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FORECASTING ACTUARIAL TIME SERIES: A PRACTICAL STUDY OF THE EFFECT OF STATISTICAL PRE-ADJUSTMENTS

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ABSTRACT

One of the most important risks in the actuarial industry is the longevity risk. The accurate prediction of mortality rates plays a crucial role in the management of the aforementioned risk. Such predictions are performed by modelling the mortality rates using mortality models. Aiming at possible improvements in such forecasts, in this work we examine the effect of data transformation and "linearization" on the quality of time series forecasts of mortality rate data. By the term time series "linearization" is meant the treatment of causes that disrupt the underlying stochastic process measured by a time series. The dataset consists of the time series of the period indices uncovering the mortality trend for England-Wales according to published mortality models. Results indicate a clear improvement in interval forecasts. However, the result on point forecasts is not as clear as is the case of interval forecasts. The documented improvement in interval forecasts can significantly affect the Solvency Capital Requirement, and subsequently the Solvency Ratio for a pension fund. Such an improvement might put some pension providers at a competitive advantage as they have less capital locked in their liabilities. In addition, it was confirmed that the transformed-linearized time series of mortality rates satisfy to a higher extent the need for normality as compared to the original series.

Keywords: Time series transformation and ‘’linearization’’, Outliers, Actuarial time series forecasts, Mortality rates, Covid-19

JEL classification: C22, C51, C53, C87, G22

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1 Introduction

In this work, we examine the possibility of improving the existing statistical approach for forecasting the trend of mortality, using mortality models reported in the literature. This is important because as the years go by, the life expectancy increases, posing challenges to the insurance industry. The longevity increment and the parallel trend of fertility decline push the funding of retirement income systems to its financial limits (Oeppen and Vaupel, 2002; Dowd et. al., 2010). The ever-increasing number of pension plans and compensations paid, as a consequence of the increased lifetime, give raise to the risk of leading the pension funds and the life insurance companies off-budget. Therefore, financial institutions such as pension funds, national governments and life insurance companies inevitably face the longevity risk. In this direction, several regulations have been introduced with the aim of maintaining the balance of an organization's reserve funds and control the inherent risks.[1] Solvency II (Directive 2009/138/EC) introduces a uniform system of calculating capital requirements among all EU Members in order to ensure organizations' solvency and risk management ability. This capital requirement is called Solvency Capital Requirement (SCR) and covers all the risks that an insurer faces. Among others, one of the most important non-diversifiable risks is the longevity risk, which is triggered due to the uncertainty in the future trend of mortality rates for annuitants (Dowd et. al., 2010; Kleinow and Richards, 2017), as the constant improvements in science and medicine render the projection of mortality rates a challenging process. In other words, the pensioners are living expectantly longer and as a result the life insurances and pension schemes are paying out compensations for much longer than it was expected, resulting in reduced profit and possibly insolvency. In light of the above, Solvency II requires that an insurer holds enough reserves to cover 99.5% of scenarios which might occur over a one-year time horizon. However, according to the literature, it has been observed that the risk of

[1] The legislative framework for the operation of domestic as well as foreign private insurance companies in Greece was first established by Law 1023/1917. Since then and until 2010 the supervision responsibility was under the jurisdiction of various authorities belonging to the General Government. However, in essence, supervision remained more rules-based, rather than risk-based. As of 1.12.2010 with Law 3867/2010, the Bank of Greece (henceforth BoG) undertook the supervision and the then formed Directorate of Private Insurance Supervision developed the expertise for proper (risk-based) supervision practices. In parallel, the Directorate of Statistics of BoG (in particular the Department of Statistics of MFIs and Insurance Companies) has developed an expertise in forecasting time series with certain characteristics. Part of this research attracted the attention of leading experts in insurance science and this manuscript is a product of collaborative work with them.

longevity unfolds over many years and hence, it does not naturally fit into the one-year time horizon required by regulators in banking and insurance (Kleinow and Richards, 2017). Indeed, longevity risk lies in the long-term trend taken by mortality rates. This trend unfolds over many years as an accumulation of small changes. While a great many insurance risks fit naturally into a one-year value-at-risk framework, not all risks do. It would be excessively dogmatic to insist that longevity trend risk only be measured over a one-year horizon (Richards and Currie, 2009). Thus, the analysis of longevity risk requires the projection of longevity related data in a time horizon of several years.

An approach to deal with the aforementioned problem is to model the mortality rates by using the mortality models reported in the literature and predict the trend of the mortality in the future. By doing so, an insurance company can reinforce the capital requirements determination process. The methodical and accurate prediction of mortality rates plays a crucial role in the management of longevity risk.

To do so, a number of stochastic models have been developed to analyse and model the mortality improvements (Lee and Carter, 1992; Renshaw and Haberman, 2006; Currie, 2006; Cairns et al., 2006; Hyndman and Ullah, 2007; Plat, 2009; Hatzopoulos and Haberman, 2011; Hatzopoulos and Sagianou, 2020). Recent developments in mortality modeling have tended to be extrapolative in nature, with the principal components (PC) approach being prominent. Thus, Bell and Monsell (1991) extended the Ledermann and Breas (1959) approach, where a PC approach is used in forecasting age-specific mortality rates. In a seminal paper, Lee & Carter (1992) explored a modified version of this approach for forecasting mortality rates. The main statistical tool used was least-squares estimation via singular value decomposition (SVD) of the matrix of the log age specific observed forces of mortality. Hyndman and Ullah (2005) use several PCs in order to capture the differential movements in age-specific mortality rates, using functional PCA. Recently, many authors have proposed new approaches to mortality forecasts, utilizing (nonparametric) smoothing. Thus, Gao and Hu (2009) introduce a Generalized Dynamic Factor method and multivariate BEKK GARCH model to describe mortality dynamics under conditional heteroskedasticity. Lazar and Denuit (2009) utilize dynamic factor analysis and the methodology of Johansen cointegration to project mortality through a linear state space representation which assumes that common factors can be modelled as a multivariate random walk with drift.

By utilizing historical mortality data, a stochastic mortality model can uncover the trend of mortality rates and provide a deeper understanding to the mortality dynamics. The extracted mortality rates can be used to extrapolate mortality behaviour in the future, with the aim to uncover the future behaviour of mortality trends.

In this paper, we use the multiple-component stochastic mortality model of Hatzopoulos-Sagianou (hereafter called HS), as developed in Hatzopoulos and Sagianou (2020), to model the mortality dynamics. The HS model uses a semi parametric estimation method. This method adopts Generalized Linear Models (GLMs) and Sparse Principal Components Analysis (SPCA). The SPCA requires the definition of a sparsity factor (s value) in order to pinpoint the optimal and most informative age–period and age–cohort components. To do so, the definition of the sparsity factor is based on a methodology tailored for the HS model and is able to measure the Unexplained Variance (UVR) of each of the age–period and age–cohort components that are incorporated in the proposed model.

In the family of age-period-cohort stochastic mortality models the dynamics of mortality are driven by the period and the cohort indices. Therefore, the forecasting of mortality rates requires the modelling of these indices using time series techniques. For the period indices, we assume the random walk with drift (henceforth RWD) model, the standard approach in the actuarial literature (Lee and Carter, 1992; Cairns et al., (2006; 2011); Pitacco et al., 2009; Haberman and Renshaw, 2011; Lovász, 2011; Villegas et al., 2018).

However, other stochastic models have also been tried in order to enhance forecasts (Lee and Miller, 2001; Plat, 2009; Villegas et al., 2018; Hatzopoulos and Sagianou, 2020). One strand of such models are the AutoRegressive Integrated Moving Average (henceforth ARIMA) models. An ARIMA model is a concise quantitative summary of the internal dynamics of a time series in a linear framework and it is useful, among several other reasons, for forecasting. However, time series from the real world need to undergo some statistical preparation and pre-adjustment as they are not usually "ready" to be used for forecasting purposes in the aforementioned context. This is so because in time series of raw data variance non-stationarity may be present. Furthermore, very often there exist outliers and other causes (such as calendar effects, etc.) that disrupt the underlying stochastic process. Their treatment is known as "linearization".

Variance non-stationarity not only increases time series variance, as does the existence of outliers, but also affects the character of the ARIMA model and the selection and type of outliers (Milionis, 2003; Milionis, 2004; Milionis and Galanopoulos, 2019). Inevitably, both variance non-stationarity and outliers affect point and interval forecasts. Hence, forecasts are less accurate with increased confidence intervals. In the actuarial context, this can adversely affect the longevity risk management process. More specifically, outliers and possible instability in the variance in longevity data, entail an increase in time series variance which in turn is expected to affect, amongst others, the uncertainty about the solvency capital requirements in a pension fund or insurance institution. However, if outliers in the actuarial time series data reflect events in the real world that occur or repeat themselves very rarely (e.g. the Spanish influenza of 1917, world wars etc), then it may be possible to improve the quality of forecasts by properly controlling for their influence, as with the adjustment for outliers the process variance is reduced and it is possible to exploit this reduction in obtaining forecasts with increased confidence. A similar fact that exists nowadays, could prove to be the Covid-19 with the increasing number of deaths due to this pandemic. In a possible future study of mortality rates, we need to consider the existence of the pandemic and its impact on mortality in order to possibly achieve a more accurate and qualitative prediction.

In spite of the importance of point, as well as of interval forecasts on actuarial time series, the possible existence and type of variance instability and outliers, and the possible implications for actuarial models' performance on mortality rates have not been systematically studied thus far. This is indeed the innovation and scope of this work. Our empirical results indicate a clear improvement in interval forecasts thus contributing to the existing literature on mortality modelling. The documented improvement in interval forecasts can significantly affect the Solvency Capital Requirement, and subsequently the Solvency Ratio for a pension fund. Such an improvement might put some pension providers at a competitive advantage as they have less capital locked in their liabilities.

Initially, the proposed methodology tests if a data transformation is needed in order to stabilize variance instability of mortality rates. Then, the time series of mortality rates are linearized for outlier adjustment. The last step of the proposed methodology is to fit forecasting models for mortality rates. The intention of this paper is clearly towards a practical approach and to this end, the RWD model will be used as a benchmark because RWD is a widely adopted forecasting model in the mortality literature. More specifically, as Deaton and Paxson (2004) point out, the simple random walk with drift model, as proposed by Lee and Carter (1992), has become "the leading statistical model of mortality forecasting in the demographic literature", even though Lee and Carter (1992) also highlighted the possibility of more general models. Furthermore, stochastic models will be employed with and without pre-adjustments in order to assess the effect of the later on the quality of forecasts.

