BANK OF GREECE

EUROSYSTEM

Working Paper

Can central banks do the unpleasant job that governments should do?

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324

DECEMBER 2023

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Published by the Bank of Greece, Athens, Greece

ISSN: 2654-1912 (online)
DOI: https://doi.org/10.52903/wp2023324

CAN CENTRAL BANKS DO THE UNPLEASANT JOB THAT GOVERNMENTS SHOULD DO?

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Abstract

We investigate what happens when the fiscal authorities do not react to rising public debt so that the unpleasant task of fiscal sustainability falls upon the Central Bank (CB). In particular, we explore whether the CB's bond purchases in the secondary market can restore stability and determinacy in an otherwise unstable economy. This is investigated in a dynamic general equilibrium model calibrated to the Euro Area (EA) and where monetary policy is conducted subject to the numerical rules of the Eurosystem (ES). We show that given the recent situation in the ES, and to the extent that a relatively big shock hits the economy, there is no room left for further quasi-fiscal actions by the ECB; there will be room only if the ES' rules are violated. We then search for policy mixes that can respect the ES's rules and show that debt-contingent fiscal and quantitative monetary policies can reinforce each other; this confirms the importance of policy complementarities.

JEL-classification: E5, E6, O5
Keywords: Central banking, Fiscal policy, Debt stabilization, Euro Area.

Acknowledgements: We thank two anonymous referees for detailed and constructive comments. We also thank participants in the conference on "Rethinking Macroeconomic Policy in Times of Turmoil", held at Aegina, Greece, 19-20 June 2023; we particularly thank Refet Gurkaynak, Evi Pappa and Frank Smets for specific comments. We have also benefited from discussions with Harris Dellas, Jim Malley, Dimitris Papageorgiou and George Tavlas. Vasiliki Dimakopoulou and Apostolos Philippopoulos clarify that the views in this paper are not necessarily those of IOBE and the Hellenic Fiscal Council respectively. Apostolos Philippopoulos is grateful to the Bank of Greece for supporting his research on the monetary-fiscal policy nexus. Any errors are ours.

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1 Introduction

In the rich literature on the interaction between fiscal and monetary policies, the conventional policy assignment is the one in which fiscal policy ensures public debt sustainability (typically meaning that tax-spending policy instruments respond to public debt imbalances), while monetary policy controls inflation (typically meaning that the central bank sets its policy nominal interest rates as a function of inflation according to a Taylor rule). This policy assignment, also known as passive fiscal policy and active monetary policy (Leeper, 1991), usually delivers macroeconomic stability and determinacy.

However, in practice, for a variety of reasons (political factors, being in a recession, etc.), fiscal authorities may not be willing, or able, to reduce public spending and/or raise tax rates as a reaction to rising public debt, so that monetary policy can be called upon to play a more direct fiscal role. Actually, this has been the case most of the time since the eruption of the global financial crisis in 2008. Since 2008, most central banks have been employing quantitative policies such as large-scale purchases of government bonds in the secondary market (known as quantitative easing, QE) financed mainly by the issuance of interest-bearing reserves held by private banks at the central bank.

Focusing on the Eurosystem (ES), the ES's cumulated net holdings of sovereign bonds were 31.8% of total Euro Area (EA) public debt at the end of 2022, while this share was negligible before 2008. At the same time, total public debt, as share of total GDP, has increased from 69% in 2008 to 92% in 2022 in the EA, while, in most member-countries, there is no evidence of systematic stabilizing reaction of national fiscal policies to rising public debt. More specifically, according to our calculations, during 2001-2022, the correlation coefficient between current public debt as share of GDP and next period's primary fiscal surplus as share of GDP has been -0.20 in the EA as a whole, while, by contrast, during 2015-2022, the correlation coefficient between current public debt as share of GDP and next period's sovereign bond holdings by the ES as share of GDP is around 0.90.[1] All this seems to imply that, practically, it is quantitative monetary policies that have been contingent on the situation of public finances. This resembles what Leeper (1991) has described as active fiscal policy and passive monetary policy or what is known as fiscal dominance.[2]

[1] That is, regarding fiscal policy, primary surpluses fall, or primary deficits rise, in response to higher inherited debt which destabilizes public finances (note however that this average masks big differences across member countries: the correlation coefficient is -0.55 in Italy, -0.44 in France and -0.21 in Spain, while, it is 0.39, namely stabilizing, in Germany). The data used for the calculation of correlation coeffcients are from Eurostat and the ECB's website. Details are available upon request.

