Implications of market and political power interactions for growth and the business cycle II: Politico-economic equilibrium

Tryphon Kollintzas
Vanghelis Vassilatos
IMPLICATIONS OF MARKET AND POLITICAL POWER INTERACTIONS FOR GROWTH AND THE BUSINESS CYCLE II:
POLITICO-ECONOMIC EQUILIBRIUM

Tryphon Kollintzas
Athens University of Economics and Business

Vanghelis Vassilatos
Athens University of Economics and Business, Econometrics Laboratory and EMOP

Abstract
Motivated by the politico-economic systems encountered in many countries all over the globe, including those of several Southern European countries. In this paper we follow a Ramsey type optimal policy approach to endogenize government policy in the two sector DSGE model with market and political power interactions developed in a companion paper. We thus obtain what we call the politico-economic equilibrium. That is, a contingency plan for the economy’s resource allocation and government policy variables that optimize the government’s objective function, subject to the private sector equilibrium. The government’s objective function seeks a balance between pursuing the interests of insiders and the interests of the representative household. The latter are in line with what Jean Tirole calls government pursuing policies for the “the common good.” We take the interests of insiders to be represented by the expected value of their income. The combination of these two defines what we call the “Hybrid” government. We then investigate the growth implications of the politico-economic equilibrium, focusing, first, on the steady state comparison of the hybrid government politico-economic equilibrium relative to the Second Best allocation implied by the Canonical Real Business Cycle economy; and second, on the asymptotic steady states of the politico-economic equilibrium of a Hybrid government in a detailed economy, for different degrees of insiders’ influence in government. We find that increasing influence of insiders in government decision making is quite bad for the economy. The degree of influence of insiders is a deep parameter of the model that can be estimated in the data and thereby rank countries accordingly. The extent of this influence may explain the different macroeconomic performance observed among countries that, ceteris paribus, enjoy a similar state of development.

JEL classification: E20, E32, H42, J51, P16

Keywords: Growth, Optimal Policy, Insiders-Outsiders, Politico-economic Equilibrium

Acknowledgements: The authors are grateful to the Bank of Greece for financial support. We wish to clarify that the views expressed herein do not necessarily express the views of the Bank of Greece. We are grateful to Tryphon Christou and Vassiliki Dimakopoulou for expert computational support. We also thank Alekos Papadopoulos, and an anonymous referee for constructive comments and suggestions. All errors are our own.

Correspondence:
Vanghelis Vassilatos
Athens University of Economics and Business
76 Patission Street, 104 34 Athens, Greece
E-mail: vvassila@aueb.gr
1. Introduction

As the title indicates, this paper is the second part of a line of research on the market and political power interactions that characterize politico-economic systems encountered in many countries all over the globe, including those of several Southern European countries. In the first part, the companion Kollintzas et al. (2021) Bank of Greece working paper No 288, based on Kollintzas et al. (2018a), we introduced certain market and political power interactions in a dynamic stochastic general equilibrium model (DSGE), to investigate the effect of these interactions on the private sector equilibrium (PSE), given policy, focusing on growth and the business cycle. In this second part we follow a Ramsey type optimal policy approach to endogenize government policy, obtaining what we call the politico-economic equilibrium (PE). Then, we study the way this politico–economic equilibrium is affected in the long run by alternative government objectives. We thereby establish a channel where potentially different policy decisions, based on alternative government objectives affect growth and the business cycle.

The private sector equilibrium, given government policy, including the share of insiders’ industries in the economy, is characterized by four distortions. The first two consist of an output price markup and a wage premium for insiders over outsiders, in all insiders’ industries. These two distortions imply lower output and lower capital and labor inputs than what would have been the case if insiders’ industries were competitive. The third distortion relates to the fact that, as different industry outputs are partial complements, the lower output in all insiders’ industries lowers the demand for output in all outsiders’ industries, lowering aggregate output. The fourth distortion is brought about by the fact that, any strictly positive share of insiders’ industries in the economy results in higher output share of government spending, reflecting the costly adjustment and maintenance of the politico-economic system, further increasing misallocation vis-a-vis a politico-economic system with no insiders. The combination of the above four distortions imply that steady state GDP per head of the model economy will be lower than what would have been the case without the underlying frictions. This is what could serve as an explanation of the relatively poor growth performance of the South European economies.
In addition, the transition dynamics and steady state properties of the PE, where the share of insiders’ industries is endogenous, are richer compared to those of the PSE where this share is exogenously fixed. Both in the PSE given policy and the PE, a greater share of insiders industries will lead to lower steady state per capita GDP and thereby deeper and longer recessions. This effect is along the lines suggested by Cole and Ohanian (2004) for the explanation of the Great Depression. Moreover, the shock propagation mechanism of the politico-economic equilibrium will be also affected, since, as the share of insiders’ industries is endogenous, insiders try to avoid painful adjustments in their sector. In this paper we limit our analysis to steady state comparisons.\(^1\)

The plan of this paper has as follows: In Section 2 we endogenize the share of insiders’ industries in the economy by defining and characterizing the economy’s politico-economic equilibrium (PE). That is, a contingency plan for the economy’s resource allocation and government policy variables that optimize the government’s objective function subject to the private sector equilibrium (PSE) constraints. The government’s objective function seeks a balance between pursuing the interests of insiders and the interests of the representative household. The latter, in our homogeneous households set up (i.e., each representative household comprises of both insiders and outsiders), is in line with what Jean Tirole calls government pursuing policies for the “the common good.”\(^2\) On the other hand, in line with Daron Acemoglu’s notion of “political elites” that influence government decision making, we take the interests of insiders to be represented by the expected value of their income (i.e., labor income and profits in all insiders’ industries).\(^3\) Further, combining the aforementioned interests and in line with the political economy tradition of Persson and Tabellini (2002), we define the government’s objective to be a weighted average welfare loss function. This is what we call the “Hybrid” government.

The underlying welfare losses are measured with respect to what would have been achieved in the long run if the government either cared only and exclusively for the representative household (as in the case of a Median Voter government); or cared only and exclusively for the welfare of insiders (as in the case of the Elite government). We

---

1. The shock propagation mechanism of the PE is currently being investigated in another paper.
2. See Tirole (2017). Although tempting to associate the interests of outsiders with those of the representative household we refrain in doing so. In our full household insurance set up, household income is the sum of incomes of both its outsiders and insiders members.
show that in the case of insiders, this reference point coincides with a situation where all agents in the economy are insiders. And, we show that reference point for the representative household coincides with a situation where there are no insiders. Despite their intuitive appeal, these reference points present challenges in solving and computing the PE allocation. However, we show that there exists a representation of the PE allocation that circumvents these problems by the introduction of a parameter representing the elasticity of substitution across insiders and representative household welfare losses in the government objective function. This elasticity is a parameter of the model that also depends on the relative weight associated with the welfare loss of insiders in the government decision making. This weight, denoted by \( \rho \), is a crucial deep parameter of the model that can serve as a bridge of our theoretical model with the data. That is, different values of \( \rho \) are associated with countries with different politico-economic structures. For example, the politico-economic structure of South European countries can be thought of as being characterized by a relatively higher value of \( \rho \) compared to the US or the European Core. Hence, we believe that an additional contribution of this paper is that this parameter can be estimated in the data and thereby rank countries accordingly. The extent of this influence may explain the different macroeconomic performance observed among countries that, ceteris paribus, enjoy a similar state of development.

Thus, we define and solve for the politico-economic equilibrium of the Hybrid government, under full commitment, using the Primal approach. We derive the underlying isoparametric constraint and we characterize the first order conditions for the equilibrium and its asymptotic steady state. We show that the asymptotic steady can be computed by a convenient semi-analytic equation system that substitutes the weights in the objective function of the Hybrid government for the elasticity of substitution across insiders and outsiders’ utility in the government objective function.

In Section 3 we investigate the growth implications of the politico-economic equilibrium established in the previous section. We are primarily interested in two comparisons. First, to compare the asymptotic steady state of the Hybrid government in a detailed economy to the Canonical RBC economy. Second, to compare the asymptotic steady states of the politico-economic equilibrium of a Hybrid government in a detailed economy, for different degrees of insiders’ influence in government. Also, we examine the sensitivity of the results of these comparisons for different values of the parameters characterizing the structure of the detailed economy (e.g., union
bargaining power, union preferences for wage premia, monopoly power in insiders’ industries and the underlying costs to maintain and adjust the insiders – outsiders structure of the economy).

We find that increasing influence of insiders in government decision making is bad for the economy. This may also serve as an explanation to why the macroeconomic performance of some countries is worse relative to that of others, that, ceteris paribus, enjoy similar state of development.

Finally, Section 4 offers some concluding remarks.

2. Politico-economic equilibrium

2.1 Constraints faced by Government

We have already assumed that the capital income tax rate, \( \tau^K_t \); and, the GDP share of conventional government spending, \( \psi_t \), are exogenously determined. Consequently, in any given period \( t \), the only endogenous government policy variables are the labor income tax rate in period \( t \), \( \tau^L_t \); and, the share of insiders’ industries in the economy at the beginning of period \( t+1 \), \( \chi_{t+1} \). A government (contingency) plan, \( \{ \tau^L_t, \chi_{t+1} \}_{t=0}^\infty \), will be formally defined later. But, it is clear that any such plan must satisfy the private sector equilibrium (PSE).

2.1.1 The Private Sector Equilibrium (PSE) Revisited

In what follows, we repeat the equations characterizing the PSE using a unifying format based on the wedges. As may have been suspected, given Proposition 2 (PSE of the detailed economy with \( \chi_t \in (0,1), \forall t \in \mathbb{N}^+ \) or the insiders-outsiders economy), Remark 8 (PSE of the prototype economy with \( \chi_t \in (0,1), \forall t \in \mathbb{N}^+ \) or the two-sector RBC economy), and Remark 9 (PSE of the prototype economy with \( \chi_t = 0, \forall t \in \mathbb{N}^+ \) or the canonical RBC economy), the same set of equations can characterize the PSE in all possible cases, given the appropriate specification of the wedges for each particular

---

4 Throughout the paper there are numerous references on the propositions (1-3) and Remarks (1-9) presented and discussed in the companion Kollintzas et al. (2021) Bank of Greece working paper No 288. To distinguish from the Propositions and Remarks of the companion paper, in this paper all propositions and remarks will be preceded by the prefix “II”. Appendix I summarizes the notation introduced therein and naturally adopted in this paper as well.
This allows for a unifying representation of the equations of the PSE for both the detailed economy and the prototype economy cases, shortening the presentation and more importantly, highlighting the differences between alternative economies.

**Remark II-1 (Unifying Representation of Private Sector Equilibrium):** Given the necessary and sufficient conditions for a PSE given in Proposition 2 and Remarks 8 and 9, the necessary and sufficient conditions for a PSE in all possible economy specifications are given by:

**Aggregate state variables:**

Aggregation production function:

\[ y_t = \bar{\omega}_t(\chi_t)k_t^{\alpha}h_t^{(1-\alpha)} \]  

(2.1)

Resource constraint:

\[ y_t = c_t + [(1+\eta)k_{t+1}-(1-\delta)k_t] + g_t \]  

(2.2)

Government budget constraint:

\[ g_t = \left\{ \tau_t^k \left[ \alpha \omega_t(\chi_t) - \delta \left( \frac{k_t}{y_t} \right) \right] + \tau_t^c (1-\alpha) \omega_t(\chi_t) \right\} y_t \]  

(2.3)

Intratemporal condition:

\[ \frac{1-\zeta}{\zeta} \frac{c_t}{1-h_t} = (1-\tau_t^c)(1-\alpha) \omega_t(\chi_t) \frac{y_t}{h_t} \]  

(2.4)

Euler condition for capital:

\[ \frac{u_t}{c_t} = \frac{\beta}{(1+\eta)} \left[ \frac{u_{t+1}}{c_{t+1}} \left[ 1 + (1-\tau_{t+1})[\omega_t(\chi_{t+1}) \alpha \frac{y_{t+1}}{k_{t+1}} - \delta] \right] \right] \]  

(2.5)

Transversality condition:

\[ \beta^T E_0 \frac{u_T k_T}{c_T} \to 0 \text{ as } T \to \infty \]  

(2.6)

Initial condition:

\( (k_0, \chi_0) \in (0, \infty) \times (0.1) \text{ given } \)  

(2.7)
**Prices and Dividends:**

**Real Rental Cost of Capital:**

\[ r_t = \omega_t(\chi_t) \frac{y_t}{k_t} \]  

(2.8)

**Dividends:**

\[ \chi_t w^i_t h^i_t + (1 - \chi_t) w^o_t h^o_t + \chi_t \bar{\pi}_t = (1 - \alpha) \delta_t(\chi_t) y_t \]  

(2.9)

where the sector wages and all functions of the share of insiders’ industries in the economy, \( \chi \), characterising the aggregate state of the PSE (i.e., Equations (2.1) – (2.7)), are given in Table 2.1, for each alternative economy specification.

