratios and the downside Sharpe ratios. This reflects an increase in risk-adjusted performance (i.e., an increase in expected return per unit of risk) and hence expands the investment opportunities of some risk-averse investors. The same is true for the UP ratio. Furthermore, we observe that portfolios with only traditional assets induce more portfolio turnover than the ones with cryptocurrencies, regardless of the choice of the traditional assets. Additionally, we can see that the return-loss measure, that takes into account transaction costs, is positive in both cases. We find a positive opportunity cost θ. One needs to give a positive return equal to θ to an investor who does not include cryptocurrencies in her portfolio so that she becomes as happy as an investor who includes cryptocurrencies. The computation of the opportunity cost requires the computation of the expected utility and hence the use of the probability density function of portfolio returns. Thus, the calculated opportunity cost has taken into account the higher order moments in contrast to the Sharpe ratios. Therefore, the opportunity cost estimates provide further convincing evidence for possible diversification benefits from the inclusion of cryptocurrencies given the large deviation from normality.

To sum up, the non-parametric stochastic dominance test, as well as the employed parametric performance measures, indicate that when the investment universe is augmented with cryptocurrencies it empirically dominates the traditional one, yielding potential diversification benefits and providing some investment opportunities.

4 Market segmentation

In this section we further investigate the reason of the empirical outperformance of the cryptocurrency assets over the traditional ones presented in the previous Section. To this end, we test whether the cryptocurrency market is integrated/ segmented with the equity and bond market.

Typical definitions of capital market integration imply that financial assets that trade in different markets will have identical expected returns as long as they have identical risk characteristics (Campbell and Hamao, 1992). An equivalent way of defining market integration is that the financial assets have at least one stochastic discount factor (SDF) in common, i.e., the same SDF prices all assets (see also, Bessembinder and Chan, 1992; Chen and Knez, 1995). Hence, in the case of market integration, there is a common stochastic discount factor which prices the various asset classes.

The existence of diversification benefits is tightly connected with spanning. A set of test assets are said to provide diversification benefits relative to a set of benchmark assets if adding these assets to the benchmark leads to a significant leftward shift in the efficient frontier (Bekaert and Urias, 1996). This is equivalent to test whether a set of benchmark assets spans the set of both benchmark and test assets. Thus, in the case of spanning, including the test assets in the benchmark asset universe does not increase the portfolio expected return per unit of risk (i.e., the Sharpe ratio) and thus diversification benefits do not exist.

In the case where markets are segmented, assets in different markets are not priced by the same stochastic discount factor. This in turn implies that assets in one market are not spanned by assets in the other markets and hence diversification benefits exist (for the case of Mean-Variance spanning, see for example Ferson et al., 1993; Bekaert and Urias, 1996; DeSantis, 1995; and for the non Mean-Variance case see, De Roon et al., 2003).

We follow the Campbell and Hamao (1992) model to test for market integration. Let a $K-$ factor asset pricing model,

$$R_{i,t+1} = E_t(R_{i,t+1}) + \sum_{k=1}^{K} \beta_{ik} f_{k,t+1} + e_{i,t+1}, \tag{22}$$

where $R_{i,t+1}$ denotes the excess return on asset i held from time t to time $t+1$, $f_{k,t+1}$ the kth factor realization, β_{ik} the factor loading with respect to the kth factor and $e_{i,t+1}$ the error term. The above equation maps to an expected return-beta representation (Cochrane, 2005),

$$E_t(R_{i,t+1}) = \sum_{i=1}^{K} \beta_{ik} \lambda_{kt}, \tag{23}$$

where λ_{kt} is the market price of risk for the kth factor at time t. The time variation in the kth factor market price of risk is modeled as

$$\lambda_{kt} = \sum_{n=1}^{N} \theta_{kn} X_{nt}, \tag{24}$$

where there is a set of N predictors X_{nt}, $n = 1, 2, ..., N$. Assuming that markets are integrated, i.e., they have the same prices of risk and the time variation in expected returns stems from time variation in the prices of risk, equations (23) and (24) imply that predictor variables that drive prices of risk should be the same across assets. Therefore, evidence that the predictors of the price of risk differ across asset classes would imply that the market price of risk is not the same across assets and hence markets are segmented.

We use predictors which have been documented to predict equity and bond markets returns: the dividend yield, the default spread (Bessembinder and Chan, 1992), the term spread (Fama and French, 1989), the money supply growth (Chen, 2007) and the growth in the Baltic Dry Index (Bakshi et al., 2011). We obtain weekly data on the dividend yield on MSCI World, the junk bond premium (or default spread, defined as the excess of the yield on long-term BAA corporate bonds rated by Moody's over the yield on AAA-rated bonds), the term spread (defined as the difference between the Aaa yield and the one-month bill rate) and the Baltic Dry Index from Bloomberg. We also obtain weekly data on money supply from the Board of Governors of the Federal Reserve System (US).