[2] For the role of central banks as quasi-fiscal actors, see Walsh (2017, chapter 4), Reis (2017), Bassetto and Sargent (2020), Buiter (2021), Hall and Sargent (2022), Leeper

In this paper, we investigate what happens when the fiscal authorities do not react to rising public debt so that the politically unpleasant task of debt stabilization and fiscal sustainability falls upon the Central Bank (CB). We explore the possibility that the CB's bond purchases in the secondary market can guarantee stability and determinacy in an otherwise dynamically unstable environment where the path of public debt would have been explosive. That is, we study whether quantitative monetary policies can be a substitute for tax-spending public debt stabilization policies and, if they can, under what circumstances. We do so in a dynamic general equilibrium model calibrated to the EA.

We deliberately employ a rather standard model. The private sector consists of households, firms and banks. Households are Ricardian. Firms are modeled as in the New Keynesian literature and in addition face a financial constraint when they borrow from private banks. Private banks are modeled as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), which means that there is an extra financial friction in the form of moral hazard in banking sector. Regarding the policy sector, following a big part of the literature since 2008, we treat the Treasury and the CB as different policy entities with separate budget constraints. The Treasury finances its spending by various taxes, a transfer from the CB, and by issuing bonds purchased by private banks in the primary market. The CB's balance sheet includes the main items in the consolidated financial statements of the ES and its monetary policy instruments include the policy nominal interest rates on reserves held by private banks and on loans to private banks, the transfer to the Treasury and the fraction of outstanding government bonds purchased from private banks in the secondary market.

Policy is conducted via "simple and implementable" feedback rules (see Schmitt-Grohè and Uribe (2007)) according to which the independently set policy instruments can react to a small number of indicators. In particular, regarding fiscal policy, tax-spending instruments can react to public debt, while, regarding monetary policy, policy interest rates can react to inflation á la Taylor and the CB's sovereign bond purchases are allowed to react to public debt similarly to fiscal instruments. Thus, we study whether, and under what circumstances, debt-contingent fiscal and QE monetary policies can be substitutable in terms of public debt stability and determinacy. We start by studying one instrument at a time so as to search which one can restore stability and determinacy in an otherwise unstable system and, if it can, at what cost. To mimic the conduct of monetary policy in the ES, we restrict the fraction of sovereign bonds in the hands of the CB not to exceed an upper limit and we also exclude the possibility of transfers from the fiscal authorities to the CB. Regarding the upper limit, our base simulations assume 33%, which is the ES's upper issuer limit for bond purchases under

(2023), Hooley et al (2023) and Belhocine et al (2023).

the Public Sector Purchase Programme (PSPP).[3] Regarding the transfer, we assume that it cannot be negative because fiscal support of the ECB is, in practice, very difficult in the ES.[4]

We solve the model by using parameter values that match specific characteristics of the EA over 2002-2022 and by setting the policy (fiscal and monetary) instruments at their recent values in the end of the year 2022. Then, departing from 2022, we shock the model by assuming an adverse supply shock (although we have experimented with various shocks and the main results do not change). In our baseline simulations, this shock is big enough to generate an economic downturn and a rise in the public debt to GDP ratio in the short term that is similar to that experienced in the global financial crisis and the pandemic crisis. We start by switching on Taylor type rules only, according to which the policy interest rates react to inflation. This macroeconomy, if left on its own, is dynamically unstable because of explosive public debt. Thus, a policy instrument needs to react to outstanding public debt to restore stability. We then experiment with different policy instruments, both fiscal and monetary, by setting the associated feedback policy coefficients on debt at the minimum value required for stability in each experiment studied.