Note, that in the prototype economy (i.e., the Canonical RBC and the two sector RBC economy specifications) the cases where \( \chi_t = 0 \), \( \forall t \) and \( \chi_t = 1 \), \( \forall t \) are essentially the same and coincide with the Canonical RBC: the case where all are outsiders \( (\chi = 0) \) is the same with the case where all insiders behave like outsiders \( (\chi = 1) \). Likewise, when it comes to the detailed economy, we do not consider the case where \( \chi_t = 0 \), \( \forall t \), because, in this case, the detailed economy is identical to the Canonical RBC specification. For that matter, in Table 2.1 as well as in Table 2.2 that follows, we report the prototype economy for \( \chi_t = 0 \), \( \forall t \) (i.e the Canonical RBC) and \( \chi_t \in (0,1) \) as well as the detailed economy for \( \chi_t = 1 \), \( \forall t \) and \( \chi_t \in (0,1) \).
Table 2.1: Functions of the Share of Insiders Industries in the Economy, $\chi$, in the PSE Representation for Alternative Economy Specifications

<table>
<thead>
<tr>
<th></th>
<th>Prototype Economy</th>
<th>Detailed Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_t = 0, \forall t \in \mathbb{R}^+$</td>
<td>$\chi_t \in (0,1), \forall t \in \mathbb{R}^+$</td>
<td>$\chi_t \in (0,1), \forall t \in \mathbb{R}^+$</td>
</tr>
<tr>
<td></td>
<td>$\chi_t = 1, \forall t \in \mathbb{R}^+$</td>
<td></td>
</tr>
<tr>
<td><strong>Canonical RBC</strong></td>
<td><strong>Two Sector RBC</strong></td>
<td><strong>Insiders – Outsiders</strong></td>
</tr>
<tr>
<td></td>
<td><strong>All Insiders</strong></td>
<td></td>
</tr>
<tr>
<td>$w^o_t$</td>
<td>$(1 - \alpha) \frac{y_t}{h_t}$</td>
<td>$(1 - \alpha) \omega_t (\chi_t) \frac{y_t}{h_t}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-$</td>
</tr>
<tr>
<td>$w^i_t$</td>
<td>$(1 - \alpha) \frac{y_i}{h_i}$</td>
<td>$(1 - \alpha) \nu \omega_t (\chi_t) \frac{y_i}{h_i}$</td>
</tr>
<tr>
<td></td>
<td>$(1 - \alpha) \nu \xi \frac{y_i}{h_i}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>1</td>
<td>$1 + \theta \Delta_t (\chi_t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>1</td>
<td>$1 + \xi \Delta_t (\chi_t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\xi$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>$A_t B_t^o \frac{\delta (1 - \phi) + \delta}{\delta (1 - \phi) + \delta} \left[1 + \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}}$</td>
<td>$A_t B_t^o \frac{1 - \chi_t}{1 + \Delta_t (\chi_t)}^{\frac{1 - \delta}{\delta}} \left[1 + \theta \Delta_t (\chi_t) \right]^{\frac{\delta}{\delta}} \left[1 + \xi \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_t B_t^o \left(1 - \chi_t \right)^{\frac{1 - \delta}{\delta}} \left[1 + \xi \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}} \left[1 + \xi \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}}$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>1</td>
<td>$1 + \theta \Delta_t (\chi_t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>1</td>
<td>$1 + \xi \Delta_t (\chi_t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\xi$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>$A_t B_t^o \frac{\delta (1 - \phi) + \delta}{\delta (1 - \phi) + \delta} \left[1 + \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}}$</td>
<td>$A_t B_t^o \frac{1 - \chi_t}{1 + \Delta_t (\chi_t)}^{\frac{1 - \delta}{\delta}} \left[1 + \theta \Delta_t (\chi_t) \right]^{\frac{\delta}{\delta}} \left[1 + \xi \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_t B_t^o \left(1 - \chi_t \right)^{\frac{1 - \delta}{\delta}} \left[1 + \xi \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}} \left[1 + \xi \Delta_t (\chi_t) \right]^{\frac{1 - \delta}{\delta}}$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>1</td>
<td>$1 + \theta \Delta_t (\chi_t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\bar{\omega}_t (\chi_t)$</td>
<td>1</td>
<td>$1 + \xi \Delta_t (\chi_t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\xi$</td>
</tr>
<tr>
<td>( \Delta_i(\chi_i) )</td>
<td>(-)</td>
<td>( \left( \frac{B'}{B^0} \right)^{\frac{i}{2}} \left( \frac{\chi_i}{1 - \chi_i} \right)^{\frac{\theta(1-\phi)+\phi}{\theta(1-\phi)}} )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( \Delta_i^\xi(\chi_i, \chi_{i+1}) )</td>
<td>( \vec{\psi}_i )</td>
<td>( \vec{\psi}_i )</td>
</tr>
</tbody>
</table>

where, given [R1] and [R2] (**):

\[
\nu = \frac{1}{(1-\theta)\lambda} > 1 \quad \xi = \frac{(1-\alpha)\theta + [1 - \alpha \theta - (1-\alpha \theta) + (1-\theta)]\lambda}{(1-\alpha)(1-\lambda) + (1-\alpha)\theta} \left( \frac{\mu}{1-\mu} \right) < 1 \quad \omega = \frac{1 - \alpha \theta}{1-\alpha} > 1
\]

(*) Since in this case there are no insiders’ industries, we adopt the convention: \( B^0_i = B^i_i \)

(**) Assumptions [R1], [R2] were introduced and discussed in the companion paper (Kollintzas, et al., 2021) and are also quoted in Appendix I.

**Note:** 1) Prototype economy: An economy without distinction between insiders and outsiders, where the only friction is distortionary factor income taxes; 1a) Canonical RBC economy: A prototype economy with one competitive final good sector; 1b) Two sector RBC economy: A prototype economy with two competitive final good sectors; 2) Detailed economy: An economy where in addition to distortionary taxation there are market power frictions; 2a) Insiders-Outsiders: A detailed economy with one competitive final good sector (outsiders) and one non-competitive final good sector (insiders); 2b) All Insiders: A detailed economy with one non-competitive final good sector.
2.1.2 The Implementability Constraint

As already mentioned, any government policy plan, \( \{ \tau^L_t, \chi_{t+1} \}^\infty_{t=0} \), is constrained by the equations characterizing the PSE as well as the government budget constraint, specified in the preceding subsection. We follow the so-called primal approach of the literature on optimal fiscal policy, whereby most of the equations characterizing the aggregate state of the PSE are summarized in the so-called implementability constraint. Simultaneously, we express consumption and the labor income tax rate in terms of the aggregate state variables. Specifically, in Part a of Proposition II-1, that follows, we show that the equations characterizing the aggregate state of the PSE, with the exception of the resource constraint and the initial condition, can be incorporated into the implementability constraint. And, by incorporating the government spending policy specification for each economy into the resource constraint, we express consumption in terms of the exogenous state, \( z_t \) and the endogenous aggregate state variables: \( k_t, \chi_t, h_t, k_{t+1}, \chi_{t+1} \). Similarly, in Part b, by incorporating the government spending policy specifications into the government budget constraint, we express the labor tax rate in terms of the exogenous state, \( z_t \) and the endogenous aggregate state variables: \( k_t, \chi_t, h_t, k_{t+1}, \chi_{t+1} \).

Proposition II-1: (a) The equations characterizing the aggregate state of the PSE, (2.1) - (2.7), are equivalent to the following three conditions:

(i) The “Implementability Constraint” (IC):

\[
E_0 \sum_{i=0}^{\infty} \beta^i \tilde{\xi} (h_t, \chi_t; z_t) u^h(c_t, h_t) = \tilde{\xi} (c_0, h_0, k_0, \chi_0; z_0) u^h(c_0, h_0) \tag{2.10}
\]

where:

\[
\tilde{\xi} (c_0, h_0, k_0, \chi_0; z_0) \equiv \frac{\alpha (1 - \tau_0^k) \tilde{\omega}_h (\chi_0) \tilde{\omega}_h (\chi_0) k_0^a h_0^{1-a} + \left[ 1 - (1 - \tau_0^k) \delta \right] k_0}{c_0} \tag{2.11}
\]

\[
\tilde{\xi} (h_t, \chi_t; z_t) \equiv 1 - \sigma_t (\chi_t) \frac{(1 - \delta) h_t}{\delta (1 - h_t)} \tag{2.12}
\]

and, given restrictions [R1] - [R2]:

\[5\text{ For the methodology, see for example, Chari et al. (1994) and Ljungqvist and Sargent (2018, Chapter 16).}\]
\( \sigma_t(\chi_t) \equiv \frac{\omega_t(\chi_t)}{\omega_i(\chi_t)} = \begin{cases} 
1, & \text{in the prototype economy with } \chi_t = 0, \forall t \in \mathbb{R}^+ \\
1, & \text{in the prototype economy with } \chi_t \in (0,1), \forall t \in \mathbb{R}^+ \\
\frac{1+\alpha_\chi(\chi_t)}{1+\xi_\chi(\chi_t)} > 1, & \text{in the detailed economy with } \chi_t \in (0,1), \forall t \in \mathbb{R}^+ \\
o > 1, & \text{in the detailed economy with } \chi_t = 1, \forall t \in \mathbb{R}^+. 
\end{cases} \)

(ii) The “government spending policy augmented resource constraint” (GSPARC):

\[ c_t = \left[ 1 - \Delta^s_t(\chi_t, \chi_{t+1}) \right] \bar{\omega}_i(\chi_t) k_t^{\alpha} h_t^{(1-\alpha)} - \left[ (1+\eta)k_{t+1} - (1-\delta)k_t \right] \]

where, the government spending policy function \( \Delta^s_t(\chi_t, \chi_{t+1}) \) is given in Table 2.1 for all specifications of the economy.

(iii) The initial condition, (2.7).

(b) Along any PSE, the labor tax rate is set according to the “government spending policy augmented government budget constraint” (GSPAGBC):

\[ \tau^L_t = \frac{\Delta^s_t(\chi_t, \chi_{t+1}) - \tau^K_t \left[ \alpha \bar{\omega}_i(\chi_t) - \frac{\delta}{\bar{\omega}_i(\chi_t)} \left(\frac{k_t}{h_t}\right)^{(1-\alpha)} \right]}{(1-\alpha)\bar{\omega}_i(\chi_t)} \]

Proof: In Appendix II.

In view of Proposition II-1, any government plan, \( \{r^L_t, \chi_{t+1}\}_{t=0}^\infty \), that adheres to the PSE equations and the government budget constraint, can be determined once the allocation \( \{h_t(z'), k_{t+1}(z'), \chi_{t+1}(z')\}_{t=0}^\infty \) has been determined. Evidently, the preceding result is useful only if the criterion function of government can also be expressed in terms of the allocation \( \{h_t(z'), k_{t+1}(z'), \chi_{t+1}(z')\}_{t=0}^\infty \). This will turn out to be the case below, despite the fact that, we account for both the interests of insiders and outsiders and these interests are expressed in terms of sectoral variables. Thus, in what follows labor taxes will be eliminated from the equations characterizing the aggregate state of the PSE, using the GSPAGBC, (2.15). Subsequently and as already mentioned, we will follow the Primal approach and choose an
allocation, $\{h_t(z'), k_{t+1}(z'), \chi_{t+1}(z')\}^\infty_{t=0}$, subject only to the three constraints: the IC, (2.10), the GSPARC, (2.14) and, the Initial condition, (2.7).

### 2.2 The Government Objective

Next, we need to characterize formally the solution to the government’s problem, or what we refer to as a politico-economic equilibrium (PE). To do so, we have to specify the government’s objective function. Naturally, this specification should capture the basic features of the situation we are interested in modeling. That is, governments that to some extent are influenced by insiders’ interests. We restrict our attention to the case where government seeks a balance between pursuing the interests of insiders and the interests of the representative household. The latter, in our homogeneous households setup, is in line with what Jean Tirole calls policies for the “the common good.”

Thus, we take the “common good” policies to be those that maximize the expected value of the utility function introduced in Section II:

$$U_0^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{\gamma_t^\gamma (1-h_t)^{1-\gamma}}{1-\gamma} \right] \right\}$$

(2.16)

Further, following Kollintzas et al. (2018a) we assume that insiders’ interests are represented by the expected value of the discounted future stream of their income. That is, labor income and profits in all insiders’ industries:

$$U_0^i = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \int_0^{\chi_i} \left\{ w_i(\xi) l_i(\xi) + \pi_i(\xi) \right\} d\xi \right] \right\}$$

(2.17)

It would appear that this specification of insiders’ interests would entail problems for the analysis, as it involves industry specific variables. However, as the following remark makes clear, using the PSE relationships, the discounted future stream of insiders’ income can be re-expressed exclusively in terms of the aggregate state variables. Moreover, this remark shows that the discounted future stream of insiders’ income is strictly increasing in the share of insiders’ industries in the economy.

---

7 This is, in a way, inconsistent with the complete household insurance hypothesis and strictly speaking, it could be justified only by appealing to some type of myopic and shellfish individuals that seek to maximize their aggregate own income or that there is an idiosyncratic benefit to individuals having their income from insiders’ industries that is not modeled explicitly.
This type of preferences for insiders as a whole is consistent with the incentive of each group of insiders to cooperate with other groups of insiders in order to influence government in maintaining their insider status and promoting their special interests. In particular, as shown in the remark that follows, ceteris paribus, insiders as a whole prefer an economy with more insiders. This in turn implies that we subsequently model government decision making in such a way so as to choose an economy with a bigger share of insiders’ industries. However, it should be pointed out at this stage that insiders as whole, by preferring a bigger share of insiders’ industries in the economy, they are responsible for the creation of a negative externality that, as will be shown later in the simulations, may prove to be detrimental to their own wellbeing, T, as well. The choice of this kind of preferences captures the essence of the political distortion induced by the very existence of insiders.\(^8\)

**Remark II-2:** (a) Given the PSE representation in Remark II-1, it follows that:

\[
U'_0 = E_0 \sum_{t=0}^{\infty} \beta^t v^t \left[ h_t(z^t), k_t(z^{t-1}), \chi_t(z^{t-1}); z_t \right]
\]

where:

\[
v^t \left[ h_t(z^t), k_t(z^{t-1}), \chi_t(z^{t-1}); z_t \right] \equiv v^t_t(h_t, k_t, \chi_t) \equiv \hat{\omega}_t(\chi_t)k_t^{a_t}h_t^{1-a_t}
\]

\[
\hat{\omega}_t(\chi_t) \equiv \hat{\omega}_t(\chi_t)\hat{\omega}_t(\chi_t)
\]

\[
\hat{\omega}_t(\chi_t) \equiv (1-\alpha \theta) \frac{\Delta_t(\chi_t)}{1+\Delta_t(\chi_t)}
\]

(b) If \( \phi \in \left( -\frac{\theta}{1-\theta}, 0 \right) \) and \([R3]\) hold, then: \( \hat{\omega}(\chi): (0,1) \to (0,1); \)

\[
\lim_{\chi \to 0} \hat{\omega}(\chi) = 0, \ \lim_{\chi \to 1} \hat{\omega}(\chi) = 1; \ \text{and} \ \hat{\omega}(\chi) > 0, \forall \chi \in (0,1).
\]