Table 4 presents the evidence on the forecastability of the weekly asset returns during the period from August 7, 2015 to December 29, 2017. The Newey-West standard errors are used to correct for autocorrelation and heteroskedasticity. We can see that the S&P 500 equity index can be predicted by the default spread and the dividend yield. The Barclays Bond index can be predicted by the term spread. On the other hand, these predictors do not forecast cryptocurrency weekly returns. This evidence is also supported by the F-statistic and the respective p-values: the hypothesis that all coefficient estimates equal to zero can be rejected only for the traditional asset classes, i.e., the S&P 500 equity index and the Barclays Bond Index.

Our results show that cryptocurrency returns cannot be forecasted by variables that predict stock and bond market returns. This suggests that the price of risk is driven by different predictors across the various asset classes which in turn implies that the prices of risk differ. As a result, cryptocurrency markets are segmented from equity and bond markets and hence the inclusion of cryptocurrencies in investors' portfolios is expected to yield some diversification benefits. Note that the market integration test offers a statistical setting on top of the economic significance setting applied in the previous sections to assess whether cryptocurrencies may offer diversification benefits. In addition, the test is applied to the full sample and hence the obtained results are meant to be discussed in light of the in-sample evidence.

5 Conclusions

Cryptocurrencies are one of the most important financial innovations in recent times. They have drawn an increasing number of critics and supporters in equal measures. To expand the analysis of cryptocurrencies as assets, we have adopted a stochasic dominance approach.

In a related empirical application using actual market data, we test whether a portfolio set originated from a traditional asset universe does not span the same set augmented by including cryptocurrencies. If spanning is rejected, then some risk averters could benefit from the augmentation of investment opportunities. We employ the S&P 500 Total Return Index, Barclays U.S. Aggregate Bond Index and the one month Libor rate to proxy the traditional asset universe, i.e. the equity market, the bond market and the risk-free rate. We also use daily and weekly prices for four cryptocurrencies namely Bitcoin, Ethereum, Ripple and Litecoin for the period August 2015 to December 2017. We conduct our analysis both in- and out-of-sample by constructing and comparing optimal portfolios derived from two respective asset universes: one that includes only the traditional asset classes (equities, bonds and cash) and one that is augmented with cryptocurrencies, too.

In the in-sample tests, we find that the optimal portfolios formed based on the investment opportunity set that also includes cryptocurrencies are not spanned by the corresponding portfolio strategies based on the traditional investment opportunity set. Thus, there may be some risk averse investors that would benefit from the augmentation, while some others would not.

In the out-of-sample analysis, using a non-parametric stochastic dominance test as well as parametric performance measures, we find that the expanded investment universe with cryptocurrencies empirically dominates traditional one with stocks, bonds and cash, yielding some diversification benefits and providing better investment opportunities.

We explain the reported diversification benefits by documenting that cryptocurrency markets are segmented from equity and bond markets.

While our results are interesting, and the cryptocurrency markets seem to be growing in popularity, investments in such markets should be considered with extreme caution due to excessive volatility. Hence, we acknowledge that there are some issues to be considered before cryptocurrencies will form an asset class of significant interest.

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Appendix: Computational strategy

For convenience we present here the computational strategy for the spanning testing procedure developed in Online Appendix of Arvanitis et al. (2017).

The test statistic can be written:

$$\eta_T = \sqrt{T} \sup_{u \in \mathscr{U}_2} \left(\sup_{\lambda \in \Lambda} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \lambda \right) \right] - \sup_{\kappa \in \mathrm{K}} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \kappa \right) \right] \right). \tag{25}$$

The term in parentheses is the difference between the solutions to two standard convex optimization problems of maximizing a quasi-concave objective function over a polyhedral feasible set. The analytic complexity of computing η_T stems from the search over all admissible utility functions ($\mathscr{U}_2$). However, the utility functions are univariate, normalized, and have a bounded domain ($\mathscr{X}$). As a result, we can approximate $\mathscr{U}_2$ with arbitrary accuracy using a finite set of increasing and concave piecewise-linear functions in the following way.