To put our work in the context of the literature, we start with something standard. We show that public debt, and hence macroeconomic, stability can be restored when at least one fiscal (tax-spending) instrument reacts to the public debt gap. This is in accordance with the conventional policy regime mentioned in the opening paragraph above, in the sense that fiscal policy instruments are assigned to debt stabilization and at least one of the policy interest rates is assigned to inflation stabilization. Noticeably, under this regime, bond purchases by the CB and the transfer from the CB to the government respect the numerical rules of the ES as defined above.

We then move on the main part of the paper. We now switch off debt stabilization through fiscal policy (i.e. fiscal policy is now "active") and investigate whether quantitative monetary policy in the form of debt-contingent sovereign bond purchases by the CB can do the unpleasant job and thereby

[3] Below we will experiment with a wide range of starting bond holdings and upper thresholds. The latter are not well defined in practice. Although the official PSPP limit of 33% is still in place, sovereign bond holdings under the Pandemic Emergency Purchase Programme (PEPP) have not counted as part of this 33%. At the same time, there is a 50% upper limit that applies to entities listed as "supranationals located in the EA" like the European Stability Mechanism (ESM), the European Investment Bank (EIB), etc.

[4] A negative transfer is interpreted as "fiscal support", or "loss of independence", or even "insolvency" of the CB by various authors (see e.g. Del Negro and Sims (2015) and Reis (2017)). Focusing on the ES, as Reis (2017) argues, the charters of the ECB have no explicit allowance for fiscal support of the ECB so that, for the ECB to receive a transfer from the national fiscal authorities, an agreement by all member-countries is needed which is politically difficult. For details, see the "Protocol of the statute of the ES of CBs and of the ECB" available at the site of the ECB.

free the hands of the Treasury. Our baseline simulations show that, given the current size and mix of the ES and the assumed shock, this can be done only when the CB becomes a long-lasting creditor of the government and, more importantly, only when the rules of the ES are violated, namely, when the fraction of bonds in the hands of the CB is unrestricted and/or fiscal support of the CB is possible at least in some periods. Sensitivity checks, on the other hand, show that this is not a generic result. In particular, and as perhaps expected, if the fraction of sovereign bonds held initially by the CB were assumed to be less than in the current situation (as said, it was 21.8% in 2022 under PSPP), or if the upper limit on sovereign bonds in the hands of the CB were allowed to be much higher than 33%, say 50%,[5] or, if we were willing to make the counter-factual - but popular in the academic literature - assumption that the CB participates in the primary sovereign bond market so it directly controls public finances, then it would be possible for the CB to stabilize the economy on its own via debt-contingent QE and still respect the numerical rules of the ES. Analogous results apply if, other things equal, the shock that triggers dynamics is relatively small, although even a small shock can result in violation of the ES's rules under QE-type stabilization when it persists over time.

It is therefore fair to claim that, given the current situation in the ES, and to the extent that relatively big shocks keep hitting the European economy (the global financial crisis, the pandemic crisis, the war in Ukraine and the resulting energy crisis, and the general geopolitical uncertainty, mean that new challenges cannot be excluded) and that fiscal policy remains practically unresponsive to public debt imbalances, there is no room left for further quasi-fiscal actions by the ECB under the self-imposed rules of the ES. And, as our results show, all this holds under loose upper limits on sovereign bond holdings. This, as we will discuss in the closing section below, can add another argument for an amendment to the existing legal framework of the ECB.