**Proof: In Appendix II.**

\(^8\) Kollintzas and Pechlivanos (2022) study the existence of a coalition of simplified versions of the unions and entrepreneurs in insiders industries of this model. First they study the conditions under which these coalitions are stable. Second, they study the conditions under which stable coalitions are electable, over a regime where there are only outsiders. They find two cases where the latter is true. First, if there is a critical fraction of individuals that perceive that the benefits from being an outsider in a society consisting only of outsiders are lower than the expected benefits of being an insider in the insiders – outsiders regime. Second, if individuals irrationally believe that the probability of being an insider in an insiders - outsiders regime is higher than the corresponding equilibrium probability. The first condition relates to situations where perceptions about a common regime could be manipulated and the second to Pasarelli and Tabellini ( 2017) individuals, who think that they are better than themselves. In both cases there is a lower bound on the share of insiders’ industries in the economy.
Further, we are interested in operationalizing the degree insiders influence governments. In so doing, we follow the political economy literature (see, e.g., Persson and Tabellini (2002, Ch.7)), considering a hybrid government objective function, where, as already mentioned, to some degree government is influenced by representative household preferences and is influenced likewise by insiders’ preferences (henceforth, “Hybrid Government”). Moreover, to avoid scale problems, we follow Kollintzas et al. (2018a) in postulating that government seeks to minimize a weighted average of the percentage deviations of: (a) the welfare of the representative household under the Hybrid Government from the household welfare that would had been achieved in the long run if government cared only about the representative household (Median Voter Government) and (b) the insiders’ welfare under the Hybrid Government from the insiders’ welfare that would had been achieved in the long run if government cared only about insiders (Insiders’ Government). In this set up the degree of insiders influence is the weight of the welfare loss of insiders in the government objective function. That is, we consider a government objective function of the form:

\[ U_0^i = \rho \frac{\bar{U}^i - \bar{U}^i_0}{\bar{U}^i} + (1 - \rho) \frac{\bar{U}^h - \bar{U}^h_0}{\bar{U}^h}; \quad \rho \in (0, 1) \]  

(2.22)

where: \( \bar{U}^h \) is the maximum value of the household utility function (i.e., the RHS of (2.16)) over all possible steady states of the PSE, given the share of insiders’ industries, and for all possible shares of insiders industries (i.e., \( \forall \chi \in [0, 1] \)); \( \bar{U}^i \) is the maximum value of the discounted future stream of insiders’ income (i.e., the RHS of (2.18)) over all possible steady states of the PSE, given the share of insiders industries, and for all possible shares of insiders industries; and \( \rho \) is the degree of insiders’ influence in government decision making. On the contrary, \( 1 - \rho \) reflects the degree government decisions are influenced by the “common good.”

\[ \text{Since we allow for the share of insiders industries to take all possible} \]

\[ \text{One of the motivations of this paper was to seek a way to operationalize the “varieties of capitalism” and} \]

\[ \text{“neo-corporatism” ideas of the political science literature and in particular the institutional complementarity} \]

\[ \text{between the political system and groups in society enjoying market power and the associated influence of the} \]

\[ \text{latter groups in government’s decision making (see Kollintzas, et al 2018 and the references therein.)} \]

\[ \text{Kollintzas et al (2018b) also provide empirical evidence on the existence and repercussions of market and} \]

\[ \text{political power interactions. The latter is captured by parameter } \rho. \text{ Here, this is as an exogenously given} \]

\[ \text{deep parameter, in accordance to the political economy literature tradition of aggregating preferences among} \]

\[ \text{different groups in society (see, e.g. Persson and Tabellini, 2002). That been said, one of the contributions of} \]

\[ \text{this paper is the development of a DSGE model where } \rho \text{ is an additional deep parameter that can be estimated} \]

\[ \text{from the data.} \]

\[ \text{15} \]
values in the definitions of \( \tilde{U}^h \) and \( \tilde{U}^i \), we may think of the steady states of the PSE along with the underlying constant value of the share of insiders, associated with \( \tilde{U}^h \) and \( \tilde{U}^i \), as a bliss point for those seeking “common good” policies and insiders’ interest, respectively, if they were allowed to choose the structure of the economy, including the extreme case of no insiders (outsiders). Moreover, the expression 

\[
\frac{\tilde{U}^h - U^h_0}{\tilde{U}^h} \left( \frac{\tilde{U}^i - U^i_0}{\tilde{U}^i} \right)
\]

reflects the percentage loss of outsiders (insiders) from their bliss point.

Now, in order to completely characterize the objective function of the Hybrid Government, it remains to show that \( \tilde{U}^h \) and \( \tilde{U}^i \) exist and are computable. And, this is where we turn our attention, next.

First, note that in view of Remark II-1, the unique steady state of the PSE for all relevant economy specifications is given by (II.28)-(II.32), with wedges as in Table 2.2.

### Table 2.2: Functions of the Share of Insiders Industries in the Economy, \( \chi \), in the Steady State of the PSE for Alternative Economy Specifications

<table>
<thead>
<tr>
<th></th>
<th>Prototype Economy</th>
<th>Detailed Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Canonical RBC</strong></td>
<td><strong>Two Sector RBC</strong></td>
</tr>
<tr>
<td>( \chi = 0 )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \chi \in (0,1)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \bar{o}(\chi) = 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \bar{o}(\chi) = 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \Delta(\chi)</td>
<td>\frac{(B^i)}{(B^o)} \left( \frac{\chi}{1 - \chi} \right) \frac{\theta(1-\phi) + \phi}{\theta(1-\phi)} \left[ 1 + \frac{\Delta(\chi)}{\chi} \right]^{\frac{1}{\phi}}</td>
<td>\frac{(B^i)}{(B^o)} \left( \frac{\chi}{1 - \chi} \right) \frac{\theta(1-\phi) + \phi}{\theta(1-\phi)} \left[ 1 + \frac{\Delta(\chi)}{\chi} \right]^{\frac{1}{\phi}}</td>
</tr>
<tr>
<td>( \Delta^*(\chi, \chi)</td>
<td>\tilde{\psi}</td>
<td>\tilde{\psi}</td>
</tr>
</tbody>
</table>

16
Second, in view of the above steady state specifications of the PSE, given the share of insiders’ industries, \( \chi \), in Table 2.2, in order to check whether the bliss point \( \bar{U}^h \) is well defined and computable, it suffices to find a vector of steady state values for the aggregate state of the economy, \((c, h, k, \chi)\), that maximizes household utility (discounted future stream of insiders’ income) across all possible steady states of the PSE.

The following proposition establishes that \( \bar{U}^h \) is attained by the steady state of the PSE in the Canonical RBC specification.

**Proposition II-2:** Suppose \( \phi \in \left(-\frac{\theta}{1-\theta}, 1\right) \) and let \((c^+, h^+, k^+)\) denote the steady state of the PSE in the prototype economy with \( \chi = 0 \); \([c^*(\chi), h^*(\chi), k^*(\chi)]\) denote the steady state of the PSE in the prototype economy with \( \chi \in (0, 1) \); \([c(\chi), h(\chi), k(\chi)]\) denote the steady state of the PSE in the detailed economy with \( \chi \in (0, 1) \); and \((c^-, h^-, k^-)\) denote the steady state of the PSE in the detailed economy with \( \chi = 1 \).

Then, given \([R1]-[R3]\),

\[
\frac{\bar{\omega}(\chi)}{1 - \bar{\psi} - \bar{\psi}_\chi} \leq \frac{[\beta^{-1}(1 + \eta) - 1] + (1 - \tau^k)\delta}{(\eta + \delta)(1 - \tau^k)} \quad \text{for all } \chi \in (0, 1); \quad \text{or,}
\]

\[
\frac{\bar{\omega}(\chi)}{1 - \bar{\psi} - \bar{\psi}_\chi} \geq \frac{[\beta^{-1}(1 + \eta) - 1] + (1 - \tau^k)\delta}{(\eta + \delta)(1 - \tau^k)} \quad \text{and } \bar{\psi} \geq \left\{\frac{\alpha}{1 - \alpha} - \frac{(\eta + \delta)(1 - \tau^k)\bar{\omega}(\chi)}{[\beta^{-1}(1 + \eta) - 1 + (1 - \tau^k)\delta]}\right\} \frac{\bar{\omega}(\chi)}{\bar{\omega}(\chi)} > 0 \quad \text{for all } \chi \in (0, 1),
\]

and

\([R5] \ h(\chi) \leq 0 \text{ and } h'(\chi) < 0 \text{ for all } \chi \in (0, 1), \]

the following are true:

(a) \( u^h(c^+, h^+) > u^h[c^*(\chi), h^*(\chi)] \)

(b) \( u^h(c^+, h^-) > u^h[c(\chi), h(\chi)] \)

(c) \( u^h(c^+, h^+) > u^h(c^-, h^-) \)

(d) \( \bar{U}^h = \sum_{t=0}^{\infty} \beta^t u^h(c^+, h^+) \)
Proof: In Appendix II.

Assumptions [R1]-[R5] constitute a set of strong sufficient conditions for the results in Proposition II-2 as well as those in Proposition II-3 that follows. Assumptions [R1]-[R3] were introduced and discussed in the companion paper (Kollintzas, et al., 2021). Both assumptions [R4]-[R5], regulate trade-offs that are consequences of a, say, rising share of insiders’ industries, in the utility of the representative household. In particular, [R4], regulates the trade-off, between $y$ and $c/y$ that are both brought about by a change in $k/y$, given $h$, so that consumption falls. And [R5] regulates a trade-off between consumption and leisure. In particular, as we have shown in assumptions [R1]-[R2], a rise in the share of insiders’ industries, $\chi$, implies lower employment. This in turn increase leisure and lowers output and consumption. [R5], then imposes an upper bound on $1 - \mathcal{I}$.

Of course, there are no surprises with respect to the results in (b) and (c), for they are simply implications of the First Fundamental Theorem of Welfare Economics. That is, the steady state of the PSE in an economy with no insiders (i.e., no frictions) yields a greater value for the utility of the representative household than in economies with insiders. The result in (a) is less straightforward. This result implies that for the assumed values of the technological parameters, the steady state of the PSE in an economy with one competitive sector (i.e., the Canonical RBC economy) yields a greater value for the utility of the representative household than the steady state of economies with two competitive sectors. This of course is a consequence of the result established in Part A, that for these parameter values, Edgeworth complementarity within sectors in the production of the final good is stronger than Edgeworth complementarity across sectors. And, although counter intuitive, this result is reasonable, for the basic difference between sectors in the model economies has to do with non-competitive frictions and not the physical nature of input complementarity, which is captured within sectors.

Finally, we will take the result of Proposition II-2 to suggest that in a government that is influenced by both insiders’ interests and the “common good”, the latter will be served by reducing the number of insiders’ industries and their associated frictions.

The following proposition establishes that $\bar{U}$ is attained by the steady state of the PSE in an all insiders economy.
**Proposition II-3:** Suppose $\Phi \in \left( -\frac{\theta}{1-\theta}, 1 \right)$ and let $[c(\chi), h(\chi), k(\chi)]$ denote the steady state of the PSE in the detailed economy with $\chi \in (0,1)$; and $(c^*, h^*, k^*)$ denote the steady state of the PSE in the detailed economy with $\chi = 1$. Then, given [R1]-[R3] and [R5], the following are true:

(a) $v'(h^*, k^*, 1) > v'[h(\chi), k(\chi), \chi]$

(b) $\bar{U}^i = \sum_{t=0}^{\infty} \beta^t v^i(h^*, k^*, 1)$

**Proof:** In Appendix II.

There are three factors that drive this result. First, in this case, ceteris paribus, the discounted future stream of insiders’ income is strictly increasing in $\chi$. Second, TFP is maximized at $\chi = 1$, due to the stronger Edgeworth complementarity within vs across sectors, as in the case of $\chi = 0$. And, third, given the assumed parameter values the inefficiency implied by the capital and labor wedges, that tend to reduce aggregate capital and labor inputs as the share of insiders’ industries rises, in the detailed economy, is dominated by the first two effects.

### 2.3 Characterization of the Politico-economic Equilibrium

We shall consider sequences of the form $\{h_t^*(z^r), k_t^*(z^r), \chi_t^*(z^r)\}_{t=0}^{\infty}$, such that:

$$\left\{ z^r \right\}_{t=0}^{\infty} = \left\{ h_t^*(z^r), k_t^*(z^r), \chi_t^*(z^r) \right\}_{t=0}^{\infty} = \arg\max_{[h_t(z^r), k_t(z^r), \chi_t(z^r)]_{t=0}^{\infty}} U_0$$

Now, we define a politico-economic equilibrium in the case of a Hybrid government as follows:

**Definition:** Given $\{z^r\}_{t=0}^{\infty}$, a politico-economic equilibrium in the case of a Hybrid Government is a sequence of the form $\{h^*_t(z^r), k^*_t(z^r), \chi^*_t(z^r)\}_{t=0}^{\infty}$, such that:

$$\left\{ h_t^*(z^r), k_t^*(z^r), \chi_t^*(z^r) \right\}_{t=0}^{\infty} = \arg\max_{[h_t(z^r), k_t(z^r), \chi_t(z^r)]_{t=0}^{\infty}} U_0^\rho$$

where $\mathcal{S}_0$ is the space of sequences of the form $\{h_t(z^r), k_t(z^r), \chi_t(z^r)\}_{t=0}^{\infty}$ such that:
The IC (2.10), the GSPARC (2.15), and the initial condition (2.7) are satisfied; and, (b) 
\((h_i(z'), k_{r+1}(z'), \chi_{r+1}(z')) \in (0,1) \times (0, \infty) \times (0,1), \forall t \geq 0.\)

Note that \(\chi_{r+1}(z') \in (0,1), \forall t \geq 0,\) means that we are only interested in equilibria of a
detailed economy with both insiders and outsiders, since the Hybrid Government
presupposes the existence of both insiders and outsiders.

