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GREEK GDP FORECASTING USING BAYESIAN MULTIVARIATE MODELS

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ABSTRACT

Building on a proper selection of macroeconomic variables for constructing a Gross Domestic Product (GDP) forecasting multivariate model (Kazanas, 2017), this paper evaluates whether alternative Bayesian model specifications can provide greater forecasting accuracy compared to a standard Vector Error Correction model (VECM). To that end, two Bayesian Vector Autoregression models (BVARs) are estimated, a BVAR using Litterman's prior (1979) and a BVAR with time-varying parameters (TVP-BVAR). Two forecasting evaluation exercises are then carried out, a 28-quarters ahead forecast and a recursive 4-quarters ahead forecast. The BVAR outperformed the other models in the first, whereas the TVP-VAR was the best-performing model in the second, highlighting the importance of having adjusting mechanisms, such as time-varying coefficients in a model.

Keywords: Bayesian VARs, Forecasting, GDP, TVP-VAR, VECM

JEL-Classification: C11, C51, C52, C53

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1. Introduction

Macro-econometrics according to Stock and Watson (2001), serve a quadruple purpose: Data description, forecasting, structural inference, and policy analysis. To that end, several types of models, from single-equation to large models with hundreds of equations have been used, like Klein's LINK model in 1980 (Klein, 1976) and more recently, Dynamic Stochastic General Equilibrium (DSGE) models (Christiano et al., 2018).

Lucas and other new classical economists were especially critical of the use of large-scale macro-econometric models to evaluate policy impacts when they were purportedly sensitive to policy changes (Lucas, 1976). Given that the optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any policy change will systematically alter the structure of econometric models.

Sim's (1980) framework of Vector Autoregressive models (VARs) came as an answer to this critique. VARs are n-equation, n-variable linear models in which each variable is in turn explained by its own lagged values, plus past values of the remaining n - 1 variables. This simple framework provides a systematic way to capture rich dynamics in multiple time series, while at the same time, the statistical toolkit that came with VARs is easy to use and interpret. As Sims (1980) and others argued in a series of influential early papers, VARs held out the promise of providing a coherent and credible approach to data description, forecasting, structural inference, and policy analysis.

Alternative to VARs, which are estimated equation by equation using OLS, are the Bayesian VARS (BVARs), initially proposed by Sims (1980) and Doan, Sims, and Litterman (1984), which through Bayesian shrinkage sought to further improve the forecasting performance of the multivariate econometric models available at the time. The BVARs' superiority in forecasting is well established as the literature is rich in Bayesian multivariate models that outperform either standard frequentist or DSGE models, for example, see Gupta and Mabundi (2010).

To estimate a BVAR, the formulation of priors is necessary, with the most popular one being the so-called Minnesota prior (Litterman, 1979). However, since its introduction several more advanced priors have been proposed, such as the one by Sims and Zha (1998), or the GLP prior (Giannone et al. 2015). Furthermore, advances in Bayesian statistics and computational capabilities have enabled the use of more complex BVARs, such as the Time-Varying Parameter VARs (TVP-VARs) with the most prominent work on such models being that of Cogley and Sargent (2002;2005), Primiceri (2005), and more recently Carriero (2015).

This advantage of inputting a researcher's belief or knowledge as a BVAR prior has an extra argument in favor of Bayesian specifications, when it comes to specifically forecasting Greek macroeconomic variables. This is because as the time series usually used in estimating Greek macroeconomic models' coefficients start at the year 2000, a significant portion of the sample is comprised of observations that occurred during the economic crisis. This may lead to obtaining coefficients that do not accurately reflect the data generating process of the economy over the long run, and hence it makes sense to limit the parameter space that OLS would have to "search" for coefficient estimation by imposing priors consistent with general macroeconomic stylized facts. Despite that, the application of BVARs in forecasting Greek macroeconomic activity is rather limited, with the most prominent work being that of Louzis about macroeconomic and credit variables forecasting using BVARs (2017) and Greek GDP nowcasting (2018).

Against this background we sought out to use a set of macroeconomic variables, suitable for use in a multivariate model to generate GDP forecasts, as specified in Kazanas (2017), to estimate alternative specifications of BVARs and examine the accuracy gains in GDP forecasting using the standard frequentist Vector Error Correction Model (VECM) estimated in Kazanas (2017) as a benchmark. To do so, we opted to use two alternative models: A BVAR estimated using the Minnesota prior under the notion of limiting the parameter space to obtain a more parsimonious model as explained earlier, and a TVP-VAR to allow the coefficients to change throughout the sample, thus generating forecasts based on the most recent state of the business cycle. The forecasting exercise comprises of two types of pseudo out-of-sample forecasts: The first one is a four-quarter rolling window and the second one is a forecast of all the out-of-sample observations, which are then evaluated.