The paper is structured as follows: in section 2 we briefly discuss the statistical treatment of variance non-stationarity and outliers in conjunction with an ARIMA model. In section 3 we describe the data and the software to be employed. In section 4 we present and comment upon our results. In section 5 we summarize and conclude.

2 Statistical pre-adjustments

By and large, statistical pre-adjustments for time series modelling comprise: (i) data transformation; (ii) data linearization. When variance is non-stationary the principal problem is the misspecification of the ARIMA model and then the identification of the various types of outliers. However, if variance is somehow functionally related to the mean level it is possible to select a transformation to stabilize the variance. Widely used transformations to tackle this problem belong to the class of the power Box and Cox transformation (Box and Cox, 1964) given by:

$$f(z_t) = \begin{Bmatrix} \frac{z_t^{\lambda}-1}{\lambda}\ if\ \lambda \neq 0 \\ ln z_t\ if\ \lambda = 0 \end{Bmatrix} \qquad (1)$$

As far as outlier adjustment is concerned, after data transformation of the raw series z_t for variance stabilization, $y_t \coloneqq f(z_t)$, is "linearized" according to the general framework (Kaiser and Maraval, 2001):

$$y_t = w_t' b + C_t' \eta + \sum_{j=1}^{m} \alpha_j \mu_j(B) I_t(t_j) + x_t \qquad (2)$$

where:

$b = (b_1, \dots, b_n)$ is a vector of regression coefficients;

$w_t' = (w_{1t}, \dots, w_{nt})$ denotes n regression or intervention variables;

C_t' denotes the matrix with columns possible calendar effect variables (e.g. trading day) and η the vector of associated coefficients;

$I_t(t_j)$ is an indicator variable for the possible presence of an outlier at period t_j;

$\mu_j(B)$ captures the transmission of the j-th effect and α_j denotes the coefficient of the outlier in the multiple regression model with m outliers;

x_t follows in general a multiplicative seasonal ARIMA(p,d,q)(P,D,Q)s model:

$$\varphi(B)\Phi(B^s)\nabla^d\nabla_s^D x_t = \theta(B)\Theta(B^s)\varepsilon_t \qquad (3)$$

where:

- $\varphi(B) = 1 - \phi_1 I \dots - \phi_p B^p$ is the so–called autoregressive polynomial of order p;
- $\theta(B) = 1 - IB - \dots - \theta_q B^q$ is the so–called moving average polynomial of order q;
- $\nabla^d \equiv (1 - B)^d$ is the arithmetic difference operator of order d;
- $\nabla_s^D \equiv (1 - B^s)^D$ is the seasonal arithmetic difference operator of order D and seasonality s;
- $\Phi(B^s) = 1 - I - \dots - \Phi_P B^{P\cdot s}$ is the so–called seasonal autoregressive polynomial of order P and seasonality s;
- $\Theta(B^s) = 1 - IB^s - \dots - \Theta_Q B^{Q\cdot s}$ is the so–called moving average polynomial of order Q and seasonality s;
- ε_t is the stochastic disturbance (white noise process).

So, there are two effects with potential influence on forecasting: «linearization» and transformation, each of which separately, as well as in combination, may play an important role on time series forecasting.

In our case, the dataset for the time series analysis consists of nine annual time series, so equation (3) can be substantially simplified. Indeed, seasonality is out of context, as annual data will be used. Hence, a non-seasonal ARIMA model will be sought for.

Moreover, $b_1, \ldots, b_n$, as well as $C_t' \eta$ in equation (2) will all be set equal to zero, as intervention variables and calendar effects are not taken account into this paper, and so the final model is $y_t = \sum_{j=1}^{m} \alpha_j \mu_j(B) I_t(t_j) + x_t$. More specifically, intervention variables (Box and Tiao, 1975) are often special and unusual events that affect the evolution of the series, such as a change of a base index, and cannot be accounted for by the ARIMA model. So, there is a need for 'intervention' in the series so as to correct for the effect of special events. In addition, calendar effects are the effect of calendar dates, such as the location of Easter effect, and are incorporated into the model through regression variables (obviously not relevant to our case).

3 Data and software-computational details

In this direction, we use the HS multiple-component stochastic mortality model in order to model the mortality dynamics. By utilizing mortality models, we estimate the death rates and, in turn, the mortality trends in terms of time series, which reveal the behaviour of mortality over time. In the family of age-period-cohort stochastic mortality models the dynamics of mortality are driven by the period and the cohort indices. The data used, in order to estimate the time series of the period and cohort indices, consist of the number of deaths, $D_{t,x}$, and the corresponding central exposures to risk, $E_{t,x}$, which are defined in rectangular arrangement (t, x) over a unit range of individual calendar years t $(t_1, \ldots, t_n)$, and individual ages, x, last birthday $(x_1, \ldots, x_n)$. Thus, we calculate the crude (unsmoothed) central death rate for any age x and calendar year t as $m_{t,x} = D_{t,x}/E_{t,x}$. $E_{t,x}$ is usually approximated by an estimate of the population aged x last birthday in the middle of the calendar year t or by an estimate of the average population aged x last birthday of the beginning and the end of the calendar year t. We model the number of deaths as independent Poisson realizations; that is, $D_{t,x}$ follow Poisson distribution with mean $E_{t,x} \cdot m_{t,x}$ (Brillinger, 1986; Brouhns et al., 2002).

Hatzopoulos and Sagianou (2020) proposed a dynamic multiple-component model that includes δ_1 age-period and δ_2 age-cohort effects. The HS model can be represented by the following generic formula:

$$\log(\widetilde{m}_{t,x}) = a_x + \sum_{i=1}^{\delta_1} \beta_x^{(i)} \kappa_t^{(i)} + \sum_{j=1}^{\delta_2} \beta_x^{c(j)} \gamma_c^{(j)} \qquad (4)$$

In equation (4) the tilt above $m_{t,x}$ indicates expected value. The term a_x reflects the main age profile of mortality by age, $\beta_x^{(i)}$ and $\beta_x^{c(j)}$ represent the age effect for each period and cohort component, respectively. The terms $\kappa_t^{(i)}$ reflect period-related effects and determine the mortality trend. The terms $\gamma_c^{(j)}$ represent the cohort-related effects, where $c = t - x$. The parameters δ_1 (>=1) and δ_2 (>=0) are indices for the number of period and cohort components included in the model structure, respectively. The number of period and cohort components vary depending on the experimental dataset, i.e., the intrinsic mortality peculiarities of the examined population in a given time frame. For the England and Wales dataset, for the period 1841-2006, δ_1 = 5 and δ_2 = 2 and for the period 1961-2006, δ_1 = 4 and δ_2 = 1. For full details of the estimation methodology, see Hatzopoulos and Sagianou (2020).

Therefore, these κ values must be projected. These period, $\kappa_t^{(i)}$, indices reveal the mortality trends of unique age clusters and can be used by a time series analysis technique in order to forecast future mortality trends.

In this spirit, the approach adopted in this paper is the traditional two-stage process: firstly, we fit the stochastic mortality model in order to estimate κ values (see Hatzopoulos and Sagianou, 2020) and then we fit a projection model to the estimated κ values for forecasting.

Therefore, considering the aforementioned and according to Hatzopoulos and Sagianou (2020) results, the dataset for the time series analysis consists of nine annual time series of period indices $\kappa_t^{(i)}$ for England and Wales dataset, of which five are "long" time series, while four are "short" time series. The long time series data cover the period from 1841 to 2016 and consist of one hundred and seventy-six (176) observations. The short time series cover the period from 1961 to 2016 (55 observations). The graphical representations of the nine time series are shown in the Appendix.

To assess the effect of statistical pre-adjustments on forecasts two statistical software approaches will be employed, namely the AUTOARIMA command and its extensions of the well-known programming software "R" and the module TRAMO of the TSW statistical package. TSW stands for TRAMO-SEATS for Windows, a Windows version of the DOS programmes TRAMO (Time Series Regression with

ARIMA Noise, Missing Observations and Outliers) and SEATS (Signal Extraction in ARIMA Time Series), and it is a software for applied time series analysis specializing, amongst others, on time series pre-adjustments, ARIMA modelling, model-based seasonal adjustment, and forecasting (Gomez and Maravall, 1996). TSW routines are also incorporated in other widely used econometric software products such as EVIEWS.

The AUTOARIMA command of "R" allows for the automatic selection of an ARIMA model. Moreover, forecasts based on the selected model may be obtained. On the other hand, TRAMO pre-tests for time series transformation to tackle with variance non-stationarity. Moreover, it offers several options for the treatment of outliers within the frames of the more general pre-adjustment procedure known as "linearization". However, transformation-wise, TSW allows for the logarithmic transformation as the only option. For this reason and for a wider selection of transformations (e.g., the square root transformation) the statistical approach and recommendations suggested by Milionis will also be used (Milionis, 2003; Milionis, 2004; Milionis and Galanopoulos, 2019). It is further noted that TSW identifies three types of outliers utilizing the Chen and Liu (1993) computational approach. These three types of outliers are discriminated according to their effect in a time series as follows: i) Additive Outliers (AO), which affect only a single observation of time series, ii) Transitory Changes (TC), in which the effect of one observation that is extremely high or low is not extinguished in the next observations but damps out gradually over a few periods, iii) Level Shifts (LS), which reflect a major change in the stochastic process and have a permanent effect as all observations subsequent to the outlier move to a new level. Outlier parameters are estimated together with the ARIMA parameters using maximum likelihood estimation. An observation is considered as an outlier according to the critical value of the appropriate statistic $\boldsymbol{\tau}$ (Milionis and Galanopoulos, 2020). More specifically, let $\hat{Y}_{T+1}/\Phi_T$ denote the optimal one-step-ahead linear forecast of Y_{T+1} given the information set Φ_T, which includes information up to time T, $e_{T+1} = Y_{T+1} - \hat{Y}_{T+1}/\Phi_T$ denote the associated forecast error, and $\sigma_{T+1}^2 = [Y_{T+1} - \hat{Y}_{T+1}/\Phi_T]^2$ denote the associated variance. The observation Y_{T+1} is considered as an outlier if the null Hypothesis: H_0: $e_{T+1} = 0$ is rejected. The appropriate statistic to test H_0 is: $\tau = \frac{e_{T+1}}{\sigma_{T+1}}$. As theory cannot predict the critical value of $\boldsymbol{\tau}$, a usual practice is to relate the critical value of $\boldsymbol{\tau}$ to the length of a time series (Fischer and Planas, 2000). In the present study the default options of TSW for the identification of outliers will be used.