### 2.3.1. First – Order Conditions:

Let \(m \) be the multiplier associated with the IC and let:

\[
M^\rho (c_i, h_i, k_i, \chi_i; z_i) \equiv \left[1 - \rho - m \zeta (h_i, \chi_i; z_i)\right] \frac{u^h(c_i, h_i)}{U^h_0} + \rho \left[\frac{v'(h_i, k_i, \chi_i; z_i)}{U^h_0}\right]
\]

where, \(c_i \) is given by the GSPARC (2.14). Clearly, if \([h_i(z'), k_{r+1}(z'), \chi_{r+1}(z')]^{\sigma}_{r=0} \) is a
politico-economic equilibrium in the case of a Hybrid government, then the IC (2.10) and
the GSPARC (2.14) must be satisfied along with the following first order conditions, for
all \(t > 0: \)

\[
M^\rho_h + (1 - \alpha)(1 - \Delta^i_t) \frac{k_{r+1}}{h_{r+1}} \left[ M^\rho_k \right] = 0
\]  
(2.24)

\[
(1 + \eta)M^\rho_{c_i} = \beta E_i \left[ M^\rho_{k_{r+1}} + M^\rho_{\chi_{r+1}} \frac{1}{\theta} \left[ 1 - \Delta^i_t \right] + \alpha (1 - \Delta^i_{r+1}) \frac{k_{r+1}}{h_{r+1}} \left[ M^\rho_{\chi_{r+1}} + M^\rho_{v_{r+1}} \right] \left[ \frac{\partial \Delta^i_{r+1}}{\partial \chi_{r+1}} + (1 - \Delta^i_{r+1}) \frac{\partial v_{r+1}}{\partial \chi_{r+1}} \right] \frac{k^a}{h^{1-a}} \right]
\]  
(2.25)

\[
M^\rho_{c_i} \frac{\partial \Delta^i_{r+1}}{\partial \chi_{r+1}} = \beta E_i \left[ M^\rho_{\chi_{r+1}} + M^\rho_{v_{r+1}} \frac{1}{\theta} \left[ 1 - \Delta^i_t \right] + \alpha (1 - \Delta^i_{r+1}) \frac{k_{r+1}}{h_{r+1}} \left[ M^\rho_{\chi_{r+1}} + M^\rho_{v_{r+1}} \right] \left[ \frac{\partial \Delta^i_{r+1}}{\partial \chi_{r+1}} + (1 - \Delta^i_{r+1}) \frac{\partial v_{r+1}}{\partial \chi_{r+1}} \right] \frac{k^a}{h^{1-a}} \right]
\]  
(2.26)

where:

\[
M^\rho_{c_i} = \frac{\theta (1 - \gamma) (1 - \rho - m \zeta_i) u^h_i}{c_i}
\]  
(2.27)

\[
M^\rho_h = -\frac{1 - \theta}{1 - h_i} \frac{m \zeta_i h_i}{h_i} \frac{u^h_i}{U^h_0} + \frac{m \zeta_i (h_i + \zeta_i)}{(1 - \gamma) (1 - \rho - m \zeta_i) \frac{u^h_i}{U^h_0}} \left[ \frac{u^h_i}{U^h_0} \right] + (1 - \alpha) \rho \left[ \frac{v_{r+1}}{U^h_0}, \frac{1}{h^i} \right] = \frac{(1 - \gamma) (1 - \rho - m \zeta_i) u^h_i}{1 - h_i} - \frac{m \zeta_i}{h_i} \frac{h_i^a}{h_i} + \frac{(1 - \gamma) (1 - \rho - m \zeta_i) \frac{u^h_i}{U^h_0}}{h_i} + (1 - \alpha) \rho \left[ \frac{v_{r+1}}{U^h_0}, \frac{1}{h^i} \right] \zeta_i \frac{h_i}{h_i}
\]
\[ M^p_{k_t+1} = \alpha \rho \left( \frac{\nu_{t-1}^i}{U_0^{i_t}} \right) \left( \frac{1}{k_t+1} \right) = \alpha \left( \frac{\rho}{U_0^i} \right) (\omega_{t+1}(\chi_{t+1}) - \omega_{t+1}(\chi_{t+1}^*) \left( k_{t+1} \right) \right) \alpha^{-1} \] (2.28)

\[ M^p_{z_{t+1}} = -m z_{t+1} \left( \frac{\partial \zeta_{t+1}}{\partial \chi_{t+1}} \right) \left( \frac{1}{z_{t+1}} \right) + \rho \left( \frac{\omega_{t+1}'(\chi_{t+1})}{\omega_{t+1}(\chi_{t+1})} \right) \left( \frac{v_i^i}{U_0^i} \right) \] (2.29)

\[ = -m z_{t+1} \left( \frac{\partial \zeta_{t+1}}{\partial \chi_{t+1}} \right) \left( \frac{1}{z_{t+1}} \right) + \rho \left( \frac{\omega_{t+1}'(\chi_{t+1})}{\omega_{t+1}(\chi_{t+1})} \right) \omega_{t+1}(\chi_{t+1}) \omega_{t+1}(\chi_{t+1}) k_{t+1} \alpha h_{t+1} \] (2.30)

### 2.3.2 The Asymptotic Steady State:

We consider the asymptotic steady state \((c, h, k, \chi, m)\) implied by the first-order conditions for \(t > 0\), (2.24)-(2.26), the IC (2.10) for \((k_0, \gamma_0) = (k, \chi)\), and the GSPARC (2.14).

In order to condense notation, we also use steady state output, \(y = \hat{\omega}(\chi)k^\alpha h^{1-\alpha}\); and, steady state output share of government expenditures in the detailed economy, \(\Delta^s = \hat{\psi} + \hat{\psi}^\chi\). Then, the asymptotic steady state of the PE in the case of a Hybrid government is characterized by the following equations:

\[
\begin{align*}
\left( -\frac{M^p_{h}}{M^p_{c}} \right) \left( \frac{h}{y} \right) &= \left( 1 - \vartheta \right) \left( \frac{c}{y} \right) h - \frac{m}{\vartheta(1 - \gamma)(1 - \rho - m z)} \left( \frac{c}{y} \right) \left( 1 \right) \\
+ (1 - \alpha) \frac{\rho}{\vartheta(1 - \gamma)(1 - \rho - m z)} &= (1 - \alpha)(1 - \Delta^s)
\end{align*}
\] (2.31)

\[
\left( \frac{k}{y} \right) = \frac{\alpha(1 - \Delta^s) + \left( \frac{M^p_{k}}{M^p_{c}} \right) \left( \frac{k}{y} \right)}{\beta^{-1}(1 + \eta) - (1 - \delta)} = \frac{(1 - \Delta^s) + \left( \frac{\rho}{\vartheta(1 - \gamma)(1 - \rho - m z)} \right) \left( \frac{c}{y} \right)}{\beta^{-1}(1 + \eta) - (1 - \delta)}
\] (2.32)

\[
\left( \frac{M^p_{h}}{M^p_{c}} \right) \left( \frac{1}{y} \right) = \frac{m}{\vartheta(1 - \gamma)(1 - \rho - m z)} \left( \frac{c}{y} \right) + \frac{\rho}{\vartheta(1 - \gamma)(1 - \rho - m z)} \left( \frac{c}{y} \right) \left( \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} \right) = \frac{\partial \Delta^s}{\partial \chi} - (1 - \Delta^s) \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)}
\] (2.33)
2.3.2 The Great Ratios

Then, the following proposition shows that, in the five-equation system (2.10), (2.14), and (2.31) – (2.33), the multiplier \( m \) can be eliminated, so that the asymptotic steady state of the politico-economic equilibrium can be characterized in terms of the four great ratios \( \left[ \left( \frac{c}{y} \right), h, \left( \frac{k}{y} \right), \chi \right] \). And, moreover, \( \left( \frac{k}{y} \right), \left( \frac{c}{y} \right) \), and \( h \) can be expressed, analytically, in terms of \( \chi \).

**Proposition II-4 (Auxiliary Variables and Great Ratios):** Suppose that \( \rho \in (0,1) \). Then, \( (c,h,k,\chi,m) \) is an asymptotic steady state of the politico-economic equilibrium with a hybrid government, if and only if, the four great ratios: \( \left( \frac{k}{y} \right), \left( \frac{c}{y} \right), h, \chi \) and the three auxiliary variables: \( \xi, \overline{\xi}, \hat{\xi} \) constitute a solution to the following seven-equation system:

\[
\left( \frac{c}{y} \right) = (1 - \overline{\psi} - \hat{\psi}\chi) - (\eta + \delta) \left( \frac{k}{y} \right) \tag{2.34}
\]

\[
\hat{\xi} = \frac{1}{1 - \frac{(1 - \theta)\sigma(\chi)}{\theta(1 - \xi)}} \tag{2.35}
\]

\[
h = \frac{1}{1 + \frac{(1 - \theta)\sigma(\chi)}{\theta(1 - \xi)}} \tag{2.36}
\]

\[
\overline{\xi} = \frac{1 - \theta}{\hat{\xi}} \left( \frac{c}{y} \right) h - (1 - \alpha)(1 - \overline{\psi} - \hat{\psi}\chi - \hat{\xi})(1 - h) \tag{2.37}
\]

\[
\left( \frac{k}{y} \right) = \frac{\alpha(1 - \overline{\psi} - \hat{\psi}\chi + \hat{\xi})}{\beta^{-1}(1 + \eta) - (1 - \delta)} \tag{2.38}
\]

\[
\xi \frac{\sigma'(\chi)}{\sigma(\chi)} + \overline{\xi} \left( \frac{\hat{\sigma}'(\chi)}{\hat{\sigma}(\chi)} \right) = \hat{\psi} - \frac{\hat{\sigma}'(\chi)}{\hat{\sigma}(\chi)} (1 - \overline{\psi} - \hat{\psi}\chi) \tag{2.39}
\]

\[
\hat{\xi} = e^\rho \left[ (1 - \xi) \left( \frac{c}{y} \right) + \overline{\xi} \right] \tag{2.40}
\]
where:

\[
\varepsilon^\rho = \frac{\psi^h}{(1-\rho)U^h} = \frac{\psi^i}{(1-\rho)U^i} = \frac{dU^h}{dU^i} \bigg|_{dU^\rho = 0}
\]  

(2.41)

**Proof: In Appendix II.**

As the last equality in (2.41) indicates, \( \varepsilon^\rho \) is the elasticity of substitution across insiders and outsiders utility in the government objective function, evaluated at the asymptotic steady state. Clearly, \( \varepsilon^\rho \) is a positive function of \( \rho \) with \( \varepsilon^\rho = 0 \) if \( \rho = 0 \) and \( \varepsilon^\rho \to +\infty \) as \( \rho \to 1 \). We conjecture that it is, also, a strictly increasing function of \( \rho \), as

\[
\left( \frac{\psi^i}{u^h} \right)
\]

that is an implicit function of \( \rho \), through the dependence of \( \left( \frac{\psi^i}{u^h} \right) \) in the asymptotic steady state \( \left[ \frac{c}{y}, h, \frac{k}{y}, \chi, \xi, \hat{\xi}, \bar{\xi} \right] \) of the hybrid politico-economic equilibrium. That is, we conjecture that the more influential are insiders the higher will be the level of utility of insiders, relative that of the median voter, in the hybrid politico-economic equilibrium. As it turns out, this is confirmed by the numerical analysis of the next section.

Although this paper focuses on theoretical aspects of politico-economic equilibria, we think that Proposition II-4 has important implications for applied research. This is the case, since \( \varepsilon^\rho \), which also depends on the relative weight associated with the welfare loss of insiders in the government decision making, \( \rho \), can be retrieved from the data. Clearly, the estimated value of \( \varepsilon^\rho \) or \( \rho \) reflect the underlying politico-economic structure of countries.
3. Growth and policy implications of alternative politico-economic equilibria: A numerical analysis

In this section we investigate the growth implications of the politico-economic equilibrium established in the previous section, for the parameter values in Table 3.1.\(^{10}\) We are primarily interested in two comparisons. First, to compare the asymptotic steady state of the Hybrid government in a detailed economy to the Canonical RBC economy. Recall that, the latter corresponds to the prototype economy for \( \chi = 0 \); and, as we have shown in Proposition II-2, the PSE of the Canonical RBC model coincides with the Median Voter PE of the prototype model across all possible economy specifications. The results are presented in Table 3.2.

Second, we are interested in examining the long run implications of alternative politico-economic equilibria, based on different degrees of insiders’ influence in government decisions. That is, we are interested on comparing the various politico-economic equilibria as \( \rho \) ranges in the (0, 1) interval. It should be emphasized that in so doing we take the existence of insiders as given and what changes is only the degree of their influence in government decision making. Note that, as Proposition II-4 shows, this is equivalent to conducting the sensitivity analysis with respect to \( \varepsilon^\rho \), that is, the elasticity of substitution across insiders’ and outsiders’ utility in the government objective function, evaluated at the asymptotic steady state of the politico-economic equilibrium with a hybrid government,. For, there is one to one correspondence between \( \varepsilon^\rho \) and \( \rho \). Thus, we conduct the sensitivity analysis for the range of values of \( \varepsilon^\rho \) that imply that \( \rho \) lies between zero and one. This range of values for \( \varepsilon^\rho \) depends, of course, on the underlying parameterization. The results of the comparison among the asymptotic steady states of the politico-economic equilibrium of a Hybrid government in a detailed economy, for different degrees of insiders’ influence in government, are summarized in Table 3.3 and Figures 3.1a, 3.1b and 3.1c.