We partition $\mathscr{X}$ into N_1 equally spaced values as $\underline{x} = z_1 < \cdots < z_{N_1} = \bar{x}$, where $z_n := \underline{x} + \frac{n-1}{N_1-1}(\bar{x} - \underline{x})$, $n = 1, \cdots, N_1$; $N_1 \geq 2$. Instead of an equal spacing, the partition could also be based on percentiles of the return distribution. Similarly, we partition the interval $[0,1]$, as $0 < \frac{1}{N_2-1} < \cdots < \frac{N_2-2}{N_2-1} < 1$, $N_2 \geq 2$. Using this partition, let

$$\underline{\eta_T} := \sqrt{T} \sup_{u \in \underline{\mathscr{U}_2}} \left(\sup_{\lambda \in \Lambda} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \lambda \right) \right] - \sup_{\kappa \in \mathrm{K}} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \kappa \right) \right] \right); \tag{26}$$

$$\underline{\mathscr{U}_2} := \left\{ u \in \mathscr{C}^0 : u(y) = \sum_{n=1}^{N_1} w_n r(y; z_n) \, w \in \mathrm{W} \right\}; \tag{27}$$

$$\mathrm{W} := \left\{ w \in \left\{ 0, \frac{1}{N_2-1}, \cdots, \frac{N_2-2}{N_2-1}, 1 \right\}^{N_1} : \sum_{n=1}^{N_1} w_n = 1 \right\}. \tag{28}$$

Every element $u \in \underline{\mathscr{U}_2}$ consists of at most N_2 linear line segments with knots at N_1 possible outcome levels. Clearly, $\underline{\mathscr{U}_2} \subset \mathscr{U}_2$ and $\underline{\eta_T}$ approximates η_T from below as we refine the partition ($N_1, N_2 \to \infty$). The appealing feature of $\underline{\eta_T}$ is that we can enumerate all $N_3 := \frac{1}{(N_1-1)!} \prod_{i=1}^{N_1-1} (N_2 + i - 1)$ elements of $\underline{\mathscr{U}_2}$ for a given partition, and, for every $u \in \underline{\mathscr{U}_2}$, solve the two embedded maximization problems in (26) using LP:

Proposition 1. *Let*

References

$$c_{0,n} := \sum_{m=n}^{N_1} \left(c_{1,m+1} - c_{1,m}\right) z_m; \tag{29}$$

$$c_{1,n} := \sum_{m=n}^{N_1} w_m; \tag{30}$$

$$\mathscr{N} := \{n = 1, \cdots, N_1 : w_n > 0\} \bigcup \{N_1\}. \tag{31}$$

For any given $u \in \underline{\mathscr{U}_2}$*,* $\sup_{\lambda \in \Lambda} \mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\lambda\right)\right]$ *is the optimal value of the objective function of the following LP problem in canonical form:*

$$\begin{aligned} &\max T^{-1} \sum_{t=1}^{T} y_t \\ \text{s.t. } & y_t - c_{1,n} X_t^{\mathrm{T}} \lambda \leq c_{0,n},\ t = 1, \cdots, T; n \in \mathscr{N}; \\ & \sum_{i=1}^{M} \lambda_i = 1; \\ & \lambda_i \geq 0,\ i = 1, \cdots, M; \\ & y_t \text{ free},\ t = 1, \cdots, T. \end{aligned} \tag{32}$$

The proof is in the Online Appendix of Arvanitis et al. (2017). The LP problem always has a feasible and finite solution and has $\mathscr{O}(T+M)$ variables and constraints, making it small for typical data dimensions. Our application is based on the entire available history of monthly investment returns to a standard set of benchmark assets ($M = 11$, $T = 1,062$), and uses $N_1 = 10$ and $N_2 = 5$. This gives $N_3 = \frac{1}{9!}\prod_{i=1}^{9}(4+i) = 715$ distinct utility functions and $2N_3 = 1,430$ small LP problems, which is perfectly manageable with modern-day computer hardware and solver software.

The total run time of all computations for our application amounts to several working days on a standard desktop PC with a 2.93 GHz quad-core Intel i7 processor, 16GB of RAM and using MATLAB with the external Gurobi Optimizer solver.

Tables and Figures

Table 1: Descriptive Statistics of daily returns

Asset	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe ratio
S&P 500	0.00045	0.00782	-0.40137	4.24918	0.05371
Bond Index	0.00010	0.00195	-0.31009	1.37703	0.037313
1m Libor	0.000003	0.00001	0.42899	-1.11955	-
Bitcoin	0.00760	0.04558	0.62145	5.64741	0.16604
Ethereum	0.01465	0.10171	0.86734	11.3222	0.14375
Ripple	0.01338	0.10145	4.73490	36.42765	0.13163
Litecoin	0.00940	0.07770	3.31961	24.30942	0.12064

Entries report the descriptive statistics on daily returns for the alternative asset classes used in this study. The average return, the standard deviation, the skewness, the kurtosis, as well as the Sharpe ratio are reported. The dataset covers the period from August 7, 2015 to December 29, 2017.