But then, if this is the case, a natural question arises. If, in practice, we do not observe any systematic fiscal reaction to debt imbalances and, at the same time QE cannot restore debt stability without violating the rules of the ES, quoting Leeper et al (2010) in their study for the US economy, it is natural to ask ourselves "Why do forward-looking agents continue to purchase bonds with relatively low interest rates and bond prices don't plummet?". A possible answer to this is that agents believe that current fiscal inaction is only temporary and it will be replaced by necessary corrections in the future. We therefore simulate an extra scenario with policy mixes where fiscal corrections start with a delay, say after 10 years from now, and this is complemented by mild debt-contingent QE policy. Now, our simulations

[5] Notably, however, an upper limit of 50% is not enough if we depart with 31.8% initial holdings (which was the sum of PSPP and PEPP holdings at the end of 2022).

show that this mix of policies can restore stability and this can happen respecting the rules of the ES. Further simulations also show that this kind of complementarity between fiscal and monetary policies becomes even more beneficial when, at the same time, there is a temporary increase in public investment like that under the Recovery and Resilience Facility (RRF) of NextGenerationEU (NGEU).

Literature and how we differ As already said, there is a rich literature on the nexus between fiscal and monetary policies.[6] Within this literature, since 2008, many authors have built dynamic (stochastic) general equilibrium, D(S)GE, models with financial frictions and quantitative monetary policies; applications to the EA include e.g. Priftis and Vogel (2016), Coenen et al (2018, 2020, 2021), Hohberger et al (2019a, 2019b), Darracq Paries et al (2019), Kabaca et al (2023), Mackowiak and Schmidt (2023) and Mazelis et al (2023).[7] Our paper differs mainly because here we address a different issue: we investigate the role of such policies as a mean of public debt stabilization and we do so within the institutional framework of the ES.[8] In particular, building a model that includes the main items in the consolidated financial statements of the ES, we show that the implications of sovereign bond purchases by the CB in the secondary market, and their ability to restore stability and determinacy, depend crucially on the institutional restrictions under which monetary policy is conducted and, specifically, on whether there are upper limits to the fraction of bonds that the CB can hold and whether a fiscal support from the Treasury to the CB is possible in case of need. Another difference between our paper and most of the above literature is that the latter assumes that the CB purchases sovereign bonds in the primary market; as we show, this counter-factual assumption makes a difference.

The rest of the paper is as follows. Section 2 presents the model. Parameterization and solution for the year 2022 are in section 3. Sections 4, 5 and 6 present simulation results when we depart from the year 2022. Section 7 closes the paper. An Appendix includes algebraic details.

[6] See e.g. Leeper (1991, 2021, 2023), Leith and Wren-Lewis (2008), Kirsanova et al (2009), Leeper et al (2010), Davig et al (2010), Davig and Leeper (2011), Canzoneri et al (2011), Canova and Pappa (2011), Reis (2013, 2017), Bassetto and Messer (2013), Del Negro and Sims (2015), Benigno and Nisticò (2020), Sims and Wu (2020, 2021), Bernanke (2020), Bassetto and Sargent (2020), Bianchi et al (2021), Buiter (2021), Chadha et al (2021), Hall and Sargent (2022), Kurovskiy et al (2022), Hooley et al (2023), etc.

[7] By contrast, focusing on D(S)GE papers on the EA, Villa (2013), Angelini et al (2019), Bankowski et al (2021), Hauptmeier et al (2022), Gomes and Seoane (2023) and many others include interest rate policy only in the toolkit of the ES.

[8] Kurovskiy et al (2022) address a similar issue in a model for the US. However, they assume that the CB purchases sovereign bonds in the primary market, and, as we show, this matters to the results. In addition, since they calibrate their model to the US economy, naturally, they do not investigate whether and when extra numerical rules like those of the ES are violated when the CB exercises its QE type policies.

2 Model

This section constructs a medium-scale micro-founded macroeconomic model that embeds most of the macroeconomic policies observed in the EA. We start with an informal description of the model.

2.1 Informal description of the model

Households Households consume, work and keep deposits at private banks. They also own the private firms and banks and so receive their profits. Households are modeled in subsection 2.2.

Private firms A single final good is produced by final good firms which act competitively using differentiated intermediate goods as inputs. The latter are produced by intermediate goods firms which act monopolistically à la Dixit-Stiglitz and face nominal rigidities à la Rotemberg as in the New Keynesian literature. Intermediate goods firms choose labor and capital and also make use of productivity-enhancing public goods. On the financial side, these firms can borrow from private banks subject to a working capital constraint. Firms are modeled in subsection 2.3.