\(^{10}\) The parameterization, as in the companion paper (Kollintzas, et al. 2021), reflects parameter values that: (a) are commonly used in the literature and (b) satisfy the restrictions imposed by theory (i.e., Assumptions [R1]-[R5]). The range of values of \( \varepsilon^\rho \) are those implied by the restriction that \( \rho \) belongs to the (0,1) interval.
### Table 3.1: Parameterization

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>capital input elasticity in industry production</td>
<td>0.33</td>
</tr>
<tr>
<td>$\tilde{\beta}$</td>
<td>constant discount factor of households</td>
<td>0.98</td>
</tr>
<tr>
<td>$1/\gamma$</td>
<td>household intertemporal elasticity of substitution</td>
<td>0.50</td>
</tr>
<tr>
<td>$\delta$</td>
<td>capital depreciation rate</td>
<td>0.07</td>
</tr>
<tr>
<td>$\eta$</td>
<td>growth rate of labor of augmenting technology</td>
<td>0.02</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>intensity of consumption in household preferences</td>
<td>0.33</td>
</tr>
<tr>
<td>$\frac{1}{1-\theta}$</td>
<td>aggregation elasticity of substitution across industries in the same sector</td>
<td>10 ($\theta = 0.9$)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>intensity of wage premium in union preferences</td>
<td>0.75</td>
</tr>
<tr>
<td>$\mu$</td>
<td>relative bargaining power of insiders’ unions</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>capital income tax rate</td>
<td>0.20</td>
</tr>
<tr>
<td>$\frac{1}{1-\phi}$</td>
<td>elasticity of substitution across sectors in final good production function</td>
<td>0.50 ($\phi = -1$)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>share of insiders’ industries</td>
<td>1/3</td>
</tr>
<tr>
<td>$\psi$</td>
<td>GDP share of government consumption in the prototype economy</td>
<td>0.225</td>
</tr>
<tr>
<td>$\tilde{\psi}$</td>
<td>such that the difference between the GDP share of government consumption in South Europe and the US is $\tilde{\psi}\chi$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\tilde{\psi}'$</td>
<td>GDP share of government spending devoted to the expansion of the insiders’ sector</td>
<td>0</td>
</tr>
<tr>
<td>$\rho$</td>
<td>relative weight associated with the welfare loss of insiders in the government decision making criterion</td>
<td>1/3</td>
</tr>
</tbody>
</table>
Table 3.2. Steady State of Politico-Economic Equilibrium: Canonical RBC Model vs Detailed Economy Hybrid Government

<table>
<thead>
<tr>
<th></th>
<th>Canonical RBC</th>
<th>Detailed Economy Hybrid Government $\rho=1/3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>0</td>
<td>0.5424</td>
</tr>
<tr>
<td>$y$</td>
<td>0.4857</td>
<td>0.1256</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.5436</td>
<td>0.5885</td>
</tr>
<tr>
<td>$i/y$</td>
<td>0.2314</td>
<td>0.1729</td>
</tr>
<tr>
<td>$g/y$</td>
<td>0.2250</td>
<td>0.2386</td>
</tr>
<tr>
<td>$h$</td>
<td>0.3029</td>
<td>0.2876</td>
</tr>
<tr>
<td>$k/y$</td>
<td>2.5713</td>
<td>1.9215</td>
</tr>
<tr>
<td>$r$</td>
<td>0.1296</td>
<td>0.1641</td>
</tr>
<tr>
<td>$w^o$</td>
<td>1.0690</td>
<td>0.2661</td>
</tr>
<tr>
<td>$w^i$</td>
<td>-</td>
<td>0.2946</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1</td>
<td>0.9462</td>
</tr>
<tr>
<td>$\bar{\omega}$</td>
<td>1</td>
<td>0.9140</td>
</tr>
<tr>
<td>$\bar{\dot{\omega}}$</td>
<td>1</td>
<td>0.4630</td>
</tr>
<tr>
<td>$\bar{\omega}$</td>
<td>1</td>
<td>1.0269</td>
</tr>
<tr>
<td>$u^h$</td>
<td>-1.9827</td>
<td>-2.9871</td>
</tr>
<tr>
<td>$u^h / u^h_{CRBC}$</td>
<td>1</td>
<td>1.5066</td>
</tr>
</tbody>
</table>
Table 3.3. Politico-Economic Equilibrium of Hybrid Government: Steady State Sensitivity with respect to $\rho$

<table>
<thead>
<tr>
<th></th>
<th>$\rho=0.1$</th>
<th>$\rho=0.25$</th>
<th>$\rho=0.33$</th>
<th>$\rho=0.5$</th>
<th>$\rho=0.66$</th>
<th>$\rho=0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^\rho$</td>
<td>0.0107</td>
<td>0.0317</td>
<td>0.0469</td>
<td>0.0901</td>
<td>0.1609</td>
<td>0.2115</td>
</tr>
<tr>
<td>$x$</td>
<td>0.5244</td>
<td>0.5347</td>
<td>0.5424</td>
<td>0.5652</td>
<td>0.6060</td>
<td>0.6381</td>
</tr>
<tr>
<td>$y$</td>
<td>0.1303</td>
<td>0.1277</td>
<td>0.1256</td>
<td>0.1192</td>
<td>0.1058</td>
<td>0.0929</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.5724</td>
<td>0.5816</td>
<td>0.5885</td>
<td>0.6089</td>
<td>0.6456</td>
<td>0.6745</td>
</tr>
<tr>
<td>$i/y$</td>
<td>0.1895</td>
<td>0.1800</td>
<td>0.1729</td>
<td>0.1520</td>
<td>0.1143</td>
<td>0.0845</td>
</tr>
<tr>
<td>$g/y$</td>
<td>0.2381</td>
<td>0.2384</td>
<td>0.2386</td>
<td>0.2391</td>
<td>0.2402</td>
<td>0.2410</td>
</tr>
<tr>
<td>$h$</td>
<td>0.2855</td>
<td>0.2876</td>
<td>0.2876</td>
<td>0.2900</td>
<td>0.2937</td>
<td>0.2963</td>
</tr>
<tr>
<td>$k/y$</td>
<td>2.1053</td>
<td>2.0000</td>
<td>1.9215</td>
<td>1.6887</td>
<td>1.2700</td>
<td>0.9390</td>
</tr>
<tr>
<td>$r$</td>
<td>0.1499</td>
<td>0.1578</td>
<td>0.1641</td>
<td>0.1866</td>
<td>0.2476</td>
<td>0.3344</td>
</tr>
<tr>
<td>$w^o$</td>
<td>0.2785</td>
<td>0.2714</td>
<td>0.2661</td>
<td>0.2500</td>
<td>0.2184</td>
<td>0.1896</td>
</tr>
<tr>
<td>$w^j$</td>
<td>0.3083</td>
<td>0.3005</td>
<td>0.2946</td>
<td>0.2767</td>
<td>0.2418</td>
<td>0.2099</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.9470</td>
<td>0.9466</td>
<td>0.9462</td>
<td>0.9452</td>
<td>0.9434</td>
<td>0.9419</td>
</tr>
<tr>
<td>$\tilde{\omega}$</td>
<td>0.9152</td>
<td>0.9145</td>
<td>0.9140</td>
<td>0.9123</td>
<td>0.9094</td>
<td>0.9070</td>
</tr>
<tr>
<td>$\bar{\omega}$</td>
<td>0.4625</td>
<td>0.4628</td>
<td>0.4630</td>
<td>0.4642</td>
<td>0.4675</td>
<td>0.4713</td>
</tr>
<tr>
<td>$\tilde{\omega}$</td>
<td>1.0265</td>
<td>1.0267</td>
<td>1.0269</td>
<td>1.0274</td>
<td>1.0283</td>
<td>1.0291</td>
</tr>
<tr>
<td>$u^h_{\rho=0.1}/u^h_{\rho=0.1}$</td>
<td>1</td>
<td>1.5032</td>
<td>1.5066</td>
<td>1.5194</td>
<td>1.5558</td>
<td>1.6050</td>
</tr>
<tr>
<td>$v^i_{\rho=0.1}/v^i_{\rho=0.1}$</td>
<td>1</td>
<td>0.9884</td>
<td>0.9783</td>
<td>0.9464</td>
<td>0.8681</td>
<td>0.7826</td>
</tr>
</tbody>
</table>
Figure 3.1. Politico-Economic Equilibrium of Hybrid Government: Steady State Sensitivity with respect to $\rho$

a. Great Ratios
b. Great Ratios (cont.) and Welfare
c. Wedges

Notation: $\omega_{crown} \equiv \hat{\omega}$
We also conduct a sensitivity analysis of the results of these comparisons for different values of parameters characterizing the structure of the detailed economy (e.g., the elasticity of substitution across sectors in final good production function, \( \phi \), the elasticity of substitution across industries in the same sector \( \theta \), union preferences for wage premia, \( \lambda \) and relative bargaining power of insiders’ unions, \( \mu \)). The respective figures are presented in Appendix III.

The steady state comparison of the hybrid government relative to the canonical RBC in Table 3.2., reveals a dramatic fall in output, wages, investment, capital and welfare, which mirrors the deterioration of all wedges and especially the efficiency wedge \( \bar{\omega} \).\(^{11}\) The latter, as already mentioned in the companion paper (Kollintzas, et al 2021), is a consequence of the U-shaped efficiency wedge \( \bar{\omega} \), implying greater input complementarity within sectors versus that across sectors of the economy. In particular, starting from relatively low values of \( \chi \), as the mass of insiders’ industries increases (i.e., \( \chi \) increases) the drop in the marginal product in the outsiders’ sector is greater compared to the increase in the marginal product in the insiders’ sector. In other words, efficiency increases when the activity of the economy is concentrated in a single sector: The positive effects of complementarity become stronger the bigger the sector. The deterioration in the labor, capital and government spending wedges reflect the frictions present in the detailed economy. Of course, these frictions exist when \( \chi > 0 \), which reflects what we refer to as, the political friction.\(^{12}\)

This comparison, however, between the detailed economy and the canonical RBC model which corresponds to the second best allocation, cannot reveal the extent of the relative importance of the political friction (i.e., the influence of insiders in government). For this reason, in Table 3.3 and Figures 3.1.a-c, we focus on the way the steady state of the PE for the Hybrid government changes when the degree of insiders influence in government decision making increases (i.e., \( \rho \) increases).

First, note that for the range of values for \( \rho \) in Table 3.3., the complementarity effect essentially plays no role, as evidenced from the values of \( \bar{\omega} \) that show no variability. This implies that the observed differences are essentially due to the political friction and the subsequent economic frictions that are what we have referred to as the interaction of political and market forces. First of all, observe that a higher value for \( \rho \) implies a higher

\(^{11}\) An increase in the value of the utility ratio, indicates a welfare deterioration.

\(^{12}\) We refer to the government spending friction here, as over and above the distortionary taxation factor.
value for $\varepsilon^o$ giving rise to a higher share of insiders in the economy. Moreover, as $\rho$ increases, output, wages in both sectors, investment, capital and welfare (both in what concerns the common good as well as insiders’ as a whole) fall. Also, with the exception of the efficiency wedge, which as mentioned above remains relatively constant, all other wedges deteriorate.

In particular, five policy implications emerge from this long run comparative statics analysis: First, the share of insiders industries in the economy increases with the degree of insiders influence in government, leading to lower aggregate output and household welfare and income from the insiders’ industries. That is, giving insiders greater power in influencing government decisions (i.e., a higher value of $\rho$) enhances the political distortion leading to a worse allocation for the economy as a whole. Second, the great ratios $k/y$, $c/y$, $i/y$, $g/y$ and $h$ are all affected by changes in rho. And, in particular, $c/y$, $g/y$ and $h$ rise, while $k/y$ and, therefore $i/y$, fall. These comparative statics results are a manifestation of the frictions inherent in the model.

The deterioration in output is the result of two opposing forces. From the one hand, aggregate labor input and total factor productivity rise, while, on the other hand, aggregate capital input falls, with the latter force been the dominant one. In particular, as can be seen in Figure 3.1.c., the rise in $\bar{\omega}$ reflects mild efficiency increases associated with stronger within sector versus across sectors complementarity gains. The rise in $h$ is a consequence of the income effect that dominates over the substitution effect associated with the underlying wage decrease, due to the fall in $\bar{\omega}$ (See Figure 3.1.c.). However, both of these forces are counterbalanced by the fall in aggregate capital, due to the increase in intertemporal distortions, incorporated in the capital wedge, $\tilde{\omega}$.

Finally, the increase in $g/y$ is of course a direct consequence of the need to support a bigger insiders’ sector, while the rising $c/y$ is, apparently, a consequence of the drop in $k/y$ dominating the increase in $g/y$ (See, the resource constraint (2.34)).

It is worth noting that, apart from household welfare which is related to the common good, the welfare of insiders also deteriorates as $\rho$ increases. As already mentioned, this a consequence of the negative externality that is created by selfish insiders that ignore the effect of their own political collective intervention on society.

The sensitivity analysis of different values of $\phi$, $\theta$, $\lambda$ and $\mu$ reaffirms a similar qualitative behavior. The respective figures are presented in Appendix III.
Consequently, the sensitivity results indicate that our theory is capable of explaining significant differences in macroeconomic performance and welfare as a consequence of the underlying politico-economic framework. Our model, then, can serve as a vehicle for assessing the importance of the market and political power interactions in the data.

4. Concluding remarks

In this paper we characterized and solved the problem of a government that makes its decisions pursuing a balance between what Jean Tirole calls policies for the “the common good” and the interests of an elite – which we call insiders - that emanates from various market frictions. In so doing we developed a government welfare objective that captures this underlying trade-off and can account for different degrees of the elites’ influence in government decision making. Such a welfare objective characterizes what we called the Hybrid government. Further, we applied a Ramsey type optimal policy approach, in the two sector DSGE model with market and political power interactions, developed in the companion paper Kollintzas, et al. (2021).

The optimal policy solution is a politico-economic equilibrium allocation that, apart from the standard macroeconomic variables, also characterizes the behavior of the share of insiders’ industries in the economy as an additional policy instrument.

We then investigated the growth implications of this politico-economic equilibrium, focusing, on steady state comparisons of the hybrid government politico-economic equilibrium, first, relative to the Second Best allocation (implied by the Canonical RBC economy); and second, for different degrees of the insiders’ influence in government.

Our numerical analysis showed that increasing influence of insiders in government decision making is bad for the economy. This may also serve as an explanation to why the macroeconomic performance of some countries is worse relative to that of others, that, ceteris paribus, enjoy similar state of development.

This influence is measured by the weight assigned by the hybrid government to insiders’ interests. This is a crucial deep parameter of our model (\(\rho\)) that can serve as a bridge of the theoretical model with the data. That is, different values of this weight can be associated with countries with different politico-economic structures. For example, the politico-economic structure of South European countries can be thought of as being characterized by a relatively higher value of \(\rho\) compared to the US or the European Core.
REFERENCES


Kollintzas Tryphon and Pechlivanos Lambros, (2022), Stable Insiders’ Coalition Governments, mimeo, Athens University of Economics and Business.