The remaining of the paper is organized as follows: In section II the data used in estimating the three models (the benchmark VECM, the BVAR, and the TVP-VAR) are presented, in section III we concisely present the models and their estimation techniques, while in section IV we present the forecasting exercise results and lastly in section V we discuss the conclusions and policy implications.

2. The data

The variable selection for the Greek GDP forecasting follows Kazanas (2017), where a VECM is constructed including data for real GDP (Y), unemployment rate (U), GDP deflator (P), 10-year government bond yield (GB), and exports as a percentage of GDP (XY). The data sample ranges from 2000Q1 to 2022Q4. All data are adjusted for seasonality and sourced from Eurostat's national accounts (Eurostat database code: na10), labor market survey (Eurostat database code: labor), and interest rates (Eurostat database code: irt) databases.

For each variable, the ADF unit root test (Dickey and Fuller, 1981) was conducted, as stationarity is a prerequisite, especially in estimating a standard VECM or a VAR model. All variables have a unit root in levels but are stationary if they are transformed into log differences.

Table 1: *ADF test p-values*

	Y	U	P	GB	XY
Levels	0.5357	0.1718	0.9976	0.1124	0.1308
Log differences	0.0000	0.0273	0.0000	0.0000	0.0000

In Figures 1 and 2 we can see the long-run evolution of the series over time. It is evident that the variables have increased volatility from 2009 to 2015, which reflects the impact of the economic crisis. Furthermore, real GDP, unemployment rate, GDP deflator, and exports (% GDP) display increased volatility after the second quarter of 2020, which reflects the impact of the pandemic and the corresponding lockdown and the following recovery of economic activity.