4 Results and discussion

4.1 Data transformation

Firstly, it is important to note that the effect of a transformation is meant both in a direct and an indirect way. The direct way is self-explanatory and refers to the transformation itself. The indirect way refers to the influence of the transformation on outlier detection. Indeed, regarding the later, it has been shown that data transformation affects the number and the type of outliers in a time series (Milionis, 2004; Milionis and Galanopoulos, 2019).

Table 1 presents the results on the decision about, transforming or not, the original time series data using both TSW and Milionis (2004) approaches. From the analysis of the nine-time series examined, both methodologies suggest no transformation of the original data in seven cases. In two cases time series need to be transformed. Both methodologies suggest the log transformation.

Table 1: Decision about data transformation

Time series	**TSW**	**Milionis (2004)**
E&W L.KT1	Levels	Levels
E&W L.KT2	Levels	Levels
E&W L.KT3	Logs	Logs
E&W L.KT4	Logs	Logs
E&W L.KT5	Levels	Levels
E&W S.KT1	Levels	Levels
E&W S.KT2	Levels	Levels
E&W S.KT3	Levels	Levels
E&W S.KT4	Levels	Levels

Note: *E&W L.KT1* means England and Wales Long (1841-2016) KT1, *E&W S.KT1* means England and Wales Short (1961-2016) KT1. Analogously for the rest of the series.

For further analysis on statistical forecasting three different approaches will be examined. More specifically, these three approaches are the following: (a) Forecasts based on the random walk with drift model, which, for simplicity, is a widely used model in relevant actuarial studies, as already mentioned, and will be used as a benchmark; (b) Forecasts using the automatic selection and forecasting procedure of the programming software “R”, specifically the “AUTOARIMA” command, as in Hatzopoulos and Sagianou (2020); (c) Forecasts based on ARIMA models after statistical pre-adjustments. The later implies Variance Reduction and will be called "VR" forecasts henceforth. VR has already been used in the analysis of twenty key macroeconomic time series for the Greek economy with data transformation and linearization (Milionis and Galanopoulos, 2019).

Table 2 shows the type of ARIMA models of methodologies (b) and (c). It is noted that the differences in the ARIMA models for those time series where no transformation was needed, should be attributed to the existence of outliers adjusted by linearization, the absence or presence of a drift, and possible differences in the algorithms of software (R versus TSW).

Table 2: ARIMA models

Time series	**"Autoarima"**	**VR**
E&W L.KT1	(0,1,0) WITH DRIFT	(0,1,1) WITH DRIFT
E&W L.KT2	(0,1,3) WITH DRIFT	(3,1,0) WITHOUT DRIFT
E&W L.KT3	(3,0,0) WITHOUT DRIFT	(0,1,0) WITHOUT DRIFT
E&W L.KT4	(1,0,3) WITHOUT DRIFT	(0,1,1) WITHOUT DRIFT
E&W L.KT5	(1,1,4) WITHOUT DRIFT	(1,2,1) WITHOUT DRIFT
E&W S.KT1	(0,2,2) WITHOUT DRIFT	(1,1,0) WITH DRIFT
E&W S.KT2	(0,1,0) WITHOUT DRIFT	(0,1,0) WITHOUT DRIFT
E&W S.KT3	(0,1,0) WITHOUT DRIFT	(0,1,0) WITHOUT DRIFT
E&W S.KT4	(0,1,0) WITHOUT DRIFT	(1,0,0) WITH DRIFT

Note: *E&W L.KT1* means England and Wales Long (1841-2016) KT1, *E&W S.KT1* means England and Wales Short (1961-2016) KT1. Analogously for the rest of the series.

It should be remarked that in the results of Table 2 there are only two cases with AUTOARIMA and three cases with VP where a statistically significant drift was found. The absence of a drift, which is atypical for such kind of actuarial series, obviously weakens the forecasting capacity especially for point forecasts and long-term forecasting.

4.2 The effect of "Linearization"

Outliers are major changes in values that stand out in time series. Inspecting visually the time series included in our dataset (*E&W L.KT3* and *E&W L.KT4* in natural logarithms), it is obvious that in some cases the increased variance can be attributed to outliers. TSW was used (in default options) to identify outliers in all time series.

The type of outlier and the order of observation in which they appear are presented in Table 3. The first number refers to the order of observation followed by the type of outlier. By way of an example 9 AO in E&W L.KT1 time series (see first row of Table 3) shows that the order (9) of the observations (years) of detected outlier is 1849, as the initial observation is 1841, and the type of outlier (AO) is Additive Outlier.

Table 3: Detected outliers and their type

Time series	Temporal Position and Type of outliers
E&W L.KT1	9 AO, 74 LS, 75 LS, 78 TC, 79 LS, 100 LS, 106 LS
E&W L.KT2	80 LS, 100 AO
E&W L.KT3	74 LS, 79 LS, 80 LS, 89 AO, 100 LS, 102 LS, 106 LS, 111 LS, 113 TC, 116 TC, 118 AO, 124 TC, 128 TC, 130 TC, 133 LS
E&W L.KT4	9 TC, 18 AO, 74 LS, 79 LS, 88 AO, 95 AO, 100 LS, 106 LS
E&W L.KT5	9 AO, 18 AO, 23 TC, 50 TC, 74 TC, 77 TC, 78 AO, 104 AO, 157 AO
E&W S.KT1	NO OUTLIERS DETECTED
E&W S.KT2	NO OUTLIERS DETECTED
E&W S.KT3	37 AO
E&W S.KT4	NO OUTLIERS DETECTED

Note: *E&W L.KT1* means England and Wales Long (1841-2016) KT1, *E&W S.KT1* means England and Wales Short (1961-2016) KT1. Analogously for the rest of the series.

4.3 The combined effect of Data Transformation and Linearization

To evaluate the combined effect of data transformation and linearization on the quality of point forecasts some typical statistics will be used. Firstly, the Mean Square Forecast Error (MSFE) which measures the average squared difference between the forecasting values (F_t) and the actual values (A_t), i.e. $MSFE = \frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2$. It is well known that optimal forecasts are those with the minimum MSFE (Hamilton, 1994). Auxiliary the following statistics will also be used:

i) the Mean Absolute Percentage Error (MAPE) statistic given by:
$MAPE = \frac{100\%}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right|$, and

ii) the Mean Absolute Error (MAE) statistic given by:
$MAE = \frac{1}{n}\sum_{t=1}^{n}|A_t - F_t|$.

Furthermore, as far as interval forecasts are concerned, the forecast standard error will be considered.

In addition, the Akaike Information Criterion (AIC) will be used as a probabilistic statistical measure that attempts to quantify the model performance on the training dataset in conjunction with the complexity of the model.

Best forecast will obviously be perceived the one with the minimum value of the each time utilized statistic from the ones mentioned above.

Table 4 presents the number of best forecasts, in terms of minimization of the corresponding statistic, when VR model is compared with the RWD model.

From these results it is apparent that forecasts are better in every single case in terms of the width of the forecast confidence interval with VR methodology. Also, VR methodology is superior based on the minimum value of the Akaike information criterion. Point forecasts with VR methodology are slightly better in terms of all three statistics (MSFE, MAPE, MAE).

Table 4: Summary table - Number of best forecasts (VR versus RWD)

(Table is read as follows: for each statistic, in the second and third column the cases with the minimum value of the statistic (i.e., the best forecasts) out of the total number of cases (i.e. the nine time series of the dataset) are presented).

Point Forecasts	**RWD**	**VR**
MSFE	3/9	6/9
MAPE	4/9	5/9
MAE	3/9	6/9
AIC	1/9	8/9
Interval Forecasts	**RWD**	**VR**
Forecast Standard Error (SE)	0/9	9/9

The results of the examination of the forecasting performance between VR methodology and “Autoarima” are presented in Table 5.

Table 5 is read in the same manner as Table 4, explaining further that when the calculated values of a statistic are found to be equal, then the arithmetic value 0.5 is assigned in both methodologies. For instance, the AIC values 2.5/9 and 6.5/9 of the fourth row of the Table 5 indicate that in two out of the nine time series the corresponding statistic value is minimum with the AUTOARIMA methodology, in six out of the nine time series the corresponding statistic value is minimum with TSW methodology, and in one time series the estimated statistic value is equal in both methodologies.

From the results of Table 5 it is seen that the VR methodology outperforms “Autoarima” in terms of the interval forecasts and is better in terms of the Akaike information criterion. However, point forecasts with the VR methodology are only slightly better in terms of the three statistics (MSFE, MAE and MAPE).

Table 5: Summary table - Number of best forecasts (VR versus “Autoarima”)

Point Forecasts	**“Autoarima” with further analysis in TSW**	**VR**
MSFE	3/9	6/9
MAPE	4/9	5/9
MAE	3/9	6/9
AIC	2.5/9	6.5/9
Interval Forecasts	**“Autoarima” with analysis further in TSW**	**VR**
Forecast Standard Error (SE)	1.5/9	7.5/9

4.4 An Ad-Hoc Evaluation of the overall Models’ Forecasting Performance

The skill of a forecast can be assessed by comparing the relative proximity of both the forecast and a benchmark to the observations. The presence of a benchmark makes it easier to compare approaches and for this reason a benchmark is proposed to establish a common ground for comparison. In the present case an obvious benchmark is the Random Walk Model with Drift (RWD) as already mentioned.

A crude, yet very simple and transparent ad-hoc forecasting evaluation for both point and interval forecasts will be used. More specifically, for the point forecasts for each time series and for each model an arithmetic value is assigned in ascending order based on the corresponding value of the MSFE statistic (i.e., 1 for the best (minimum) MSFE value, 2 for the second best MSFE value, 3 for the worst (maximum) MSFE value). Then, adding up the arithmetic values for all series for a particular model their sum will represent the performance of the model. Models will be ranked according to the value of the corresponding sum. Apparently, the model with the lowest sum will be considered as the best one. For interval forecasts the same procedure will be followed replacing the value of the MSFE statistic with the value of the corresponding standard error around point forecasts. The results are presented in Tables 6 and 7.

From the results of Tables 6 it is evident that the performance of VR methodology for point forecast is better than that of RWD model and "Autoarima". It is noteworthy that the RWD model outperforms the "Autoarima" one.