APPENDIX I. Notation

\( y_t \): output of representative final good producer in period \( t \)

\( Y_t^i \): composite input of intermediate goods produced in the insiders’ sector

\( Y_t^o \): composite input of intermediate goods produced in the outsiders’ sector

\( A_t \): (static) total factor productivity

\( \chi_t \): share of insiders’ industries in the economy

\( 1 / (1 - \phi) \): input elasticity of substitution across sectors

Composite inputs are defined by the Dixit-Stiglitz aggregator functions:

\[
Y_t^i = \left[ \int_0^{\zeta} y_t^i(\zeta) ^\theta d\zeta \right] ^{(1/\theta)} \quad \text{and} \quad Y_t^o = \left[ \int_0^{\zeta} y_t^o(\zeta) ^\theta d\zeta \right] ^{(1/\theta)}
\]

where:

\( y_t^i(\zeta) \): input of the intermediate good of the \( \zeta \) industry in insiders’ sector

\( y_t^o(\zeta) \): input of the intermediate good of the \( \zeta \) industry in outsiders’ sector

\( 1 / (1 - \vartheta) \): aggregation elasticity of substitution across industries

\( p_t^i(\zeta) \): real price of the intermediate good of the \( \zeta \) industry in the insiders’ sector

\( p_t^o(\zeta) \): real price of the intermediate good of the \( \zeta \) industry in the outsiders’ sector

\( k_t^i(\zeta) \): capital input

\( l_t^i(\zeta) \): labor input

\( B_t^o \): total factor productivity in outsiders’ industries

\( \alpha \): capital input elasticity

\( r_t \): real rental cost of capital

\( w_t^o \): real wage rate of outsiders

\( B_t^i \): total factor productivity in insiders’ industries

\( w_t^i(\zeta) \): real wage rate in the \( \zeta \) industry of the insiders’ sector

Union preferences are characterized by a utility function of the form:

\[
u_t^i(\zeta) = \left[ w_t^i(\zeta) - w_t^o \right] ^\lambda \left[ l_t^i(\zeta) \right] ^{(1-\lambda)} \quad \lambda \in (0,1)
\]
\( \lambda \): preference intensity of wage premium over union membership / employment

\[
(w'_i(\varphi'), l'_i(\varphi')) = \arg \max_{(w'_i(\varphi), l'_i(\varphi))} \left\{ \mu \left[ u'_i(\varphi) - \bar{w}'_i(\varphi) \right]^\mu \left[ \pi'_i(\varphi) - \bar{\pi}'_i(\varphi) \right]^{1-\mu} \right\}; \mu \in (0,1)
\]

\( \bar{w}'_i(\varphi) \): union’s reservation option

\( \bar{\pi}'_i(\varphi) \): producer’s reservation option

\( \mu \): union’s relative bargaining power

\( c_i \): (final good) consumption

\( k_t \): capital stock at the beginning of period \( t \)

\( \chi, h'_i \): labor supply to the insiders’ sector

\( (1-\chi)h^o_i \): labor supply to the outsiders’ sector

\( \chi, \pi'_i \): dividends from outsiders’ industries

\( \tau^k_i \): capital income tax rate

\( \tau^l_i \): labor income tax rate

\( \delta \): fixed (geometric) capital depreciation rate

\( h_i \): fraction of available household time devoted to work

\( 1-h_i \): fraction of available household time devoted to leisure

\[
u^k_i = \left[ c_i^\gamma (1-h_i)^{1-\gamma} \right]^\gamma; \gamma > 0
\]

\( \beta \in (0,1) \) is the efficient household constant discount factor

\( 1/\gamma \) is the intertemporal elasticity of substitution

\( \theta \) is the preference intensity for consumption

\[ [R1] \quad \lambda < \frac{(1-a\theta)}{(1-a\theta) + (1-\theta)} + \frac{(1-a)\theta}{(1-a\theta) + (1-\theta)} \left( \frac{1-\mu}{\mu} \right), \]

\[ [R2] \quad \lambda > \frac{\alpha}{1+\alpha} - \frac{1-a}{1+\alpha} \left( \frac{1-\mu}{\mu} \right), \]

\[ [R3] \quad \frac{1-\nu}{\alpha \xi + (1-a)\theta} < \phi < \frac{1-\nu}{[a\theta + (1-a)\xi] - \nu}, \text{ where } \nu \equiv \left( \theta^a \xi^{(1-a)} \right)^{\frac{1}{1-a}} \]

36
APPENDIX II. Mathematical Appendix

Proof of Proposition II-I:

Part a: First, note that combining the resource constraint (2.2) and the government budget constraint (2.3), we have:

\[ y_i = c_i + (1+\eta)k_{r+1} - (1-\delta)k_i + \left[ \tau_i^k \left[ \alpha \varphi_i(\chi_i) - \delta \left( \frac{k_i}{y_i} \right) \right] + \tau_i^t (1-\alpha)\omega_i(\chi_i) \right] y_i \]  \hspace{1cm} (A.1)

Second, note that:

\[ \alpha \varphi_i(\chi_i) + (1-\alpha)\omega_i(\chi_i) = \alpha \frac{1+\theta\Delta_i(\chi_i)}{1+\Delta_i(\chi_i)} + (1-\alpha) \frac{1+\alpha\Delta_i(\chi_i)}{1+\Delta_i(\chi_i)} \]

\[ = \alpha + \alpha\theta\Delta_i(\chi_i) + 1 - \alpha + (1-\alpha\theta)\Delta_i(\chi_i) \]

\[ = \frac{1+\Delta_i(\chi_i)}{1+\Delta_i(\chi_i)} = 1 \]

Hence, (A1) can be re-written as:

\[ c_i - (1-\tau_i^k)(1-\alpha)\varphi_i(\chi_i)y_i = -(1+\eta)k_{r+1} + \left[ 1 + (1-\tau_i^k) \left[ \alpha \varphi_i(\chi_i) \left( \frac{y_i}{k_i} \right) - \delta \right] \right] k_i \]

Further, in view of the intratemporal condition (2.4), the preceding equation yields:

\[ c_i \left[ 1 - \sigma_i(\chi_i) \frac{(1-\beta)h_i}{\beta(1-h)} \right] = -(1+\eta)k_{r+1} + \left[ 1 + (1-\tau_i^k) \left[ \alpha \varphi_i(\chi_i) \left( \frac{y_i}{k_i} \right) - \delta \right] \right] k_i \]  \hspace{1cm} (A.2)

where, as defined in (2.42),

\[ \sigma_i(\chi_i) \equiv \frac{\varphi_i(\chi_i)}{\omega_i(\chi_i)} = \begin{cases} 1, & \text{in the prototype economy} \\ \frac{1+\alpha\Delta_i(\chi_i)}{1+\alpha\Delta_i(\chi_i)} > 1, & \text{in the detailed economy} \end{cases} \]

The last result follows from the facts: (i) \( \varphi_i(\chi_i) = 1 \) and \( \omega_i(\chi_i) = 1 \), in the prototype economy; and \( \varphi_i(\chi_i) = \frac{1+\alpha\Delta_i(\chi_i)}{1+\Delta_i(\chi_i)} > 1 \) and \( \omega_i(\chi_i) = \frac{1+\alpha\Delta_i(\chi_i)}{1+\Delta_i(\chi_i)} < 1 \), given [R1] and [R2], in the detailed economy. Facts (i) and (ii) combined imply (2.13).

Now, multiplying both hand sides of (A.2) by \( \beta_i \frac{u_i^h}{c_i} \) and summing up from period 0 to any finite period T, gives:
\[
\sum_{t=0}^{T} \beta^t u_t^h \left[ 1 - \sigma_t(\chi_t) \frac{(1-\partial h_t)}{\partial(1-h_t)} \right] = \sum_{t=0}^{T} \beta^t \frac{u_t^h}{c_t} \left[ -(1+\eta)k_{t+1} + \left( 1 + (1-\tau_t^K) \left[ \alpha \bar{\omega}_t(\chi_t) \left( \frac{y_t}{k_t} \right) - \delta \right] \right) k_t \right]
\]

Further, note that:

\[
\sum_{t=0}^{T} \beta^t \frac{u_t^h}{c_t} \left[ -(1+\eta)k_{t+1} + \left( 1 + (1-\tau_t^K) \left[ \alpha \bar{\omega}_t(\chi_t) \left( \frac{y_t}{k_t} \right) - \delta \right] \right) k_t \right] =
\]

\[
\frac{u_0^h}{c_0} \left[ 1 + (1-\tau_0^K) \left[ \alpha \bar{\omega}_0(\chi_0) \left( \frac{y_0}{k_0} \right) - \delta \right] \right] k_0 - \sum_{t=0}^{T-1} \beta^t \left[ \frac{u_{t+1}^h}{c_{t+1}} \left( 1 + (1-\tau_{t+1}^K) \left[ \alpha \bar{\omega}_{t+1}(\chi_{t+1}) \left( \frac{y_{t+1}}{k_{t+1}} \right) - \delta \right] \right) k_{t+1} \right] - \beta^T \frac{u_T^h}{c_T} (1+\eta)k_{T+1}
\]

By taking expectations, based on the information available at the beginning of period 0, applying the Iterated Expectations Theorem in view of the fact that the information of economic agents about the state of the economy is increasing, using the fact that \(c_t, h_t, k_{t+1}\) are chosen by economic agents as functions of the information available at the beginning of period \(t, \epsilon_t^t\), and taking the limit as \(T \to \infty\), the preceding equation gives:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1+\eta)k_{t+1} + \left( 1 + (1-\tau_t^K) \left[ \alpha \bar{\omega}_t(\chi_t) \left( \frac{y_t}{k_t} \right) - \delta \right] \right) k_t \right] =
\]

\[
\frac{u_0^h}{c_0} \left[ 1 + (1-\tau_0^K) \left[ \alpha \bar{\omega}_0(\chi_0) \left( \frac{y_0}{k_0} \right) - \delta \right] \right] k_0 - E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{u_{t+1}^h}{c_{t+1}} \left( 1 + (1-\tau_{t+1}^K) \left[ \alpha \bar{\omega}_{t+1}(\chi_{t+1}) \left( \frac{y_{t+1}}{k_{t+1}} \right) - \delta \right] \right) k_{t+1} \right] - \lim_{T \to \infty} \beta^T E_0 \frac{u_T^h}{c_T} (1+\eta)k_{T+1}
\]

Then, it follows from the household Euler condition for capital (2.5) that the second term in the RHS of the above equation is zero. Moreover, it follows from the Transversality condition (2.6) that the fourth term of the above expression is zero. Therefore, it follows from (A.2), that along any PSE, we must have:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u_t^h \left[ 1 - \sigma_t(\chi_t) \frac{(1-\partial h_t)}{\partial(1-h_t)} \right] = \frac{u_0^h}{c_0} \left[ 1 + (1-\tau_0^K) \left[ \alpha \bar{\omega}_0(\chi_0) \left( \frac{y_0}{k_0} \right) - \delta \right] \right] k_0
\]

(A.3)
Clearly, then, in view of definitions (2.11) and (2.12), (A.3) implies the IC (2.10).

Collecting results, the resource constraint (2.2) and the IC are equivalent to all other conditions characterizing the aggregate state of the PSE, except the Initial condition, (2.7). That is, the IC incorporates the government budget constraint (2.3), the Intratemporal condition (2.4), the Euler condition (2.5) and the Transversality condition (2.6).

Finally, given the government spending policy function, \( \Delta^G(\mathcal{X}_t, \mathcal{X}_{t+1}) \), specified in Table 2.1, the aggregate production function, (2.1) and the resource constraint, (2.2), imply the GSPARC, (2.14).

**Part b:** The GSPAGBC, (2.15), simply follows from the government budget constraint (2.3), in view of the facts that:

\[
\frac{g_t}{y_t} = \Delta^G(\mathcal{X}_t, \mathcal{X}_{t+1}) \quad \text{and} \quad \frac{k_t}{y_t} = \frac{1}{\omega_t(\mathcal{X}_t)} \left( \frac{k_t}{h_t} \right)^{(1-a)}.
\]

This completes the proof of Proposition II-1.
**Proof of Remark II-2:**

(a) In view of the definition of the objective function of insiders’ government in (2.17) and the symmetry that characterizes the PSE, we have:

\[
U^i_0 = E_0 \left[ \sum_{r=0}^{\infty} \beta^r X_i \left( w_i^r h_i^r + \pi_i^r \right) \right]
\]

(A.4)

Then, note that:

\[
X_i \left( w_i^r h_i^r + \pi_i^{r+1} \right) = X_i \left( p_i^r y_i^r - r_i k_i^r \right) = X_i (1 - \alpha \theta) p_i^r y_i^r
\]

where the last equality follows from (I.18).

Moreover, it follows from PSE conditions (I.21), (II.16) and (II.17), that:

\[
(1 - \alpha \theta) \Delta_i(\chi_i) k_i^a h_i^{1-a} = \Delta_i(\chi_i) k_i^a h_i^{1-a}
\]

where:

\[
\omega_i(\chi_i) \equiv (1 - \alpha \theta) \frac{\Delta_i(\chi_i)}{1 + \Delta_i(\chi_i)}
\]

(A.5)

**Part (b):** Recall from Proposition 3 in Kollintzas et al (2021), that if \( \phi \in \left( - \frac{\theta}{1-\theta}, 0 \right) \) and [R3] hold, the following are true:

(i) \( \omega(\chi) > 0, \forall \chi \in (0,1) \)

(ii) \( \omega(0) = 1 \)

(iii) \( \omega(1) = 1 \)

(iv) \( \exists \ a unique \ \bar{\chi} \in (0,1) \ s.t. \ \omega'(\chi) = 0, \chi = \bar{\chi} \)

Accordingly, if \( \phi \in \left( - \frac{\theta}{1-\theta}, 0 \right) \) and [R3] hold,

(v) \( \hat{\omega}(\chi) > 0, \forall \chi \in (0,1) \)

(vi) \( \lim_{\chi \to 0} \hat{\omega}(\chi) = 0 \)

(vii) \( \lim_{\chi \to 1} \hat{\omega}(\chi) = 1 \)

(viii) \( and \ \hat{\omega}(\chi) \ is \ differentiable, \forall \chi \in (0,1) \).
Moreover, it follows after some algebra that:

\[
\hat{\omega}'(\chi) > 0 \text{ if and only if } 1 - \frac{1 - \phi}{\phi} \chi > \frac{1 - \phi}{\phi} \frac{1}{1 + \frac{1}{\Delta(\chi)}} + \alpha \theta \frac{1}{\theta + \frac{1}{\Delta(\chi)}} + (1 - \alpha) \xi \frac{1}{\xi + \frac{1}{\Delta(\chi)}}
\]

Note then that the LHS of the above inequality is a strictly increasing linear function of \(\chi\) that takes the value \(\chi = 0\) and the value \(1 - \frac{1 - \phi}{\phi}\) for \(\chi = 1\). But, it follows from the properties of \(\Delta(\parallel)\), established in Proposition 3 in Kollintzas et al (2021); that, for \(\phi \in \left(\frac{-\theta}{1-\theta}, 0\right)\), the RHS of the last inequality is a strictly positive, differentiable, and strictly increasing function of \(\chi\), that approaches zero as \(\chi \to 0\) and \(\alpha \theta + (1 - \alpha) \xi \frac{1 - \phi}{\phi} < 1 - \frac{1 - \phi}{\phi}\) as \(\chi \to 1\). Therefore, the above inequality is always satisfied for all \(\chi \in (0,1)\). Therefore, it follows that:

(i) \(\hat{\omega}'(\chi) > 0, \forall \chi \in (0,1)\).