Figure 1: Macroeconomic variables in levels

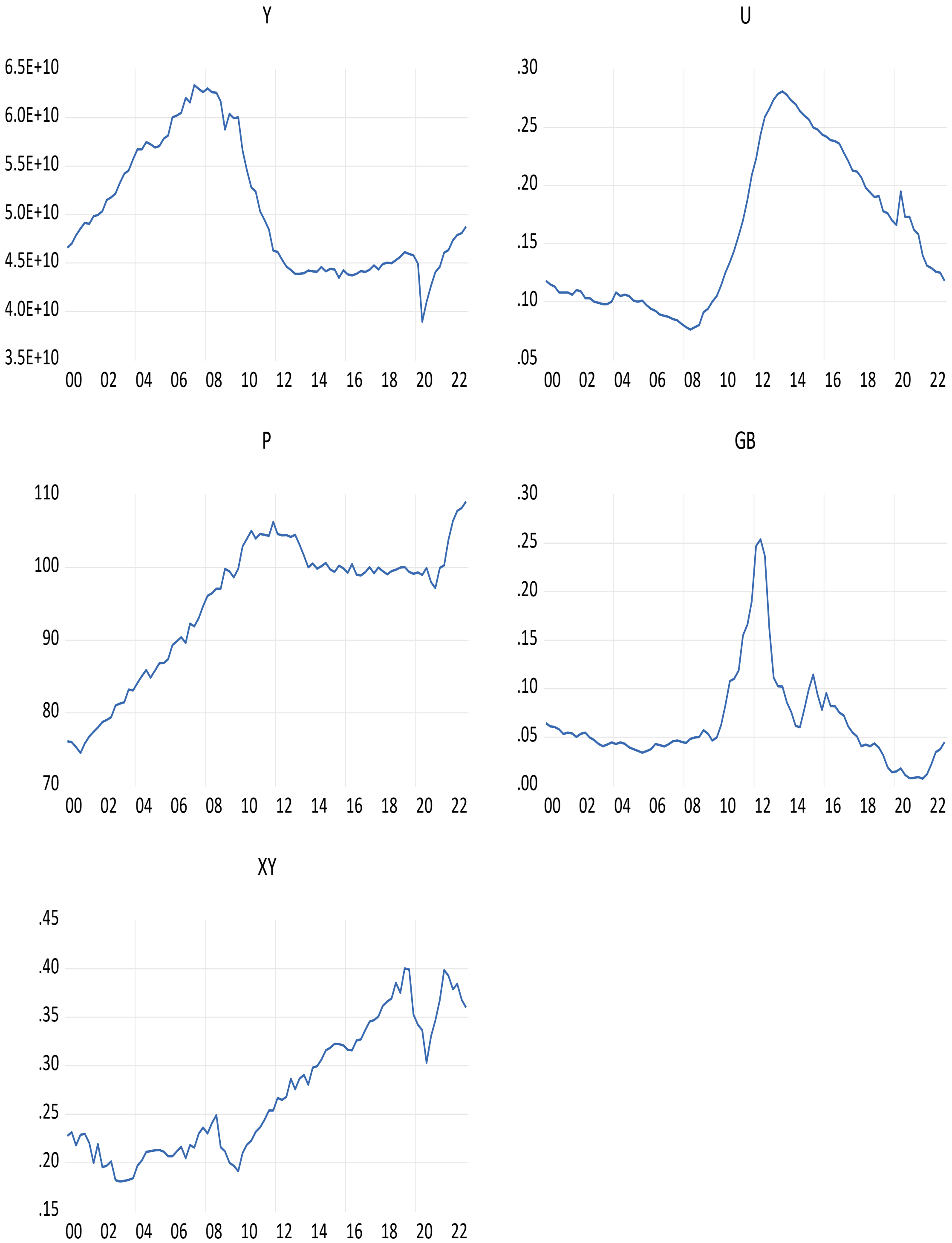
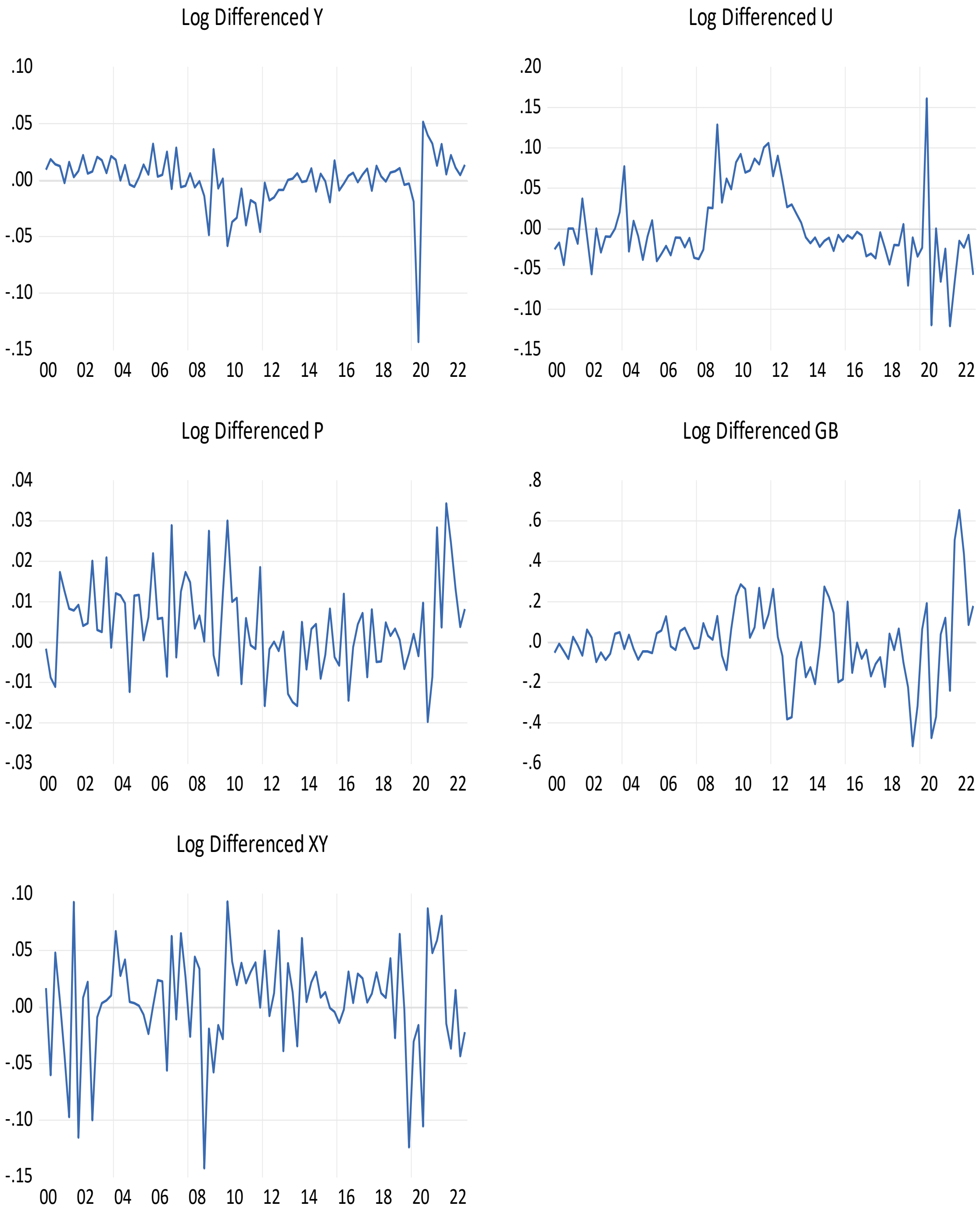