Table 6: Ranking of forecasting performance according to MSFE (points forecasts)

Time series	**RWD**	**"Autoarima"**	**VR**
E&W L.KT1	*1.5*	*1.5*	*3*
E&W L.KT2	*1*	*3*	*2*
E&W L.KT3	*2*	*3*	*1*
E&W L.KT4	*2*	*3*	*1*
E&W L.KT5	*2*	*3*	*1*
E&W S.KT1	*3*	*1*	*2*
E&W S.KT2	*1*	*2.5*	*2.5*
E&W S.KT3	*3*	*1.5*	*1.5*
E&W S.KT4	*2*	*3*	*1*
Total	*17.5*	*21.5*	*15*

Note: *E&W L.KT1* means England and Wales Long (1841-2016) KT1, *E&W S.KT1* means England and Wales Short (1961-2016) KT1. Analogously for the rest of the series.

As far as interval forecasts are concerned, the results of Table 7 show that the VR methodology has a clearly better performance than RWD model and "Autoarima". In this case "Autoarima" clearly outperforms RWD model.

Table 7: Ranking of forecasting performance according to SE (intervals forecasts)

Time series	**RWD**	**"Autoarima"**	**VR**
E&W L.KT1	*2.5*	*2.5*	*1*
E&W L.KT2	*3*	*2*	*1*
E&W L.KT3	*3*	*2*	*1*
E&W L.KT4	*3*	*2*	*1*
E&W L.KT5	*3*	*2*	*1*
E&W S.KT1	*3*	*1*	*2*
E&W S.KT2	*3*	*1.5*	*1.5*
E&W S.KT3	*3*	*2*	*1*
E&W S.KT4	*3*	*2*	*1*
Total	*26.5*	*17*	*10.5*

Note: *E&W L.KT1* means England and Wales Long (1841-2016) KT1, *E&W S.KT1* means England and Wales Short (1961-2016) KT1. Analogously for the rest of the series.

In a slightly different approach, VR methodology is compared again with "Autoarima", but at this stage ARIMA models derived from "R" were used for forecasting following the "R" procedure. The results are presented in Table 8.

The results of Table 8 show that forecasts are better in terms of the forecast standard error with VR methodology. However, point forecasts with VR methodology are only slightly better in terms of the three statistics (MSFE, MAPE, MAE). According to the Akaike information criterion VR methodology is superior.

Table 8: Summary table - Number of best forecasts (VR versus "Autoarima")

Point Forecasts	**"Autoarima" with further analysis in R**	**VR**
MSFE	3/9	6/9
MAPE	4/9	5/9
MAE	3/9	6/9
AIC	1/9	8/9
Interval Forecasts	**"Autoarima" with further analysis in R**	**VR**
Forecast Standard Error (SE)	2/9	7/9

It is worthy to present some representative cases in more detail. This is done with the aid of Figures 1 and 2. More specifically, Figure 1 refers to the case of series in which the VR methodology deploys its full strength, as the series not only is log-transformed but also there exist several outliers (see Table 3). As is evident from Figure 1, with the method of VR there is a substantial reduction in forecasts confidence interval and a clear improvement in point forecasts.

Figure 1: Forecasts and Confidence intervals with both methods for the series E&W L.KT3

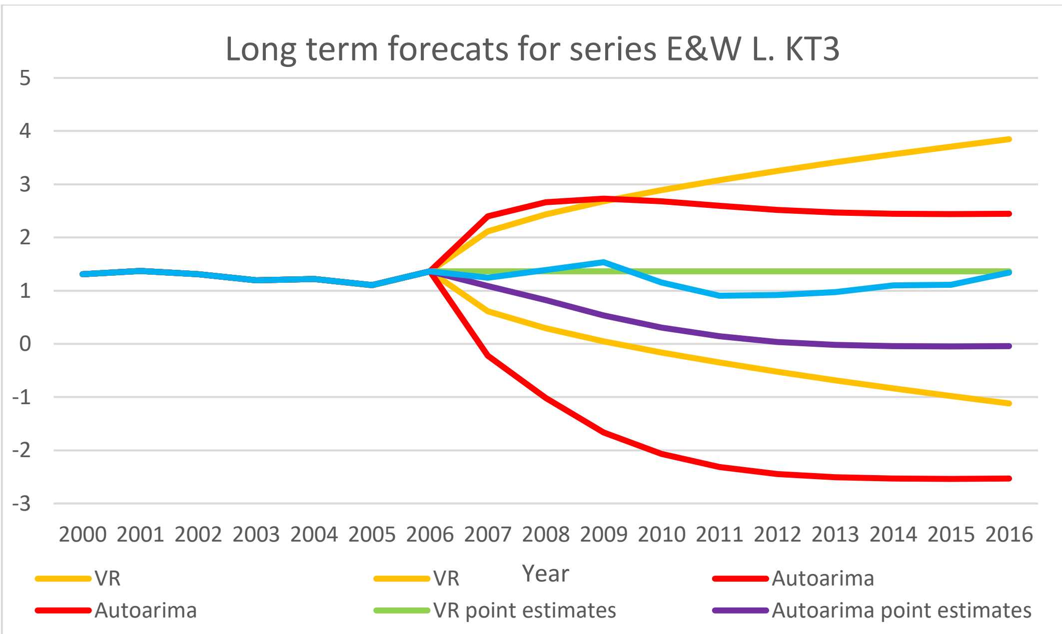
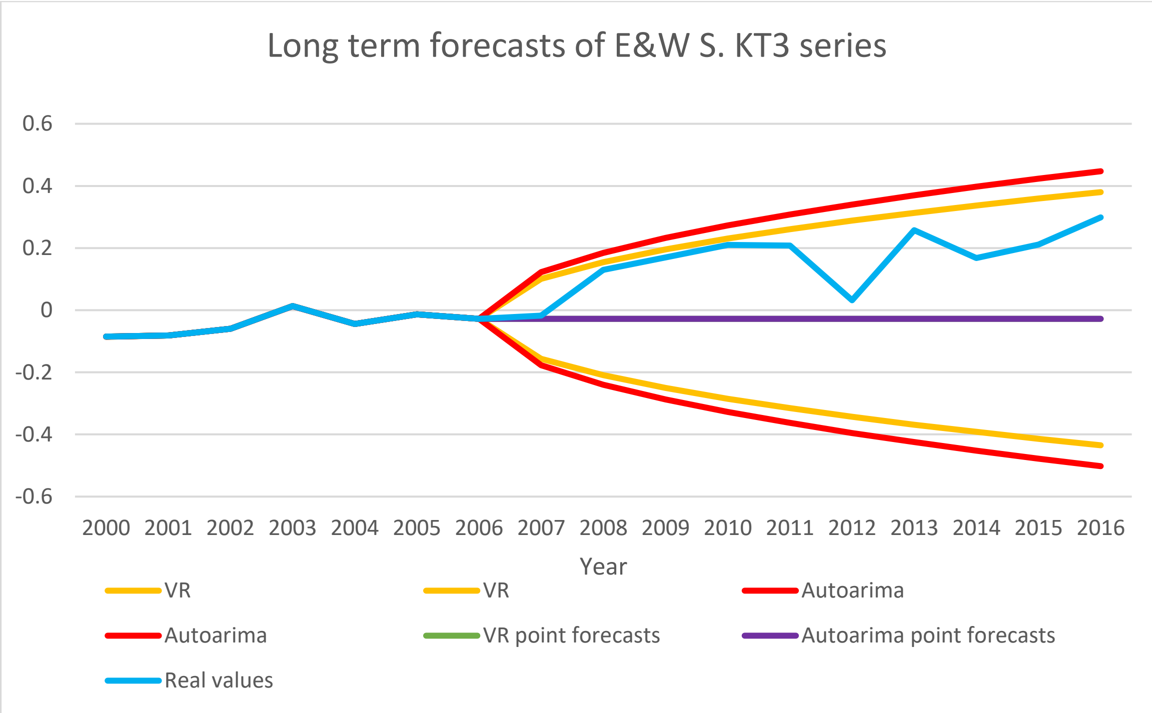


Figure 2 shows the point, as well as the interval forecasts, for the series E&W S.KT3. As presented in Table 2 the stochastic model with both methods is a simple random walk without drift. Therefore, both methods give exactly the same point forecasts, which are actually trivial, as they are all equal to the last observation. However, as presented in Table 3, an additive outlier is detected in the 37th observation with the VR method. Even in this case, forecasts quality is improved with the VR methodology. It is exactly due to the detected outlier that the forecast confidence interval is reduced with the VR method. As explained, this reduction is very important in actuarial science.

Figure 2: Forecasts and Confidence intervals with both methods for the series E&W S.KT3



4.5 The Shift towards Normality

Another serious concern is the problem of acute non-normal distributions in many time series. In the research conducted by Nelson and Granger (1979), they found that in most macroeconomic time series they analyzed, this problem was restored only very little by their use of data transformations.

Table 9 presents the results for the Jarque-Bera statistic for normality for the nine time series we examine in this work. This statistic is distributed as chi-square with two degrees of freedom. An asterisk right next to an arithmetic value of Table 9 indicates a rejection of the null hypothesis of normality at the 5% significance level (critical value = 5.99). From the arithmetic values of the Jarque-Bera statistic, as showed in Table 9, at first sight it is evident that there is a clear general tendency towards normality from the benchmark and "Autoarima" models to VR transformation-linearization procedure. Similar results were also found by Milionis and Galanopoulos (2019) in their analysis of Greek macroeconomic time series. A more detailed look at the results reveal that all short time series are well – behaved in terms of normality in every approach, again with better values of the Jarque-Bera statistic for the VR model. The big difference though, appears in the long series for which in all series there exists acute non-normality in RWD and Autoarima. The phenomenon on some occasions is very pronounced indeed (e.g., in the series of E&W L.KT1.). This is reasonable as outliers reflecting real events (e.g., world wars) occurred mainly before 1959. In contrast, with the VR approach normality in the long series is not rejected in all but two cases in which there is a weak departure from normality. This allows for computational algorithms such as maximum likelihood estimation, to be more legitimately employed with transformed-linearized data.