Table 2: Out-of-sample performance: Non-parametric stochastic dominance test

	Panel A	Panel B
	Traditional vs Augmented	Traditional vs Augmented
Quartile		
25% p-value	0.031	0.028
50% p-value	0.050	0.056
75% p-value	0.126	0.144

Entries report quartile p-values from the distribution of p-values across 233 out-of-sample periods with the null hypothesis that the augmented with cryptocurrencies optimal portfolio does not second order stochastically dominate the optimal portfolio of only traditional assets. Panels A reports the results when the traditional set includes the S&P 500 Equity Index, the Barclays US Aggregate Bond Index and the 1-month Libor, while panel B reports the results when the traditional set includes additionally the SMB and HML indices, as well as the Russell 2000 equity index, the Vanguard value and the Vanguard small-cap index funds.

Figure 1: Cumulative performance of the traditional optimal portfolio as well as the optimal augmented portfolio with cryptocurrencies for the entire sample period from 09/01/2016 to 12/29/2017.

Figure 2: Cumulative performance of the traditional optimal portfolio when the traditional asset class also includes the SMB and HML indices, the Russell 2000 equity index, the Vanguard value and the Vanguard small-cap index funds, as well as the optimal augmented portfolio with cryptocurrencies, for the entire sample period from 01/09/2016 to 12/29/2017.

Table 3: Out-of-sample performance: Parametric portfolio measures

	Panel A		Panel B	
	Traditional	Augmented	Traditional	Augmented
Performance Measures				
Sharpe ratio	0.11762	0.18728	0.14468	0.18728
Downside Sharpe Ratio	0.12986	0.39570	0.16437	0.39570
UP ratio	0.67490	1.04188	0.68556	1.04188
Portfolio Turnover	9.566%	4.530%	13.247%	4.530%
Return Loss	0.1856%		0.1778%	
Opportunity Cost				
Exponential Utility				
ARA=2	1.125%		1.211%	
ARA=4	1.127%		1.255%	
ARA=6	1.126%		1.279%	
Power Utility				
RRA=2	1.134%		1.278%	
RRA=4	1.140%		1.256%	
RRA=6	1.139%		1.249%	

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss and Opportunity Cost) for the traditional and the augmented with cryptocurrencies asset classes. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) and different degrees of relative risk aversion (RRA=2,4,6). The dataset spans the period from August 7, 2015 to December 29, 2017. In Panel A the traditional set includes the S&P 500 Equity Index, the Barclays US Aggregate Bond Index and the 1-month Libor. In Panel B the traditional set includes additionally the SMB and HML indices, the Russell 2000 equity index, the Vanguard value and the Vanguard small-cap index funds.

Table 4: Test for Market Integration

Predictor	Dividend Yield	Junk Bond Yield	Term Spread	Money Growth	Baltic Dry	adjusted R^2	F
Asset							
Bitcoin	-0.210	0.111	-0.083	-0.009	0.081	0.01	1.004
	(-1.621)	(0.721)	(-0.549)	(0.094)	(0.814)		(0.418)
Ethereum	-0.102	0.087	-0.059	-0.010	0.033	0.03	0.185
	(-0.662)	(0.622)	(-0.345)	(-0.134)	(0.027)		(0.968)
Ripple	-0.290	0.097	-0.099	0.011	-0.007	0.05	1.758
	(-1.914)	(0.538)	(-0.502)	(0.185)	(-0.073)		(0.128)
Litecoin	-0.232	0.087	-0.101	-0.029	0.051	0.02	1.542
	(-1.197)	(0.643)	(-0.715)	(-0.336)	(0.601)		(0.182)
S&P 500	0.448	-0.362	-0.057	-0.072	0.064	0.11	3.047
	(3.359**)	(-2.230**)	(-0.429)	(-0.815)	(0.706)		(0.013)
Barclays	-0.130	0.088	0.181	-0.171	-0.006	0.22	14.551
Bond Index	(-0.249)	(0.307)	(1.921*)	(-0.697)	(-1.453)		(0.008)

Predictive regressions are run to implement Campbell and Hamao (1992) test for market integration. Dependent variables for the regressions are the weekly excess returns on cryptocurrencies, equity and bond indices. The independent variables are the instrumental variables lagged one week: the dividend yield, the junk bond premium (or default spread) defined as the excess of the yield on long-term BAA corporate bonds rated by Moody's over the yield on AAA-rated bonds, the term spread defined as the difference between the AAA yield and the one-month Bill rate, the money supply growth, and the growth in the Baltic Dry Index. A constant term is included in all regressions. The Table reports standardized coefficient estimates with autocorrelation and heteroskedasticity-robust t-statistics in parentheses. The adjusted R^2 is reported and the F-statistic with the respective p-values in parentheses. The dataset spans August 2015- December 2017. * and ** asterisks indicate that the coefficient estimates are statistically significant at 10% and 5% significance level .

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