Figure 2: Macroeconomic variables in log differences



3. Models presentation and estimation

The three models mentioned earlier are estimated using the abovementioned variables in log differences. The models are estimated over sixteen years (2000:Q1 to 2015:Q4), whereas the remaining sample (2016:Q1 to 2022:Q4) is to be used for evaluating their forecasting performance. The VECM and the BVAR model are estimated using EViews 10, while the TVP-VAR is estimated using the BEAR Toolbox 4.2 (Dieppe et al. 2016).

The VECM benchmark model

Based on the works of Granger (1981), Engle and Granger (1987), Vector error correction models are essentially restricted VARs, which contain a set of variables both in differences and in levels. The differences of the variables included in the model represent the short-run interrelations of the variables, whereas the linear combination of the levels of the variables, commonly referred to as the cointegrating vector (or vectors, as more than one linear combination of a set of variables can be included), represents the long-run dynamics of the variables. Mathematically, a representative VECM model can be written as follows:

$$\Delta y_t = m + \sum_{i=1}^{p-1} B_i \Delta y_{t-i} + A y_{t-1} + \varepsilon_t \tag{1}$$
$$\varepsilon_t \sim N(0, \Sigma)$$

Where *y* is the vector containing the variables (in our case *y'= [y u p gb xy]* [1]), m is the vector containing the constants of the equations system, B_i is the matrix that contains the coefficients that describe the short-run impact of the variables' lag i and A is the matrix that contains the coefficients that describe the long-run relationship between the variables. The model can also be expanded to include exogenous variables. VECMs are very useful in modeling non-stationary time-series without having to exclude their long-run behavior, but, like their unrestricted counterparts (VARs), they suffer from the "curse of dimensionality", as the addition of a variable significantly increases the number of coefficients to be estimated.

The estimation of this model follows the Johansen procedure (Johansen, 1995). A VAR is estimated in levels (including a constant and a trend) and by incorporating the lag length criteria it is found that two lags are optimal. The existence of a cointegrating relation between the variables must be confirmed in order to use a VECM specification rather than a simple VAR in differences. The max eigenvalue cointegration test is therefore used, which indicates the existence of two cointegrating

[1]Lowercase letters denote the natural logarithms of the variables in question.

vectors at the 5% level[2]. Hence a VECM is estimated, with 1 lag per variable and 2 cointegrating vectors.

Table 2: Maximum Eigenvalue Cointegration Test

Hypothesized Number of Cointegrating equations	Eigenvalue	Max-Eigenvalue statistic	5% Critical value	P-value
None *	0.567460	51.96106	38.33101	0.0008
At most 1 *	0.450144	37.08209	32.11832	0.0114
At most 2	0.296067	21.76650	25.82321	0.1571
At most 3	0.255644	18.30464	19.38704	0.0713
At most 4	0.123745	8.190059	12.51798	0.2366

* Denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

The BVAR model

Under the Bayesian approach to econometrics, the estimated coefficients of a model are not an attempt to estimate their true value, but instead, they are perceived as a summary of the posterior distribution, which in its turn is proportional to the likelihood function times the prior distribution. Priors represent any knowledge the researcher has beforehand about the coefficients. Following this technique results in the coefficients being essentially a matrix-weighted average between the imposed priors and a regular OLS estimation (Ouliaris et al, 2016), which leads the variables to behave as if they were random walks (Del Negro and Schorfheide, 2010):

$$\hat{b} = [V^{-1} + \Sigma_e^{-1} \otimes (X'X)]^{-1}[V^{-1}\bar{b} + (\Sigma_e^{-1} \otimes X')Y] \tag{2}$$

where $\hat{b}$ is the matrix of the estimated VAR coefficients, V is the variance matrix of the prior distribution of the model's coefficients, Σ_e is the variance-covariance matrix of the model's residuals and $\bar{b}$ is a diagonal matrix containing the prior means of each variable's own first lag coefficients. X and Y are the variables included in the model.