Table 9: Values of the Jarque –Bera statistic (statistically significant values are indicated with an asterisk)

Time series	**RWD**	**"Autoarima" with further analysis in TSW**	**VR**
E&W L.KT1	4736.*	4736.*	2.840
E&W L.KT2	13.639*	12.23*	7.912 *
E&W L.KT3	0.1057E+05*	4113.*	0.9121
E&W L.KT4	8583.*	2563.*	27.79 *
E&W L.KT5	115.5 *	140.7*	1.747
E&W S.KT1	0.9577	0.9921E-01	0.7816
E&W S.KT2	0.3713E-01	0.3267E-01	0.3267E-01
E&W S.KT3	2.956	3.308	0.3408
E&W S.KT4	2.368	2.584	2.135

5 Summary and conclusions

In this work we examined the effect of statistical pre-adjustments (data transformation and linearization) on the quality of time series forecasts of mortality rate data. It was found that there is a substantial improvement in interval forecasts which on average are shortened by approximately 35.4% when comparing VR and RWD and 20.4% when comparing VR and "Autoarima" (detailed results are not presented but are available by the authors on request). Moreover, there was a less clear improvement in point forecasts. Furthermore, it was confirmed that the transformed linearized series satisfy the need for normality to a clearly larger extent as compared to the other alternatives.

The above statistical findings have important implication for the actuarial science. More specifically, the improvement in interval forecasts can significantly affect the Solvency Capital Requirement, and subsequently the Solvency Ratio for a pension fund. Such an improvement might put some pension providers at a competitive advantage as they have less capital locked in their liabilities.

As a prospect for further research, we intend to explore the effect of statistical pre-adjustments to the financial impact on Solvency Capital Requirement, under different model structures and forecast methods. As has been noted previously, the most useful tool for investigating uncertainty over longevity risk is a stochastic mortality projection model. Since, there is a wide choice of such models in the literature, the choice of model can lead to material changes in the best-estimate reserves, while even within a model family there can be major differences (Richards and Currie, 2009). For those models we aim to study the uncertainty over future mortality rates, which is measured as the variance of the mortality forecast values. In particular, we will investigate their respective contributions to the capital requirements for longevity trend risk. Our investigation will be based on the Hatzopoulos and Sagianou (2020) model, but the basic conclusions will apply to any model which uses time-series methods to project a mortality index. We will also quantify the respective contributions to capital requirements using VaR calculations. Last but not least it is apparent that the methodology presented in this work may be used in due course to adjust for the possible effect of the COVID-19 virus on the forecasting of longevity trends.

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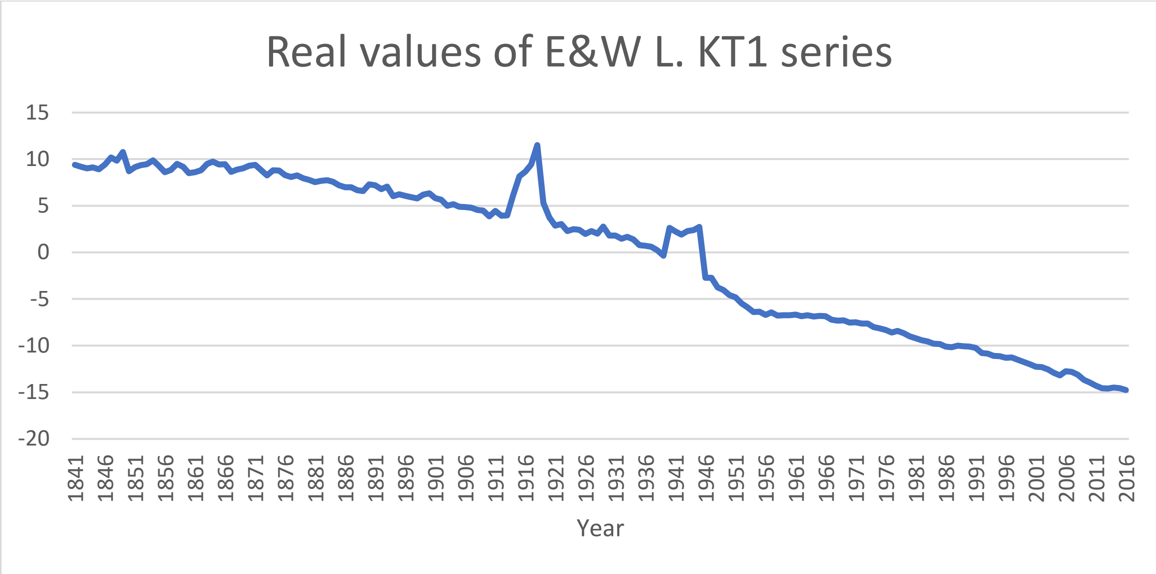
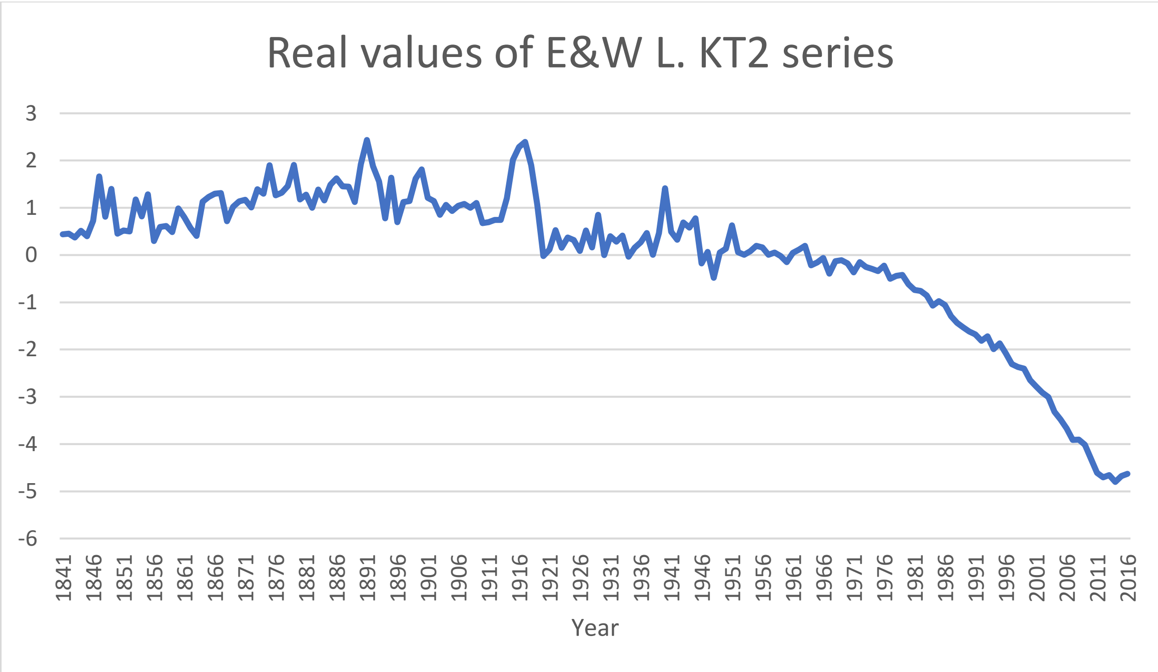
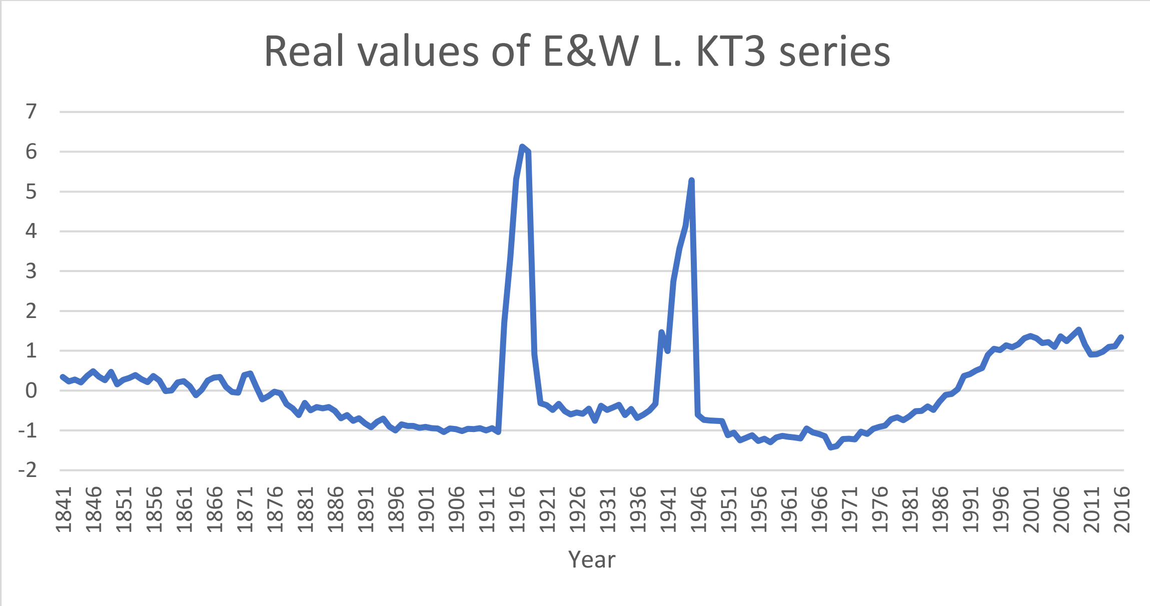
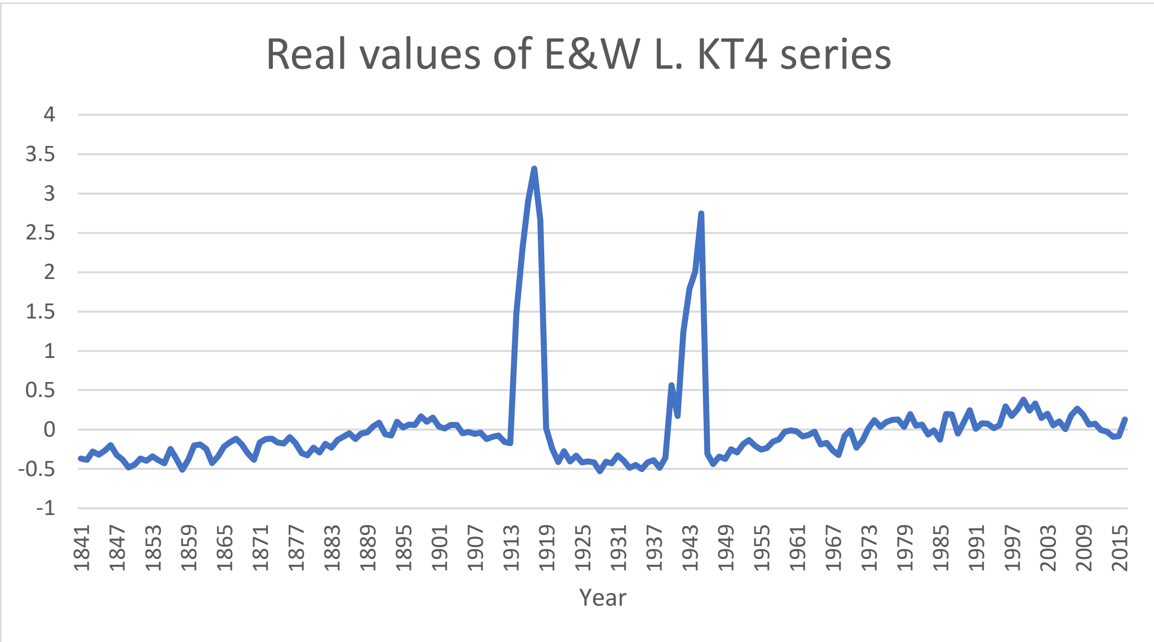
Appendix