Private banks On their asset side, private banks make loans to private firms, hold interest-bearing reserves at the CB and purchase government bonds in the primary market. On the side of liabilities, they receive deposits from households and loans from the CB. Also, when we allow for a secondary market for government bonds, private banks can sell to the CB a fraction of the bonds they have previously purchased in the primary market. This asset-liability mix is embedded into the banking model of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013).[9] Private banks are modeled in subsection 2.4 in the case in which both private banks and the CB participate in the primary sovereign bond market and in subsection 2.7 in the main case in which it is only private banks that participate in this market.

Treasury On its revenue side, the Treasury, or the government, taxes households' income and consumption as well as firms' and banks's profits, receives a transfer from the CB and issues bonds. On the expenditure side, the Treasury spends on public investment, public consumption and income transfers to households. The Treasury is modeled in subsection 2.5.

Central Bank On the side of assets, the Central Bank (CB) makes loans to private banks and holds government bonds. On the side of liabilities, it issues interest-bearing reserves. Given this balance sheet, the policy

[9] As is known, quantitative monetary policies, and QE in particular, can have real effects if there are financial frictions that overturn Wallace's (1981) irrelevance proposition. The model by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) is one of the main devices that open the door for such effects. Also, Villa (2013) augmented the Smets and Wouters (2003) model with the Gertler-Kiyotaki-Karadi model and found that it was empirically relevant for the EA. See e.g. Beck et al (2014) and Walsh (2017, chapter 11.5) for reviews of this literature and other popular models.

instruments of the CB include the nominal interest rate on reserves held by private banks at the CB, the nominal interest rate on loans to private banks, the transfer to the government, as well as its holdings of sovereign bonds. The CB is modeled in subsection 2.6 in the counter-factual case in which it participates in the primary sovereign bond market and in subsection 2.7 in the case in which this takes place in the secondary market.

2.2 Households

There are N identical households indexed by subscript $h = 1, 2, ..., N$. Each h maximizes discounted lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{h,t}, l_{h,t}; g_t^c\right) \tag{1a}$$

where $c_{h,t}$ and $l_{h,t}$ are h's consumption and work time respectively, g_t^c is per capita public consumption spending and $0 < \beta < 1$ is households' time discount factor.

For our numerical solutions, following Davig and Leeper (2011) and many others, we assume that preferences are of the CRRA variety:

$$u\left(c_{h,t}, l_{h,t}; g_t^c\right) = \frac{c_{h,t}^{1-\mu_1}}{1-\mu_1} - \chi^l \frac{l_{h,t}^{1+\mu_2}}{1+\mu_2} + \chi^g \frac{(g_t^c)^{1-\mu_1}}{1-\mu_1} \tag{1b}$$

where $1/\mu_1$ is the elasticity of intertemporal substitution, $1/\mu_2$ is the Frisch labor supply elasticity, and χ^l, $\chi^g > 0$ measure the relative importance of work effort and consumption of public goods in households' utility.

The period budget constraint of each h written in real terms is:

$$(1+\tau_t^c)c_{h,t} + j_{h,t} = (1-\tau_t^y)w_t l_{h,t} + (1+i_t^d)\frac{p_{t-1}}{p_t} j_{h,t-1} + \pi_{h,t} + g_t^t \tag{2}$$

where $j_{h,t}$ is the real value of end-of-period bank deposits earning a nominal interest rate i_{t+1}^d at $t+1$, w_t is the real wage rate, p_t is the price level of the final good, $\pi_{h,t}$ is net funds transferred from firms and banks to the household, g_t^t is a transfer from the government, and $0 \leq \tau_t^c$, $\tau_t^y < 1$ are tax rates on consumption and labor income income respectively.[10]

Each h chooses $\{c_{h,t}, l_{h,t}, j_{h,t}\}_{t=0}^{\infty}$ to maximize (1a-b) subject to (2). The first-order conditions are in Appendix A.1.