[2]In Kazanas (2017) the existence of the second cointegrating vector is rejected as the hypothesis of at most 1 cointegrating vector is marginally accepted with a P-value of 0.0505, but since then a major benchmark revision of the Greek macroeconomic data has occurred causing the maximum eigenvalue cointegration test to indicate the existence of a second cointegrating vector.

The error variance-covariance matrix Σ_e necessary for the coefficient estimation is either estimated by fitting an AR(1) model on every variable and getting the error variances, by estimating an AR(1) and a VAR to obtain the diagonal elements of the variance-covariance matrix, or by estimating all variances-covariances as implied by a full VAR (an option not commonly used, as it can lead to a singular matrix). Under the Minnesota prior, the researcher is required to specify a set of hyperparameters in order to formulate the priors to obtain the model's coefficients: μ_1, λ_1, λ_2, and λ_3.

μ_1 is used as the prior mean of the coefficients in the matrix $\bar{b}$ and it usually takes the value of 0 (if the variables of the model are stationary) or 1 (if the variables of the model have a unit root). λ_1, λ_2 and λ_3 are used to formulate diagonal elements of the V matrix (with non-diagonal elements being set to 0). More specifically, each diagonal element of the V matrix for the j-th variable in the i-th equation at lag k is formulated as follows:

$$\left(\frac{\lambda_1}{k^{\lambda_3}}\right)^2 \text{ for } i = j, \tag{3}$$

$$\left(\frac{\lambda_1 \lambda_2 \sigma_i}{k^{\lambda_3} \sigma_j}\right)^2 \text{for } i \neq j \tag{4}$$

where σ_i, σ_j are the square roots of the corresponding elements of the Σ_e matrix.

This way λ_1 determines how binding the restrictions are. The closer to zero the value of λ_1 is, the more binding the restrictions are in the estimation of the coefficients. A value over 10 implies an uninformative prior. λ_2 determines the cross-variable effects in the equations and is set between 0 and 1. The closer the value is to 1 the more lags of variable j impact variable i (for $j \neq i$) in the BVAR. Finally, λ_3 determines the decay rate of the own lags of a variable, excluding the first lag. As this hyper-parameter approaches zero, higher order lags decay at a slower rate.

Having obtained the coefficients, the resulting functional form of the model is expressed as follows:

$$\Delta y_t = B_0 + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + \ldots + B_p \Delta y_{t-p} + \varepsilon_t \tag{5}$$

$$\varepsilon_t \sim N(0, \Sigma)$$

Where (as in the previous model) y is the vector containing the variables (in our case *y'= [y u p gb xy]* [3]), B_0 is the vector containing the constants of the equations system, B_i is the matrix that contains the coefficients and ε_t is the vector containing the error terms.

To obtain the coefficients in our model, apart from μ_1, which is set to zero as the model is estimated in log differences, which are stationary, the rest hyper-parameters through forecasting sensitivity checks as suggested by Canova (2007) (see Appendix). The resulting set of hyper-parameters is the following: $\lambda_1 = 7$, $\lambda_2 = 0.2$, $\lambda_3 = 0.1$. Furthermore, to obtain an estimation of the variance-covariance matrix an AR (1) model is fitted through each variable to estimate the variances of the residuals, while the covariances (the diagonal elements of the matrix) are obtained from the equivalent matrix of the corresponding OLS VAR. The lag length of the model is set to 2, as suggested by the lag length criteria.

The TVP-VAR

The time-varying parameter VAR is a model that allows model coefficients to change over time. This is particularly useful in capturing nonlinear relationships in the data as any model with time-varying parameters can successfully represent any nonlinear functional form (Swamy, 1975 and Granger, 2008). Macroeconomic variables are known to impact differently each other across the business cycle or after structural changes, hence the TVP-VAR is an interesting approach to econometric modeling. The functional form of a TVP-VAR is expressed as:

$$\Delta y_t = B_{0,t} + B_{1,t}\Delta y_{t-1} + B_{2,t}\Delta y_{t-2} + \ldots + B_{p,t}\Delta y_{t-p} + \varepsilon_t \qquad (6)$$

Where $\varepsilon_t \sim N(0, \Sigma_t)$

With y_t[4] being the matrix containing the variables and $B_{i,t}$ being the matrix containing the time-varying coefficients. Elements of the $B_{i,t}$ matrices are assumed to follow a random walk process:

$$\beta_t = \beta_{t-1} + v_t \qquad (7)$$

[3]Lowercase letters denote the natural logarithms of the variables in question.
[4]Lowercase letters denote the natural logarithms of the variables in question.