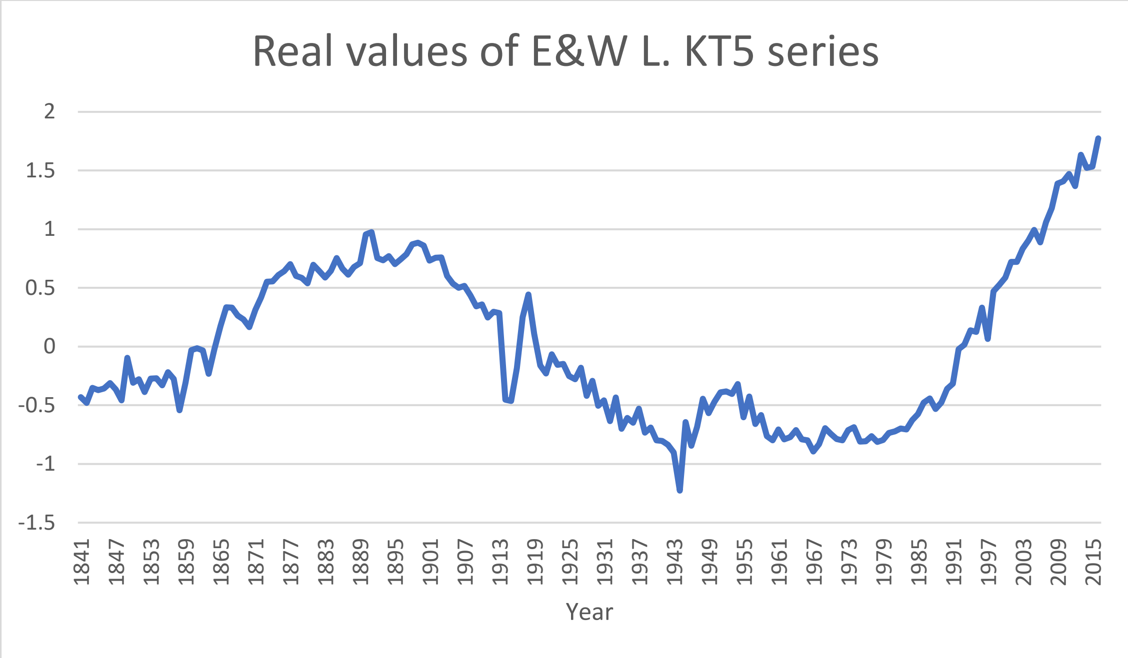
Graphical representations of nine time series (real values from Hatzopoulos and Sagianou, 2020)

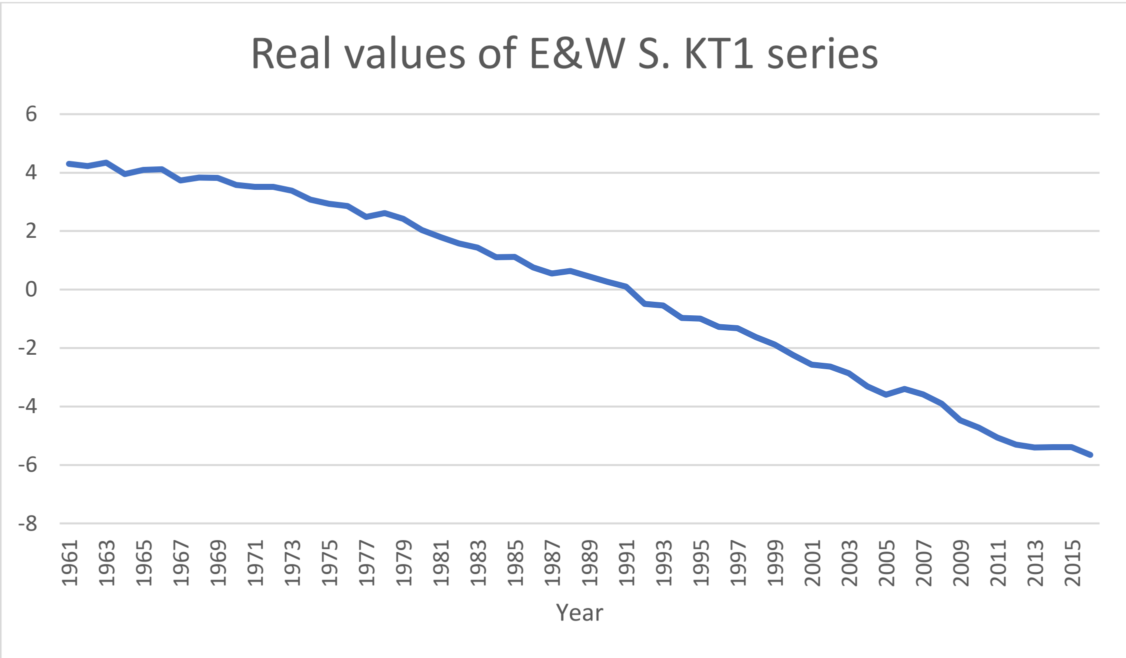





Real values of E&W L. KT3 series
7
6
5
4
3
2
1
0
-1
-2
1841 1846 1851 1856 1861 1866 1871 1876 1881 1886 1891 1896 1901 1906 1911 1916 1921 1926 1931 1936 1941 1946 1951 1956 1961 1966 1971 1976 1981 1986 1991 1996 2001 2006 2011 2016
Year


Real values of E&W L. KT4 series
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1841 1847 1853 1859 1865 1871 1877 1883 1889 1895 1901 1907 1913 1919 1925 1931 1937 1943 1949 1955 1961 1967 1973 1979 1985 1991 1997 2003 2009 2015
Year


Real values of E&W L. KT5 series
2
1.5
1
0.5
0
-0.5
-1
-1.5
1841
1847
1853
1859
1865
1871
1877
1883
1889
1895
1901
1907
1913
1919
1925
1931
1937
1943
1949
1955
1961
1967
1973
1979
1985
1991
1997
2003
2009
2015
Year