[10] An earlier version of the paper also allowed households to hold currency via a standard cash-in-advance constraint. Since this does not affect our main results, we now assume it away to simplify the model. See e.g. subsection 2.6 below for data in the ES.

2.3 Private firms and production

Firms are modeled as in the New Keynesian literature. That is, there is a single final good produced by competitive final good firms which use differentiated intermediate goods as inputs à la Dixit-Stiglitz. Then, each differentiated intermediate good is produced by an intermediate good firm that acts as a monopolist in its own product market facing Rotemberg-type nominal price fixities and a financial constraint.

2.3.1 Final good firms

There are N identical final good firms indexed by subscript $f = 1, 2, ..., N$. Each f produces $y_{f,t}$ by using intermediate goods according to the Dixit-Stiglitz aggregator:

$$y_{f,t} = \left[\sum_{i=1}^{N} \frac{1}{N^{1-\theta}} (y_{f,i,t})^{\theta} \right]^{\frac{1}{\theta}} \tag{3}$$

where $y_{f,i,t}$ is the quantity of intermediate good of variety $i = 1, 2, ..., N$ used by each firm f and $0 \leq \theta \leq 1$ is a parameter, where $1/(1-\theta)$ measures the degree of substitutability between intermediate goods. Note that we use $\frac{1}{N^{1-\theta}}$ to avoid scale effects in equilibrium (for similar modelling, see e.g. Blanchard and Giavazzi (2003)).[11]

The firm's real profit is:

$$\pi_{f,t} = y_{f,t} - \sum_{i=1}^{N} \frac{p_{i,t}}{p_t} y_{f,i,t} \tag{4}$$

where $p_{i,t}$ is the price of each intermediate good i.

Each f chooses $y_{f,i,t}$ to maximize (4) subject to (3). The familiar first-order condition is in Appendix A.2.

2.3.2 Intermediate goods firms

There are N differentiated intermediate goods firms indexed by the subscript $i = 1, 2, ..., N$. These firms own the stock of capital, make investment and other factor decisions, and face Rotemberg-type price adjustment costs.[12] New investment is financed by retained earnings and loans from private banks where these loans are subject to a working capital constraint.

Firm i's real net dividend, $\pi_{i,t}$, is defined as:

[11] That is, since $y_{f,i,t} = \frac{y_{i,t}}{N}$, where $y_{i,t}$ is the output of each i, in a symmetric equilibrium $y_{f,t} = y_{i,t}$.

[12] Using Calvo-type nominal rigidities would not affect our main results (see also the calibration subsection 3.1 below). Davig et al (2010, 2011) also employ Rotemberg-type quadraric costs to model sluggish price adjustment in related papers.

$$\pi_{i,t} = (1-\tau_t^\pi)\left(\frac{p_{i,t}}{p_t}y_{i,t} - w_t l_{i,t}\right) - x_{i,t} -$$

$$-\frac{\xi^p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 \overline{y}_{i,t} + \left(L_{i,t} - \left(1+i_t^l\right)\frac{p_{t-1}}{p_t}L_{i,t-1}\right) \tag{5}$$

where $l_{i,t}$ is units of labor input used by firm i, $x_{i,t}$ is i's investment in capital goods, $L_{i,t}$ is the real value of end-of-period bank loans on which the firm pays a nominal interest rate, i_{t+1}^l, in the next period, $0 \leq \tau_t^\pi < 1$ is the tax rate on gross profits, $\xi^p \geq 0$ is a parameter measuring Rotemberg-type price adjustment costs, and $\overline{y}_{i,t}$ is average output.[13]

The law of motion of each i's capital stock, $k_{i,t}$, is (without capital adjustment costs, the relative price of capital will be 1):

$$k_{i,t} = (1-\delta)\, k_{i,t-1} + x_{i,t} \tag{6}$$

where $0 \leq \delta \leq 1$ is the capital depreciation rate.