Where $v_t \sim N(0, \Omega)$

Apart from time-varying parameters, TVP-VARs include stochastic volatility (hence the Σ_t). This approach makes the model heavily parametrized but is necessary to avoid bias in the coefficients across potential volatility clusters, falsely attributing variance shocks to coefficient variation (Sims, 2002). The formulation of the Σ_t matrix is based on Cogley and Sargent (2005) in the BEAR toolbox. Under this approach the Σ_t matrix has $f_{n,m}$ non-diagonal elements which are time invariant and are assumed to follow a multivariate normal distribution.

The diagonal elements of the Σ_t matrix are of the form $\bar{s}_i e^{\lambda_{i,t}}$, with $\bar{s}_i$ being a time invariant scaling factor. On the other hand, $\lambda_{i,t}$ follows an AR (1) process:

$$\lambda_{i,t} = \gamma \lambda_{i,t-1} + u_{i,t} \quad (8)$$

Where $u_{i,t} \sim N\ (0, \varphi_i)$

Thus, to formulate the prior for the $\lambda_{i,t}$ the hyperparameter γ has to be determined by the researcher. Furthermore, for the prior of φ_i holds that:

$$\varphi_i \sim IG(\frac{\alpha_0}{2}, \frac{\delta_0}{2})$$

With α_0, δ_0 being scaling factors that also need to be determined by the researcher. As the posterior for the $f(B, \Omega, f^{-1}, \lambda, \varphi | y)$ cannot be analytically solved, once the abovementioned hyperparameters have been chosen, the Gibbs sampler must be used to obtain results. For more detailed presentations of TVP-VARs, one can look up Primiceri (2005), Chan and Jeliazkov (2009), Lubik and Matthes (2015), or Dieppe et al. (2018).

The above equations imply that there is no mechanism in the model to produce future values of the coefficients of the model, as in the absence of new shocks, coefficients remain the same. It is an interesting approach, however, to attempt a forecast based on the most recent interrelations between the variables and neglect coefficient values of the past, which may not adequately represent the dynamics of the system anymore.

In our estimation of the model, we follow Primiceri (2005) in choosing the number of lags, which is set to 2. We also set $\alpha_0 = \delta_0 = 0.001$ implying a rather uninformative prior. Furthermore, we set γ= 0.95, implying strong persistence of variance shocks thus limiting the possibility of explosive behavior in the model's coefficients[5] (strong persistence of shocks is also a valid macroeconomic assumption). Finally, we set the Gibbs sampler to perform 10000 iterations, out of which 2000 are burn-in iterations.

4. Forecasting evaluation

To evaluate the forecasting performance of the models two different forecasting exercises are carried out. The first one consists of estimating the models up to 2015:Q4 (as described in the previous sections) and then conducting a forecast of 28 quarters ahead (up to 2022:Q4). The second one is a recursive forecasting exercise, where the three models are estimated up to time t and perform a forecast of the quarters *t+1* to *t+4*, they are then estimated up to *t+1* and perform a forecast of the quarters *t+2* to *t+5*, and so on and so forth. Overall, 26 recursive estimations are performed from 2015:Q4 up to 2021:Q4, with the last forecast being that of 2022:Q1-2022:Q4.

The forecasts are then evaluated using the Mean Absolute Percentage Error (MAPE), the Mean Absolute Error (MAE), and the Root Mean Squared Error (RMSE):

$$MAPE = \left(\frac{1}{n}\sum_{t=1}^{n}\frac{|Y_t - \widehat{Y}_t|}{Y_t}\right) * 100 = \left(\frac{1}{n}\sum_{t=1}^{n}\frac{|e_t|}{Y_t}\right) * 100 \tag{9}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|Y_t - \widehat{Y}_t\right| = \frac{1}{n}\sum_{t=1}^{n}|e_t| \tag{10}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t - \widehat{Y}_t)^2} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2} \tag{11}$$

Where n is the period of the forecasting horizon, $\widehat{Y}_t$ is the forecasted value of GDP and Y_t is the actual value of GDP at time t. The forecast evaluation results can be found in Table 3.