Real values of E&W S. KT1 series
6
4
2
0
-2
-4
-6
-8
1961
1963
1965
1967
1969
1971
1973
1975
1977
1979
1981
1983
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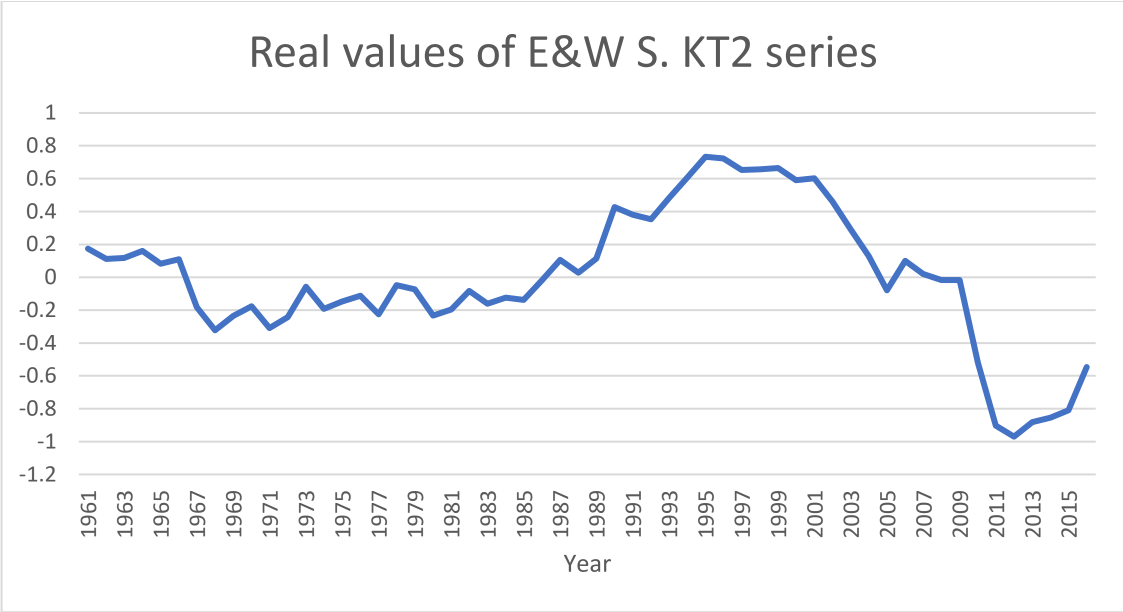
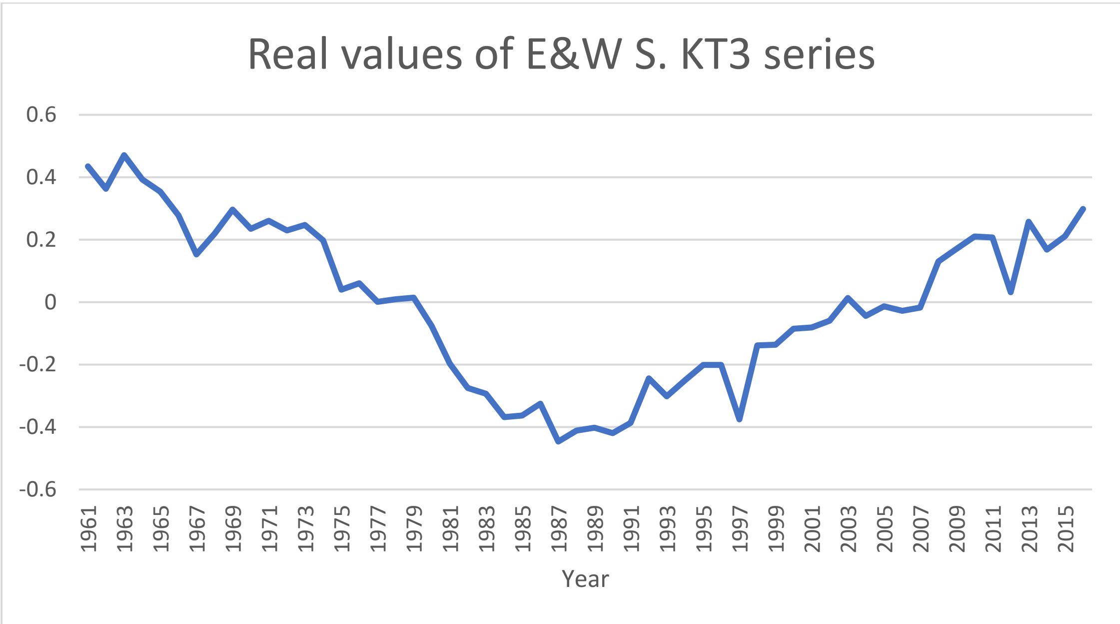
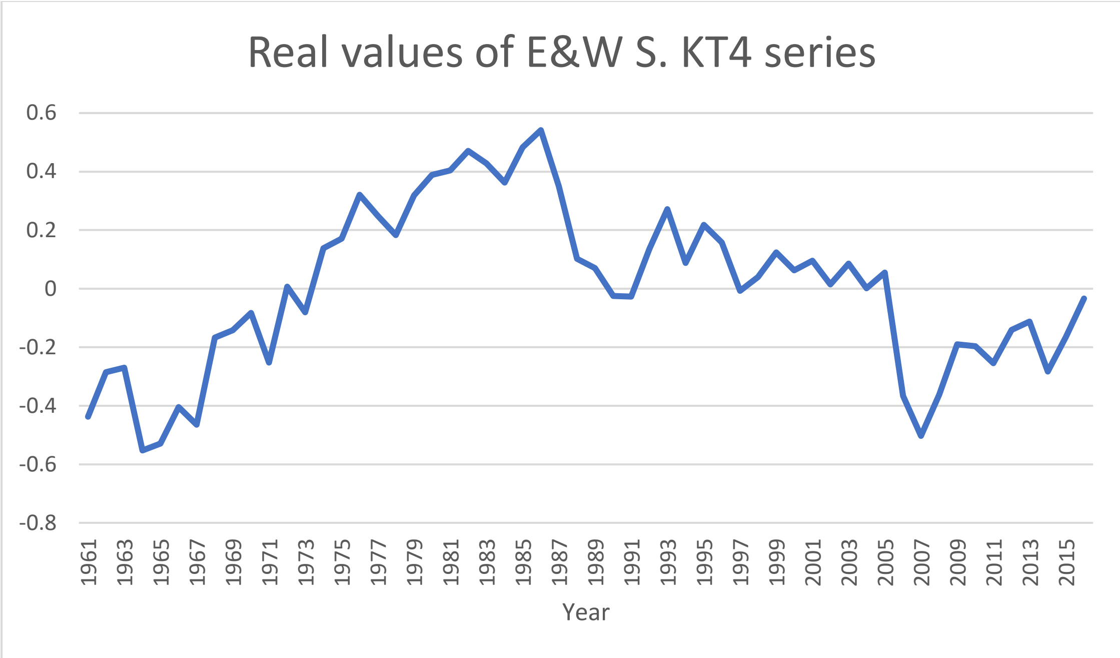
2009
2011
2013
2015
Year


Real values of E&W S. KT2 series
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015
Year


Real values of E&W S. KT3 series
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015
Year



Real values of England and Wales (Long series)

Year	kt1	kt2	kt3	kt4	kt5
1841	9.406559	0.441273	0.340534	-0.37004	-0.43177
1842	9.185418	0.451976	0.234247	-0.38388	-0.47836
1843	9.025542	0.373392	0.274886	-0.27821	-0.3515
1844	9.134703	0.512097	0.205567	-0.31802	-0.37055
1845	8.930115	0.400126	0.364794	-0.26504	-0.35828
1846	9.436748	0.724931	0.48608	-0.19798	-0.31001
1847	10.19779	1.663768	0.347568	-0.323	-0.36626
1848	9.845169	0.811043	0.261165	-0.37868	-0.45839
1849	10.76936	1.402152	0.469259	-0.4813	-0.09612
1850	8.694909	0.454182	0.154583	-0.44346	-0.30924
1851	9.154752	0.516714	0.267628	-0.3677	-0.279
1852	9.357793	0.50101	0.315246	-0.39722	-0.38585
1853	9.473418	1.175795	0.387429	-0.33907	-0.27168
1854	9.861733	0.818543	0.29066	-0.38867	-0.27006
1855	9.273063	1.281446	0.217067	-0.42728	-0.32935
1856	8.619208	0.296136	0.364109	-0.24562	-0.21752
1857	8.85541	0.591006	0.251139	-0.36611	-0.27649
1858	9.498629	0.615806	-0.008	-0.50852	-0.54278
1859	9.195068	0.488738	0.009252	-0.37918	-0.30554
1860	8.504109	0.987727	0.204166	-0.20315	-0.03014
1861	8.600771	0.800649	0.236005	-0.18802	-0.01434
1862	8.808835	0.575686	0.113554	-0.24498	-0.03381
1863	9.494197	0.407489	-0.11469	-0.42614	-0.23245
1864	9.753063	1.128629	0.018184	-0.34084	-0.02248
1865	9.413651	1.22894	0.257283	-0.21902	0.174604

1866	9.454567	1.298288	0.326037	-0.16082	0.336073
1867	8.62634	1.311769	0.345652	-0.11553	0.332153
1868	8.866177	0.715735	0.093229	-0.19001	0.264057
1869	9.021065	1.022769	-0.03262	-0.30392	0.232102
1870	9.275569	1.137637	-0.04804	-0.38286	0.16601
1871	9.378483	1.171261	0.388549	-0.16458	0.31376
1872	8.838388	1.007421	0.430325	-0.11858	0.419703
1873	8.248653	1.391788	0.105687	-0.11617	0.55161
1874	8.820999	1.298686	-0.21994	-0.16356	0.55461
1875	8.778074	1.902735	-0.13935	-0.17713	0.609171
1876	8.298597	1.263753	-0.02645	-0.09708	0.642026
1877	8.083424	1.319041	-0.06821	-0.17353	0.702001
1878	8.250983	1.461638	-0.33963	-0.30013	0.602108
1879	7.962873	1.90462	-0.44797	-0.32862	0.585402
1880	7.790545	1.176741	-0.61555	-0.22931	0.538167
1881	7.536324	1.278653	-0.30623	-0.28935	0.695925
1882	7.68757	1.001852	-0.49416	-0.18114	0.645095
1883	7.730816	1.384802	-0.40862	-0.22977	0.587299
1884	7.567067	1.157141	-0.44836	-0.1304	0.646775
1885	7.211885	1.488529	-0.41508	-0.09351	0.753182
1886	6.983221	1.626087	-0.5123	-0.04722	0.664236
1887	6.977585	1.450969	-0.69416	-0.11939	0.612982
1888	6.696472	1.448489	-0.61736	-0.05108	0.677279
1889	6.583169	1.123837	-0.75676	-0.0335	0.709787
1890	7.303726	1.924172	-0.69422	0.042348	0.956831
1891	7.213832	2.435306	-0.82434	0.087952	0.974675
1892	6.773146	1.879575	-0.91581	-0.05818	0.755528
1893	7.054049	1.556615	-0.78255	-0.07737	0.735361
1894	6.036646	0.774698	-0.70089	0.100924	0.770429
1895	6.241745	1.635662	-0.90523	0.026139	0.702308
1896	6.061251	0.69854	-0.99776	0.064694	0.742947
1897	5.936003	1.12211	-0.85026	0.058154	0.787506
1898	5.782496	1.139784	-0.88322	0.169316	0.870427
1899	6.191783	1.619429	-0.88359	0.1002	0.886344
1900	6.32239	1.812161	-0.93506	0.152807	0.86006
1901	5.808919	1.211604	-0.90691	0.040134	0.733774
1902	5.640219	1.139506	-0.94612	0.015877	0.757742
1903	5.000967	0.84857	-0.94962	0.059382	0.758822
1904	5.166713	1.059286	-1.03914	0.056941	0.602794
1905	4.887616	0.931438	-0.94981	-0.04491	0.537325
1906	4.876333	1.040887	-0.96836	-0.03127	0.502177
1907	4.79737	1.08037	-1.01265	-0.0567	0.518601
1908	4.538492	0.998843	-0.95924	-0.03849	0.436703
1909	4.50152	1.101402	-0.96904	-0.12158	0.342371
1910	3.865284	0.678271	-0.94067	-0.09284	0.359704
1911	4.450961	0.697328	-1.00168	-0.07523	0.246783
1912	3.937987	0.740601	-0.94288	-0.15508	0.297615
1913	3.978869	0.743672	-1.03578	-0.17289	0.28592