This completes the proof of Remark II-2.
Proof of Proposition II-2:

Part (a): First note that, in view of Table 2.2, the steady state of the PSE for all economy specifications is characterized by the following system of equations:

\[ \frac{k}{y}(\chi) = K \bar{\omega}(\chi) \quad \text{(A.6)} \]

\[ \frac{c}{y}(\chi) = 1 - \Delta^x(\chi) - \Lambda \bar{\omega}(\chi) \quad \text{(A.7)} \]

\[ h(\chi) = \frac{\Psi(\chi)}{1 + \Psi(\chi)} \quad \text{(A.8)} \]

\[ y(\chi) = \bar{\omega}(\chi)^{1-a} \left[ \frac{k}{y}(\chi) \right]^{\frac{a}{1-a}} h(\chi) \quad \text{(A.9)} \]

where:

\[ \Psi(\chi) \equiv \frac{1 - M\Phi(\chi)}{[1 - \Lambda \Phi(\chi)] N \sigma(\chi)} \quad \text{(A.10)} \]

\[ \Phi(\chi) \equiv \frac{\bar{\omega}(\chi)}{1 - \Delta^x(\chi)} \quad \text{(A.11)} \]

\[ K \equiv \frac{\alpha(1 - \tau^k)}{\beta^{-1}(1 + \eta) - 1 + (1 - \tau^k)\delta} \quad \text{(A.12)} \]

\[ \Lambda \equiv \frac{\alpha(\eta + \delta)(1 - \tau^k)}{\beta^{-1}(1 + \eta) - 1 + (1 - \tau^k)\delta} = (\eta + \delta)K \in (0,1) \quad \text{(A.13)} \]

\[ M \equiv \frac{\alpha \left[ \beta^{-1}(1 + \eta) - 1 \right] + \delta}{\beta^{-1}(1 + \eta) - 1 + (1 - \tau^k)\delta} = \frac{\left[ \beta^{-1}(1 + \eta) - 1 \right] + \delta}{\eta + \delta} \Lambda > \Lambda \quad \text{(A.14)} \]

\[ N \equiv \frac{1 - \vartheta}{\vartheta} \quad \text{(A.15)} \]

Now, let \((c^+, h^+, k^+)\) be the unique steady state of the PSE in the case of the prototype economy with \(\chi = 0\) and let \([c^*(\chi), h^*(\chi), k^*(\chi)]\) be the unique steady state of the PSE in the case of the prototype economy with \(\chi \in (0,1)\). It is straightforward that:

\[ u^h(c^+, h^+) > u^h[c^*(\chi), h^*(\chi)] \quad \text{if and only if} \quad u^h \left[ \left( \frac{c^+}{y^+} \right) y^+, h^+ \right] > u^h \left[ \frac{c^*(\chi)}{y(\chi)} y^*(\chi), h^*(\chi) \right] \]

But, in view of Table 2.2, it follows from (A.6)-(A.9) that:
\[
\left( \frac{k^+}{y^+} \right) = \frac{k^*(\chi)}{y^*(\chi)} = K
\]
\[
\left( \frac{c^+}{y^+} \right) = \frac{c^*(\chi)}{y^*(\chi)} = 1 - \bar{\psi} - \Lambda
\]
\[
h^* = h^*(\chi) = \frac{1}{1 - M \frac{1}{1 - \bar{\psi}}} \frac{1}{1 + \frac{N(1 - \Lambda)}{1 - \bar{\psi}}}
\]
\[
y^* = \bar{\omega}(0)^{1-a} \left( \frac{k^+}{y^+} \right)^{\frac{a}{1-a}} h^*
\]
\[
y^*(\chi) = \bar{\omega}^*(\chi)^{1-a} \left[ \frac{k^*(\chi)}{y^*(\chi)} \right]^{\frac{a}{1-a}} h^*(\chi) = \left[ \frac{\bar{\omega}^*(\chi)}{\bar{\omega}^*} \right]^{1-a} \left( \frac{k^+}{y^+} \right)^{\frac{a}{1-a}} h^* = \left[ \frac{\bar{\omega}^*(\chi)}{\bar{\omega}^*} \right]^{\frac{a}{1-a}} y^*
\]

Moreover, it follows from Part (c) of Proposition 3 in Kollintzas et al (2021), that given [R3] and \( \phi \in \left\{ \frac{-\theta}{1 - \theta}, 1 \right\}, \left[ \frac{\bar{\omega}^*(\chi)}{\bar{\omega}^*} \right] < 1, \ \forall \chi \in (0, 1) \). Hence, given the stated assumptions, it follows that,

\[
u^h \left[ \frac{c^*(\chi)}{y^*(\chi)}, h^*(\chi) \right] = u^h \left[ \frac{c^*(\chi)}{y^*(\chi)} \left[ \frac{\bar{\omega}^*(\chi)}{\bar{\omega}^*} \right]^{\frac{a}{1-a}} y^+, h^* \right] < u^h \left( c^+, h^* \right)
\]
since, \( u^h(\cdot, h^*) \) is strictly increasing.

**Part (b):** Let \( (c(\chi), h(\chi), k(\chi), \chi) \) be the steady state associated with the PSE of the detailed economy for \( \chi \in (0, 1) \). Clearly then,

\[
u^h \left( c^+, h^* \right) > u^h \left[ c(\chi), h(\chi) \right] \text{ if and only if:}
\]
\[
u^h \left[ \frac{c^*(\chi)}{y^*(\chi)}, h^* \right] > u^h \left[ \frac{c(\chi)}{y(\chi)}, h(\chi) \right]
\]

or, in view of the specific form of the utility function,
\[
\left[\left(\frac{c^+}{y^*}\right)^{y^*} \right]^\alpha \left(1 - h^+\right)^{1-\alpha} > \left[\frac{c(x)}{y(x)} \right]^{y(x)} \left[1 - h(x)\right]^{1-\alpha}
\] or, with an obvious simplification of notation:

\[
\left[\left(\frac{c^+}{y^*}\right)^{y^*} \right]^\alpha \left(1 - h^+\right)^{1-\alpha} > \left[\frac{c^+}{y^*} \right]^{y^*} \left(1 - h^+\right)^{1-\alpha} > 1 \text{ or, in view of the steady state aggregate production function, (A.9):}
\]

\[
\left[\left(\frac{c^+}{y^*}\right)^{y^*} \right]^\alpha \left(1 - h^+\right)^{1-\alpha} > 1 \text{ or}
\]

\[
\left[\frac{1}{\omega^\alpha} \left(\frac{k^+}{y^{1-a}}\right)^{y^*} \left(1 - h^+\right)^{1-\alpha} \right] < 1
\]

However, in view of the specifications of the great ratios characterizing the steady state of the PSE in the case of the detailed economy, (A.6)- A.9; and Table 2.2, we have:

\[
\left(\frac{k^+}{y^*}\right) = K, \quad \left(\frac{k}{y}\right) = K\omega(x) \left(\frac{c^+}{y^*}\right)^{y^*} = 1 - \omega - \Lambda, \quad \frac{c}{y} = 1 - \omega - \omega x - \Lambda \omega(x)
\]

\[
\bar{\omega} = AB^x, \quad \bar{\omega}(x) = AB^x \left(1 - \chi\right) \left(1 + \chi \Lambda(x)\right)^{\alpha} \left[1 + \chi \Lambda(x)\right]^{1-\alpha}, \quad \bar{\omega} = 1 - \omega(x) = \frac{1 + \alpha \Lambda(x)}{1 + \Lambda(x)}
\]

\[
\frac{h^+}{1 - h^+} = \Psi(0) = \frac{1 - \omega - M}{N(1 - \omega - \Lambda)}, \quad \frac{h}{1 - h} = \frac{1 - \omega - \omega x - M \omega(x)}{1 - \omega - \omega x - \Lambda \omega(x)}
\]

\[
1 - h^+ = \frac{1}{1 + \Psi(0)} \text{ and } 1 - h = \frac{1}{1 + \Psi(x)}
\]

In view of the preceding results, (2.38) implies that:

\[
u^h\left(c^+, h^+\right) > u^h\left[c(x), h(x)\right] \text{ if and only if:}
\]

\[
\left[\left(\frac{\omega(x)}{\bar{\omega}(x)}\right)^{\alpha} \left(1 - \omega - \omega x - \Lambda \omega(x)\right)^{y(x)} \left[1 - \omega - \Lambda \omega(x)\right]^{1-\alpha} \right] < 1
\]
\[
\left[ \frac{\tilde{\omega}(\chi)}{\omega(\chi)} \right]^{\sigma} \left[ \frac{1 - \overline{\psi} - \overline{\psi} \chi - \Lambda \tilde{\omega}(\chi)}{1 - \overline{\psi} - \Lambda} \right]^{\alpha} \left[ \frac{\Omega(\chi)}{\Omega(0)} \right] < 1
\]  
(A.17)

where
\[
\Omega(\chi) \equiv \frac{\Psi(\chi)^{\sigma}}{1 + \Psi(\chi)}
\]  
(A.18)

To show that under the stated conditions the LHS of (A.17) is less than one for all \( \chi \) in (0,1), we show that, under these conditions, each one of the three terms in the LHS of (A.17) is less than on for all \( \chi \) in (0,1). The following arguments are based on the assumption that \( \phi \in \left( -\frac{\theta}{1-\theta}, 1 \right) \). Thus, first note that Part c of Proposition 3 in Kollintzas et al (2021) implies that, given [R1] – [R3], the first term in the LHS of (A.17) is less than on for all \( \chi \) in (0,1). Next, note that the second term in the LHS of (A.17) is, by virtue of Part a of Proposition 3 in Kollintzas et al (2021), one for \( \chi = 0 \), strictly positive and differentiable function for all \( \chi \) in (0,1). Moreover, this function is non-increasing if and only if,
\[
[\alpha(1 - \overline{\psi} - \overline{\psi} \chi) - \Lambda \tilde{\omega}(\chi)] \frac{\tilde{\omega}'(\chi)}{\tilde{\omega}(\chi)} \leq (1 - \alpha)\overline{\psi}, \text{ or in view of (A.11), if and only if,}
\]
\[
[\alpha - \Lambda \Phi(\chi)] \frac{\tilde{\omega}'(\chi)}{\tilde{\omega}(\chi)} \leq \frac{(1 - \alpha)\overline{\psi}}{1 - \overline{\psi} - \overline{\psi} \chi}
\]  
(A.19)

Then, observe that, since \( \frac{(1 - \alpha)\overline{\psi}}{1 - \overline{\psi} - \overline{\psi} \chi} > 0 \) and given [R1] and [R2], \( \frac{\tilde{\omega}'(\chi)}{\tilde{\omega}(\chi)} < 0 \), there are two possible cases for (A.19) to be satisfied. That is, either \( \Phi(\chi) \leq \frac{\alpha}{\Lambda} \) for all \( \chi \) in (0,1) or \( \Phi(\chi) > \frac{\alpha}{\Lambda} \) but \( \overline{\psi} \geq \frac{1 - \overline{\psi} - \overline{\psi} \chi [\alpha - \Lambda \Phi(\chi)] \tilde{\omega}'(\chi)}{1 - \alpha \tilde{\omega}(\chi)} > 0 \) for all \( \chi \) in (0,1). That is, the second term in the LHS of (A.17) is non-increasing if and only if, [R1] and [R2] and [R4] hold. Hence, given [R1]-[R2] and [R4], the second term in the LHS of (A.17) is no greater than one, for all \( \chi \) in (0,1). Finally, note that the third term in the LHS of (A.17) is one for \( \chi = 0 \), by construction. Clearly, this term is a strictly positive and differentiable function for all \( \chi \) in (0,1); and, this function is non-increasing if and only if,
\[
\left[ \frac{\sigma - \Psi(\chi)}{1 + \Psi(\chi)} \right] \frac{\Psi'(\chi)}{\Psi(\chi)} \leq 0
\]  
or, in view of (A.8), if and only if,
\[
[\sigma - h(\chi)]h'(\chi) \leq 0
\]  
(A.20)
Then, observe that there are two possible cases for \((A.19)\) to be satisfied. That is, either \(h(\chi) \leq \theta\) and \(h'(\chi) < 0\) for all \(\chi\) in \((0,1)\) or \(h(\chi) > \theta\) and \(h'(\chi) \geq 0\).

\[
\hat{\psi} \geq \frac{1 - \bar{\psi}}{1 - \alpha} \left[ \alpha - \Lambda \Phi(\chi) \right] \frac{\bar{\psi}'(\chi)}{\bar{\psi}(\chi)} > 0 \text{ for all } \chi \text{ in } (0,1). \]

Hence, given \([R1]-[R2]\) and \([R5]\), the third term in the LHS of \((A.17)\) is no greater than one, for all \(\chi\) in \((0,1)\). We conclude that given \([R1] - [R5]\), \(u^h(c^+, h^+) > u^h[c(\chi), h(\chi)]\), for all \(\chi\) in \((0,1)\).

**Part (c):** Let \(c^-, h^-, k^-, y^-\) denote the steady state levels of aggregate consumption, labor input, capital input and output associated with the steady state of the PSE of the detailed economy, in the case of no outsiders; i.e., when the share of insiders’ industries is equal to 1. Moreover, as in the parts above, we will use this notation for all variables characterizing the PSE, as well as the variables used in the characterization of the objective function of the common good government. For example, \(\omega^-\) denotes the steady state levels of the capital wedge associated with the steady state of the PSE of the detailed economy when the share of insiders’ industries is equal to 1.

Now, using the notation introduced above and in view of Table 2.2, it follows from \((A.6) - (A.9)\) that:

\[
\begin{align*}
\left( \frac{k^-}{y^-} \right) &= \theta K < K = \left( \frac{k^+}{y^+} \right) \\
\left( \frac{c^-}{y^-} \right) &= 1 - \bar{\psi} - \hat{\psi} - \theta \Lambda \\
h^- &= \frac{1}{1 + N(1 - \bar{\psi} - \Lambda)} \\
y^+ &= \omega(0)^{1 - \alpha} \left( \frac{k^+}{y^+} \right)^{1 - \alpha} h^+ \\
\bar{U}^h &\equiv \sum_{t=0}^{\infty} \beta^t u^h(c^+, h^+) > \sum_{t=0}^{\infty} \beta^t u^h(c^+, h^+) \text{ if and only if } u^h(c^+, h^+) > u^h(c^+, h^+). 
\end{align*}
\]
Clearly, then or \( u^h \left[ \left( \frac{c^+}{y^+} \right) y^+, h^+ \right] > u^h \left[ \left( \frac{c^+}{y^+} \right) y^+, h^+ \right], \) or in view of the explicit form of the household temporal utility function, \( y^+ > y^+, \) which is always true under the stated conditions. Part c, follows from Part b, by continuity as \( \chi \) tends to one.