[5]To further check for such behavior in the model's coefficients, upon estimation we performed stationarity test. All coefficients are found to be stationary within a 10% level of significance (most in levels and a few in first differences).

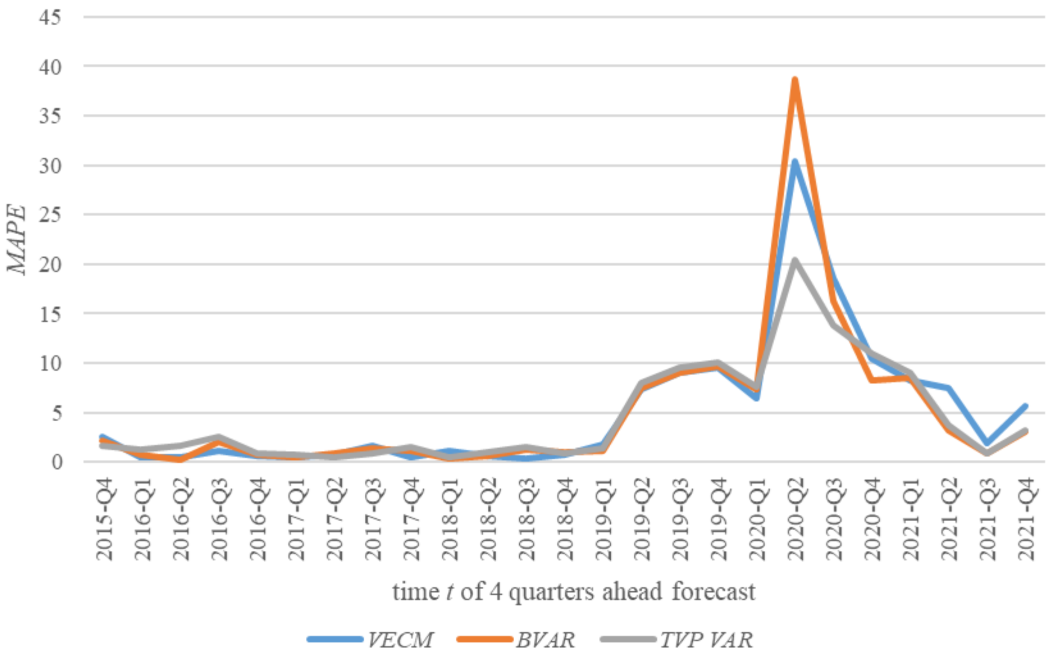
Table 3: Forecast evaluation criteria

	28 quarters ahead forecast			Recursive forecasts*		
	MAPE	MAE	RMSE	MAPE	MAE	RMSE
VECM	4.362820	2.06E+09	2.77E+09	5.122833	2.08E+09	2.34E+09
BVAR	**3.247458**	**1.47E+09**	**2.08E+09**	5.044736	2.00E+09	2.23E+09
TVP-VAR	4.335012	1.98E+09	2.67E+09	**4.548460**	**1.98E+09**	2.23E+09

**Average of 26 recursive iterations*

Overall, the Bayesian models outperform our benchmark standard VECM model. More specifically, the BVAR with the Minnesota prior provides a more accurate forecast over the long run, as evidenced by the first forecasting exercise, whereas the TVP-VAR is more accurate when it comes to short-term forecasts.

Figure 3: MAPE of recursive forecasts



A further interesting point arises when we examine the temporal distribution of the forecasting errors of the models from the recursive forecast. As shown in Figure 3, during the period 2016-2019, when GDP demonstrated limited variability, all models performed similarly. However, during the quarters impacted by COVID-19 and the lockdown, even though forecast errors spiked across models, the two models containing adjusting mechanisms, namely the cointegrating vector for the VECM and the time-varying coefficients of the TVP VAR, outperformed the BVAR. In the post COVID-19 quarters, the BVAR returns to outperforming the VECM, while the TVP-VAR seems to be the best of both worlds, as it has a smaller spike in forecast errors when forecasting COVID-19 quarters and performs similarly to the BVAR in the post COVID-19 quarters.