1914	6.170658	1.201515	1.716216	1.4731	-0.45262
1915	8.15206	2.018453	3.358247	2.289043	-0.46462
1916	8.661044	2.283439	5.313023	2.907828	-0.182
1917	9.432515	2.39614	6.129154	3.315929	0.251035
1918	11.51175	1.898203	6.001916	2.657597	0.442517
1919	5.315279	1.06339	0.902875	0.018555	0.107934
1920	3.716747	-0.02096	-0.31527	-0.24116	-0.16009
1921	2.862487	0.115241	-0.3607	-0.41199	-0.22834
1922	3.041811	0.52983	-0.48469	-0.27436	-0.06676
1923	2.286655	0.152629	-0.33327	-0.40494	-0.15495
1924	2.482614	0.372654	-0.51565	-0.33099	-0.14785
1925	2.430198	0.316273	-0.59659	-0.41685	-0.25126
1926	1.979997	0.089274	-0.55183	-0.40306	-0.27857
1927	2.301217	0.522808	-0.58205	-0.41718	-0.17893
1928	2.008469	0.160733	-0.4561	-0.52929	-0.42107
1929	2.771887	0.850997	-0.75639	-0.40741	-0.29298
1930	1.799792	-0.00376	-0.37841	-0.42927	-0.50496
1931	1.804766	0.395562	-0.48835	-0.32625	-0.45825
1932	1.46909	0.280754	-0.41704	-0.39462	-0.63566
1933	1.661632	0.413327	-0.35585	-0.48597	-0.43326
1934	1.388786	-0.03403	-0.61326	-0.44903	-0.70109
1935	0.791657	0.151618	-0.45709	-0.50276	-0.60664
1936	0.713726	0.26713	-0.68197	-0.41665	-0.64905
1937	0.618431	0.467885	-0.6071	-0.38954	-0.52798
1938	0.245895	0.004281	-0.50422	-0.4854	-0.73199
1939	-0.36163	0.463447	-0.33325	-0.35792	-0.69028
1940	2.646005	1.415078	1.471524	0.563477	-0.79731
1941	2.244741	0.4912	0.991133	0.172332	-0.80518
1942	1.900382	0.321668	2.745127	1.241869	-0.83795
1943	2.294636	0.68922	3.576003	1.788774	-0.90296
1944	2.395504	0.581522	4.136504	2.007307	-1.22717
1945	2.745823	0.779309	5.282485	2.748276	-0.6432
1946	-2.7172	-0.17772	-0.60922	-0.30731	-0.84606
1947	-2.72332	0.064538	-0.73305	-0.43568	-0.68358
1948	-3.7621	-0.47867	-0.7513	-0.34505	-0.44298
1949	-4.02084	0.052817	-0.76158	-0.37323	-0.56745
1950	-4.56546	0.132936	-0.76415	-0.25141	-0.47245
1951	-4.80681	0.625382	-1.11593	-0.29189	-0.38948
1952	-5.4683	0.062722	-1.05252	-0.18504	-0.38205
1953	-5.87511	0.005491	-1.24555	-0.13211	-0.40449
1954	-6.39709	0.082742	-1.18337	-0.20465	-0.31997
1955	-6.35091	0.195901	-1.12252	-0.25565	-0.60258
1956	-6.70679	0.164256	-1.26594	-0.23553	-0.42501
1957	-6.43109	0.005169	-1.21032	-0.15327	-0.66013
1958	-6.75689	0.051391	-1.2967	-0.12613	-0.58454
1959	-6.72447	-0.02479	-1.17236	-0.0252	-0.76233
1960	-6.72705	-0.14804	-1.13449	-0.01204	-0.79822
1961	-6.65983	0.047267	-1.16252	-0.02078	-0.70547

1962	-6.84888	0.115819	-1.17445	-0.08893	-0.79053
1963	-6.73692	0.197496	-1.19858	-0.06925	-0.77103
1964	-6.879	-0.21909	-0.9472	-0.02506	-0.71012
1965	-6.8069	-0.15696	-1.04916	-0.18911	-0.78913
1966	-6.85151	-0.06466	-1.08837	-0.16814	-0.79967
1967	-7.21064	-0.39096	-1.14349	-0.26227	-0.89387
1968	-7.32584	-0.13121	-1.42857	-0.32261	-0.83184
1969	-7.29896	-0.10728	-1.39548	-0.07854	-0.69477
1970	-7.51336	-0.17779	-1.2168	-0.01106	-0.74333
1971	-7.50395	-0.36641	-1.20528	-0.22815	-0.7887
1972	-7.62895	-0.14834	-1.22724	-0.13628	-0.79921
1973	-7.62446	-0.25376	-1.02808	0.013367	-0.71051
1974	-8.00161	-0.28968	-1.08664	0.118042	-0.68558
1975	-8.14969	-0.34069	-0.95963	0.034711	-0.81064
1976	-8.30339	-0.22803	-0.91412	0.095531	-0.80545
1977	-8.58128	-0.50509	-0.88166	0.125285	-0.76378
1978	-8.4072	-0.4384	-0.71414	0.127387	-0.81114
1979	-8.64474	-0.41917	-0.66825	0.033385	-0.7949
1980	-9.01421	-0.62059	-0.74597	0.198943	-0.73614
1981	-9.22566	-0.73613	-0.6489	0.052348	-0.72156
1982	-9.42058	-0.76237	-0.52091	0.063178	-0.69805
1983	-9.55867	-0.85289	-0.51287	-0.06509	-0.70552
1984	-9.77568	-1.07025	-0.39783	-0.01057	-0.62602
1985	-9.82972	-0.97433	-0.48559	-0.12647	-0.57455
1986	-10.1173	-1.05619	-0.27786	0.19742	-0.47653
1987	-10.1679	-1.29381	-0.10663	0.19114	-0.44125
1988	-9.98528	-1.43431	-0.08285	-0.05216	-0.53152
1989	-10.0785	-1.53101	0.040934	0.093767	-0.47621
1990	-10.1049	-1.61907	0.369533	0.24628	-0.36086
1991	-10.2546	-1.67876	0.418532	0.009004	-0.31579
1992	-10.7882	-1.81329	0.505137	0.079882	-0.02262
1993	-10.8684	-1.722	0.571948	0.07589	0.016795
1994	-11.0929	-1.98943	0.895742	0.018029	0.138065
1995	-11.1349	-1.86725	1.048564	0.056401	0.124574
1996	-11.3115	-2.07952	1.014923	0.294269	0.331763
1997	-11.2649	-2.30658	1.13591	0.171559	0.066509
1998	-11.5026	-2.37197	1.089622	0.255254	0.470111
1999	-11.7518	-2.40715	1.16309	0.378784	0.524498
2000	-11.9917	-2.65065	1.311659	0.241736	0.588484
2001	-12.2612	-2.78466	1.371264	0.329238	0.721289
2002	-12.2861	-2.91364	1.311825	0.146448	0.720663
2003	-12.5278	-3.00412	1.196263	0.200314	0.832253
2004	-12.8994	-3.31672	1.222413	0.053387	0.905025
2005	-13.1779	-3.48302	1.102644	0.103143	0.99397
2006	-12.7414	-3.6672	1.36395	0.0049	0.888468
2007	-12.8195	-3.91046	1.242347	0.179729	1.059078
2008	-13.1153	-3.90469	1.388815	0.267243	1.180214
2009	-13.6654	-4.01599	1.535442	0.192011	1.389146

2010	-13.9478	-4.31287	1.153829	0.064858	1.4079
2011	-14.2987	-4.6122	0.904879	0.073327	1.469949
2012	-14.5702	-4.70672	0.917311	-0.0046	1.36855
2013	-14.6129	-4.65473	0.974383	-0.02484	1.634103
2014	-14.5094	-4.80226	1.09826	-0.093	1.522885
2015	-14.5705	-4.67693	1.112332	-0.08259	1.534016
2016	-14.7577	-4.63156	1.33984	0.126033	1.773912

Real values of England and Wales (Short series)

Year	kt1	kt2	kt3	kt4
1961	4.301986	0.173685	0.435003	-0.43754
1962	4.219873	0.11226	0.36352	-0.28493
1963	4.342125	0.116885	0.470901	-0.27036
1964	3.950505	0.161053	0.392699	-0.55262
1965	4.091849	0.081477	0.353517	-0.52961
1966	4.120349	0.110272	0.278281	-0.405
1967	3.731637	-0.18194	0.15341	-0.46456
1968	3.835809	-0.32251	0.219849	-0.16765
1969	3.816762	-0.23526	0.296697	-0.14183
1970	3.576063	-0.17655	0.234857	-0.08304
1971	3.507844	-0.30937	0.26014	-0.25278
1972	3.515438	-0.2425	0.229906	0.006964
1973	3.377782	-0.05737	0.24756	-0.08048
1974	3.078935	-0.19185	0.197596	0.138187
1975	2.935989	-0.14818	0.040272	0.170607
1976	2.854365	-0.11285	0.060466	0.320502
1977	2.48594	-0.22531	0.000728	0.248742
1978	2.613882	-0.04822	0.009545	0.183038
1979	2.419256	-0.07265	0.014707	0.318338
1980	2.04045	-0.23253	-0.07554	0.388876
1981	1.795774	-0.19619	-0.19695	0.404295
1982	1.575832	-0.08339	-0.27488	0.470878
1983	1.435656	-0.16096	-0.29296	0.427511
1984	1.106022	-0.12391	-0.36795	0.362439
1985	1.12158	-0.13853	-0.3632	0.482955
1986	0.757176	-0.01873	-0.32501	0.541472
1987	0.546843	0.106123	-0.44649	0.35066
1988	0.637189	0.026912	-0.41174	0.101845
1989	0.452606	0.11414	-0.40177	0.070313
1990	0.268212	0.426132	-0.41998	-0.02476
1991	0.102058	0.380843	-0.38682	-0.02752
1992	-0.4909	0.352328	-0.24451	0.136162
1993	-0.54009	0.481956	-0.30123	0.271602
1994	-0.97026	0.606846	-0.25046	0.088189
1995	-0.99123	0.73321	-0.20102	0.218154

1996	-1.27728	0.722824	-0.2008	0.157256
1997	-1.32404	0.653018	-0.37522	-0.00691
1998	-1.62939	0.656834	-0.13804	0.039213
1999	-1.88015	0.665137	-0.13682	0.123897
2000	-2.24544	0.5912	-0.08498	0.063012
2001	-2.57008	0.602276	-0.08091	0.095174
2002	-2.63104	0.460418	-0.05989	0.014817
2003	-2.85969	0.290645	0.013011	0.085745
2004	-3.31022	0.128415	-0.04411	0.00112
2005	-3.59952	-0.08002	-0.01349	0.054481
2006	-3.39632	0.100054	-0.02741	-0.36655
2007	-3.5893	0.020258	-0.01741	-0.50316
2008	-3.89948	-0.01687	0.13	-0.36177
2009	-4.46801	-0.01694	0.17	-0.19038
2010	-4.71937	-0.5194	0.21	-0.19618
2011	-5.06415	-0.90282	0.20764	-0.25425
2012	-5.3052	-0.96981	0.031566	-0.14096
2013	-5.4069	-0.8819	0.257574	-0.11263
2014	-5.39659	-0.85365	0.168262	-0.28288
2015	-5.39539	-0.80913	0.211596	-0.16422
2016	-5.65576	-0.54585	0.298683	-0.03388

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