**Part (d):** It is a direct implication of Parts (a) –(c) and the definition of \( \bar{U}^h, \)

\[
\bar{U}^h \equiv \max \left\{ \sum_{t=0}^{\infty} \beta^t u^h(c^+, h^+), \sup_{\chi \in (0,1)} \sum_{t=0}^{\infty} \beta^t u^h(c^+(\chi), h^+(\chi)), \sup_{\chi \in (0,1)} \sum_{t=0}^{\infty} \beta^t u^h(c(\chi), h(\chi)), \sum_{t=0}^{\infty} \beta^t u^h(c^-, h^-) \right\}
\]

\[
= \sum_{t=0}^{\infty} \beta^t u^h(c^+, h^+).
\]

*This completes the proof of Proposition II-2.*
Proof of Proposition II-3:

Part a: Using the notation introduced above, it follows from Remark II-2, that:

\[ v'(h^*, k^-, 1) > v'[h(\chi), k(\chi), \chi], \quad \forall \chi \in (0, 1), \text{ if and only if}, \]

\[ \bar{\omega} \bar{\omega} (k^-)^{\alpha} (h^-)^{1-\alpha} > \bar{\omega}(\chi)\bar{\omega}(\chi)k(\chi)^{\alpha} h(\chi)^{1-\alpha} k(\chi)^{\alpha} h(\chi)^{1-\alpha}; \quad \text{or, in view of the steady state} \]

version of the aggregate production function, (A.9), if and only if:

\[ \frac{\bar{\omega}(\chi)}{\bar{\omega}} \left( \frac{k^-}{y^-} \right)^{1-\alpha} h^- > \bar{\omega}(\chi)\bar{\omega}(\chi) \left( \frac{k(\chi)}{y(\chi)} \right)^{1-\alpha} h(\chi); \]

or

\[ \left[ \frac{\bar{\omega}(\chi)}{\bar{\omega}} \right]^{1-\alpha} \left[ \frac{k(\chi)}{y(\chi)} \right]^{1-\alpha} h(\chi) < 1, \quad \forall \chi \in (0, 1) \]  

(A.21)

Now, in view Remark II-2 and the representation of the steady state of the PSE (i.e.,
equations (A.6) – (A.15)) and Table 2.2, it follows that: \( \bar{\omega}(\chi) = (1-\alpha\theta)\frac{\Delta(\chi)}{1+\Delta(\chi)} \),

\[ \bar{\omega} = (1-\alpha\theta), \quad \bar{\omega}(\chi) = \frac{1+\theta\Delta(\chi)}{1+\Delta(\chi)}, \quad \bar{\omega} = \Theta, \quad \left[ \frac{k(\chi)}{y(\chi)} \right] = K \bar{\omega}(\chi) \quad \text{and} \quad \left( \frac{k}{y} \right) = \theta K. \]

It follows that,

\[ v'(h^*, k^-, 1) > v'[h(\chi), k(\chi), \chi], \quad \forall \chi \in (0, 1), \text{ if and only if}, \]

\[ \left[ \frac{\bar{\omega}(\chi)}{\bar{\omega}} \right]^{1-\alpha} \left[ \frac{\Delta(\chi)[1+\theta\Delta(\chi)]}{\Theta^{1-\alpha}[1+\Delta(\chi)]} \right]^{1-\alpha} h(\chi) < 1 \]

(A.22)

Next, we show that, under these conditions, each one of the three terms in the LHS of (A.22)
is less than one, for all \( \chi \) in (0, 1). The following arguments are based on the assumption
that \( \phi \in \left(-\frac{\theta}{1-\theta}, 1\right) \). Thus, first note that Part c of Proposition 3 in Kollintzas et al (2021)
implies that, given [R1] – [R3], the first term in the LHS of (A.22) is less than one, for all
\( \chi \) in (0, 1). Next, note that the second term in the LHS of (A.22) is, by virtue of Part a of
Proposition 3 in Kollintzas et al (2021), zero for \( \chi = 0 \) and one for \( \chi = 1 \). Moreover, this
term is a strictly positive, differentiable, and strictly increasing function of $\chi$ in $(0,1)$. It follows that the second term in the LHS of (A.21) is less than one, for all $\chi$ in $(0,1)$. Finally, note that the third term in the LHS of (A.22) is one for $\chi = 1$, by construction; and, this term is, a strictly positive, differentiable, and, by virtue of [R1]-[R2] and [R5], strictly increasing function of $\chi$, for all $\chi$ in $(0,1)$. Hence, given [R1]-[R3] and [R5], the third term in the LHS of (A.21) is less than one, for all $\chi$ in $(0,1)$. We conclude that given [R1]-[R3] and [R5], $\nu'(h^{-},k^{-},1) > \nu'[h(\chi),k(\chi),\chi]$, $\forall \chi \in (0,1)$.

**Part b:** It is a direct implication of Parts (a) and the definition of $\overline{U}^i$, that:

$$\overline{U}^i =\max\left\{\sup_{\chi \in (0,1)} \beta_i^{\nu'}[c(\chi),h(\chi),\chi], \sum_{t=0}^{\infty} \beta^i v'(c^{-},h^{-},1) \right\}$$

$$\overline{U}^i =\sum_{t=0}^{\infty} \beta^i v'(c^{-},h^{-},1).$$

*This completes the Proof of Proposition II-3.*
Politico-economic Equilibrium with a Hybrid Government:

First Order Conditions for $t = 0$

\[
M_{\rho_h}^\rho + N_{\rho_h} + (1 - \alpha)(1 - \Delta_h^\rho) \bar{\phi}_i \left( \frac{k_0}{h_0} \right)^\alpha (M_{\rho_u}^\rho + N_{\rho_u}) = 0
\]  
\tag{A.23}

\[
(1 + \eta)(M_{\rho_u}^\rho + N_{\rho_u}^\rho) = \beta E_0 \left\{ M_{\rho_h} + M_{\rho_u}^\rho \left[ (1 - \delta) + \alpha(1 - \Delta_h^\rho) \bar{\phi}_i \left( \frac{k_i}{h_i} \right)^{\alpha-1} \right] \right\}
\]  
\tag{A.24}

\[
(M_{\rho_u}^\rho + N_{\rho_u}^\rho) \frac{\partial \Delta_i^\rho}{\partial \chi_i} \bar{\phi}_h k_0^{\alpha} h_0^{1-\alpha} = \beta E_0 \left\{ M_{\rho_h}^\rho + M_{\rho_u}^\rho \left[ - \frac{\partial \Delta_i^\rho}{\partial \chi_i} \bar{\phi}_h \left( \frac{k_i}{h_i} \right)^{\alpha} \right] \bar{\phi}_h k_0^{\alpha} h_0^{1-\alpha} \right\}
\]  
\tag{A.25}

where:

\[
N_{\rho_u} = \frac{\theta(1 - \gamma) - 1}{c_0} m \left( \frac{\tilde{z}_0}{c_0} \right) \left( \frac{u_0^h}{\tilde{U}_0^h} \right)
\]  
\tag{A.26}

\[
N_{\rho_h} = \left[ - \frac{(1 - \theta)(1 - \gamma) + \frac{\partial \tilde{z}_0}{\partial h_0} \frac{h_0}{h_0}}{1 - h_0} + \frac{\partial \tilde{z}_0}{\partial h_0} \frac{h_0}{h_0} \right] m \left( \frac{\tilde{z}_0}{c_0} \right) \left( \frac{u_0^h}{\tilde{U}_0^h} \right)
\]  
\tag{A.27}
Proof of Proposition II-4:

By definition, \( (c,h,k,\chi,m) \) is an asymptotic steady state of the politico-economic equilibrium with a hybrid government, if and only if, it is a solution to the following five-equation system: (2.31) – (2.33) and the GSPARC, (2.14) and the IC, (2.10), evaluated at \( (c,h,k,\chi,m) \):

\[
\frac{c}{y} = (1-\bar{\psi} + \hat{\psi}\chi) - (\eta + \delta)\left(\frac{k}{y}\right)
\]

(A.28)

and

\[
\hat{\xi} = \frac{(1 - \beta) \left\{ a(1 - \tau^k)\bar{\phi}\chi + [1 - (1 - \tau^k)\delta]\right\}}{\left(\frac{c}{y}\right) - \left(\frac{k}{y}\right)}
\]

(A.29)

Here, in going from (2.10) and (2.14) to (A.28) and (A.29), respectively, we have used the definitions:

\[
\Delta^i = \bar{\psi} + \hat{\psi}\chi \quad \text{(A.30)}
\]

\[
y = \bar{\phi}(\chi)k^\alpha h^{1-\alpha} \quad \text{(A.31)}
\]

\[
\hat{\xi} = [1 - \sigma(\chi)]\left(\frac{(1 - \sigma)h}{\sigma(1-h)}\right) \quad \text{(A.32)}
\]

It is apparent that (A.28) is (2.1), (A.29) is (2.35) and (A.32) yields (2.36) or

\[
h = \frac{1}{1 + \frac{(1 - \sigma)\bar{\phi}(\chi)}{\sigma(1 - \hat{\xi})}} \quad \text{(A.33)}
\]

Then, note that, in view of (A.30) – (A.32), conditions (2.31) – (2.33) can be re-written as follows:

\[
\left(\frac{1}{1-h}\right)\left[\frac{(1-\sigma)h}{\sigma} - \frac{m(1-\hat{\xi})}{\sigma(1-\gamma)(1-\rho-m\xi)}\right]\left(\frac{c}{y}\right) + (1-\alpha)\frac{\rho}{\sigma(1-\gamma)}\left(\frac{v}{u}\right)\left(\frac{U^h}{u^h}\right)\left(\frac{c}{y}\right) = (1-\alpha)(1-\bar{\psi} - \hat{\psi}\chi)
\]

(A.34)
\[
\alpha \left( 1 - \tilde{\psi} - \tilde{\psi}' \chi \right) + \frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})}
\]

\[
\left( \frac{k}{y} \right) = \frac{\beta^{-1}(1+\eta) - (1-\delta)}{\beta^{-1}(1+\eta) - (1-\delta)}
\]

(A.35)

\[
\frac{m(1 - \xi)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})} + \frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})} = \tilde{\psi} - \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} (1 - \tilde{\psi} - \tilde{\psi}' \chi)
\]

(A.36)

Now, since by assumption \( \rho \in (0,1) \), observe that:

\[
\frac{m(1 - \xi)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})} = \frac{m(1 - \xi)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})}
\]

(A.37)

and

\[
\frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})} = \frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})}
\]

(A.38)

However, since \( m \neq 0 \) is an unknown real number, so is \( \left( \frac{m}{1 - \rho} \right) \). For that matter, without loss of generality, we can re-write (A.34) - (A.36) as follows:

\[
\left( \frac{1}{1 - h} \right) \left[ \left( 1 - \vartheta \right) m \left( 1 - \xi \right) \right] \left[ \frac{1}{\vartheta (1 - \gamma)(1 - m \tilde{\xi})} \right] \left( \frac{c}{y} \right) + (1 - \alpha) \frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - m \tilde{\xi})} = (1 - \alpha)(1 - \tilde{\psi} - \tilde{\psi}' \chi)
\]

(A.39)

\[
\alpha \left( 1 - \tilde{\psi} - \tilde{\psi}' \chi \right) + \frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - m \tilde{\xi})}
\]

\[
\left( \frac{k}{y} \right) = \frac{\beta^{-1}(1+\eta) - (1-\delta)}{\beta^{-1}(1+\eta) - (1-\delta)}
\]

(A.40)

\[
\frac{m(1 - \xi)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})} + \frac{\rho \left( \frac{v'}{U'} \left( \frac{\hat{U}^h}{u^h} \right) \left( \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} \right) \left( \frac{c}{y} \right) \right)}{\vartheta (1 - \gamma)(1 - \rho - m \tilde{\xi})} = \tilde{\psi} - \frac{\hat{\omega}'(\chi)}{\hat{\omega}(\chi)} (1 - \tilde{\psi} - \tilde{\psi}' \chi)
\]

(A.41)
Clearly then, if we set:

\[ \xi = \frac{m(1 - \xi)}{\vartheta(1 - \gamma)(1 - m\xi)} \left( \frac{c}{y} \right) \]  \hspace{1cm} (A.42)

\[ \hat{\xi} = \frac{\rho}{1 - \rho} \left( \frac{\varphi_i}{\bar{U}^h} \right) \left( \frac{U^h}{u^h} \right) \left( \frac{c}{y} \right) = \frac{\varepsilon^\rho}{\vartheta(1 - \gamma)(1 - m\xi)} \left( \frac{c}{y} \right) \] \hspace{1cm} (A.43)

where, the last equality in (A.43) follows from the definition of \( \varepsilon^\rho \) in (2.41). Then, it is straightforward that, in view of definitions (A.42) and (A.43): (A.39) yields (2.37), (A)

Finally, observe that the auxiliary variables \( \xi, \hat{\xi}, \bar{\xi} \) are not independent of each other and their very definitions imply that they should satisfy (2.40), as a consistency requirement. To see this note that (A.42) and (A.43) can be solved for the multiplier, \( m \), to give:

\[ m = \frac{\vartheta(1 - \gamma)\xi - \varepsilon^\rho \left( \frac{c}{y} \right)}{(1 - \xi) \left( \frac{c}{y} \right) + \vartheta(1 - \gamma)\bar{\xi} \bar{\xi}} \] \hspace{1cm} (A.44)

Then, it is straightforward to show that the second equality in (A.44) yields (2.40).

This completes the proof of Proposition II-4.
APPENDIX III.

Figure A.III.1. Sensitivity analysis with respect to $\phi$
Figure A.III.2. Sensitivity analysis with respect to $\lambda$
Figure A.III.3. Sensitivity analysis with respect to $\mu$
Figure A.III.4. Sensitivity analysis with respect to $\Theta$
BANK OF GREECE WORKING PAPERS


58