5. Conclusions

Three VARs were estimated using a given set of variables aiming to examine whether Bayesian estimation could provide real GDP forecasting gains. Using two different Bayesian VAR estimation methods, namely Bayesian estimation using a Minnesota-Litterman prior and a TVP-VAR it is found that Bayesian estimation methods outperform a corresponding VECM model estimated by standard methods. Specifically, the BVAR that was estimated using a Minnesota prior outperformed the VECM model in a 28-quarters ahead forecast, whereas the TVP VAR was superior in providing forecasts over the short run, as evidenced by the 26 4-quarter ahead recursive forecasts.

This exercise also yielded an interesting policy point, as it demonstrated that forecasting at times of high uncertainty can be more accurate when it is done using models with adjusting mechanisms as the TVP VAR's time-varying coefficients allowed the model to adjust more efficiently to the state of the economy at that moment, and to re-adjust in order to capture the post-Covid recovery, thus showing that the model's performance compensates for its intensive parameterization.

This forecasting exercise demonstrated that even the most basic of Bayesian priors provided forecasting gains when it comes to Greek GDP forecasting, but this is only one of the available priors a researcher is available to choose from. One could extend this research to include more advanced Bayesian priors such as the Sims-Zha prior (Sims and Zha, 1998) that incorporates the existence of unit roots and cointegrating relationships in the priors (as it is found in Table 2 that cointegration relationships exist between the variables of the given set); or the GLP prior (Giannone et al., 2015) that treats hyperparameters not as arbitrary inputs of the user but as parameters to be determined from an optimization procedure. Another way this research could be extended is by using the TVP-VAR estimated above (possibly using a larger sample if available, to account for the model's intensive parameterization), to compute the variation in the relations between macroeconomic variables, as expressed by the time-varying coefficients, and thus examine structural changes of the Greek economy over time. This model can also be used to perform impulse response analysis on specific dates, which allows examining how differently exogenous shocks would affect the Greek economy, at different points in time.

Appendix: BVAR prior determination

Table 4: BVAR prior determination criteria

λ3			λ2			λ3		
Value	Theil's U	MAPE	Value	Theil's U	MAPE	Value	Theil's U	MAPE
0.1	**0.007687**	**1.04284**	0.1	0.007687	1.042835	10	0.007686	1.042888
0.2	0.007687	1.04288	**0.2**	**0.007686**	**1.042888**	9	0.007686	1.042888
0.3	0.007688	1.04293	0.3	0.007686	1.042960	8	0.007686	1.042888
0.4	0.007688	1.04299	0.4	0.007686	1.042990	**7**	**0.007686**	**1.042887**
0.5	0.007688	1.04305	0.5	0.007686	1.042900	6	0.007687	1.042885
0.6	0.007688	1.04312	0.6	0.007686	1.042901	5	0.007687	1.042880
0.7	0.007688	1.04321	0.7	0.007686	1.042901	4	0.007689	1.042864
0.8	0.007689	1.04330	0.8	0.007686	1.042902	3	0.007691	1.042802
0.9	0.007689	1.04341	0.9	0.007686	1.042902	2	0.007701	1.042465
1	0.007689	1.04354	1	0.007686	1.042902	1	0.007767	1.041715

To determine the priors for the model a series of one step ahead in-sample forecasts are performed (2000:Q3- 2015:Q4). Each time a prior value changes until the optimal value is reached, starting from λ_3 (with the rest hyperparameters set to uninformative values). After the value of λ_3 is determined we move to λ_2 and then to λ_1. A further check is performed to make sure that optimal values have not changed for λ_1 once values for λ_2 and λ_3 have been determined or for λ_2 once the value for λ_3 has been determined.

As the Mean Absolute Percentage Error (MAPE) seems to favor extreme hyperparameter values, we implemented Theil's inequality coefficient using MAPE to decide among equal Theil's values:

$$MAPE = (\frac{1}{n}\sum_{t=1}^{n}\frac{|Y_t-\hat{Y}_t|}{Y_t}) * 100 = (\frac{1}{n}\sum_{t=1}^{n}\frac{|e_t|}{Y_t}) * 100 \quad (12)$$

$$Theil's\ inequality\ coefficient = \frac{\sqrt{\sum_{t=1}^{n}(Y_t-\hat{Y}_t)/n}}{\sqrt{\sum_{t=1}^{n}\hat{Y}_t^2/n}+\sqrt{\sum_{t=1}^{n}Y_t^2/n}} \quad (13)$$

Where n is the period of the forecasting horizon, $\hat{Y}_t$ is the forecasted value of GDP and Y_t is the actual value of GDP at time t. This process resulted in determining the following hyperparameter set: λ_1=7, λ_2=0.2, and λ_3= 0.1.

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