For the firm's production function, we adopt the form: :

$$y_{i,t} = A_t(k_{i,t-1}^{\alpha} l_{i,t}^{1-\alpha})^{1-\varepsilon}\left(k_{t-1}^g\right)^{\varepsilon} \tag{7}$$

where k_{t-1}^g is per firm public infrastructure capital, $0 < a < 1$ and $0 \leq \varepsilon < 1$ are technology parameters, and A_t obeys a deterministic stationary $AR(1)$ process defined below.

These firms are subject to a working capital constraint.[14] That is, they have to finance a fraction of their payments to labor with loans from private banks:

$$L_{i,t} \geq \nu^l w_t l_{i,t} \tag{8}$$

where the parameter $\nu^l \geq 0$ measures the tightness of borrowing conditions faced by firms.

Each firm i maximizes the discounted sum of net-of-tax dividends distributed to households:

$$\sum_{j=0}^{\infty} \beta_{t,t+j}\pi_{i,t+j} \tag{9}$$

where, since firms are owned by households, we will ex post postulate that $\beta_{t,t+j}$ equals households' marginal rate of substitution between consumption

[13] Thus, Rotemberg-type costs associated with price changes are assumed to be proportional to average output, $\overline{y}_{i,t}$, which is taken as given by each i. This is not important but helps in producing smooth dynamics. See also e.g. Davig et al (2010, 2011), Sims and Wolff (2017) and Leeper et al (2019) for similar modeling.

[14] See also e.g. Walsh (2017, section 5.3) and Uribe and Schmitt-Grohé (2017, section 6.4). That is, the idea is that firms pay wages before selling their product. Note that we could assume different types of financial constraints, like collateral borrowing constraints as in e.g. Gertler and Karadi (2011) and Sims and Wu (2021); we report that our main results do not depend on this.

at t and $t+j$. That is, $\beta_{t,t} \equiv 1$, $\beta_{t,t+j} \equiv \beta^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}}$, etc., where $\lambda_{h,t}$ is the Lagrange multiplier associated with households' budget constraint above.

Each i chooses $\{l_{i,t},\ k_{i,t},\ L_{i,t}\}_{t=0}^{\infty}$ to maximize (9) subject to (5)-(8) and the demand function for its product coming from the final good firm's optimization problem (see Appendix A.2 for the latter). The first-order conditions are in Appendix A.3.

2.4 Private banks

There are N identical private banks indexed by the subscript $p = 1, 2, ..., N$. In addition to their standard role, which is the provision of intermediation by converting deposits by households into loans to firms, we allow private banks to hold interest-bearing reserves at the CB, to purchase government bonds and to borrow from the CB. In other words, on the asset side of banks, we have loans to firms, reserves and government bonds, while, on the liability side, we have deposits obtained from households and loans taken from the CB. Hence, each private bank p enters period t with predetermined assets in the form of loans to firms, $L_{p,t-1}$, reserves, $m_{p,t-1}$, and government bonds, $b_{p,t-1}$, as well as with preexisting obligations in the form of deposits from households, $j_{p,t-1}$, and loans from the CB, $z_{p,t-1}$.

For expositional convenience, we start with modelling the counter-factual case in which both private banks and the CB participate in the primary market for sovereign bonds; the modelling of the case in which only private banks participate in the primary market, while the CB purchases sovereign bonds from private banks in the secondary market, is postponed to subsection 2.7 below.

This financial mix is embedded into the banking model of Gertler-Karadi-Kiyotaki. The key ingredients of this popular model are as follows.[15]

The balance sheet of each private bank p at the end of t is:

$$L_{p,t} + b_{p,t} + m_{p,t} = j_{p,t} + z_{p,t} + n_{p,t} \tag{10}$$

where $n_{p,t}$ is p's after-tax net worth defined as:

$$n_{p,t} = \frac{p_{t-1}}{p_t}\{[1+(1-\tau_t^{\pi})\, i_t^{l}]L_{p,t-1} + [1+(1-\tau_t^{\pi})\, i_t^{b}]b_{p,t-1} +$$

$$+[1+(1-\tau_t^{\pi})\, i_t^{r}]m_{p,t-1} - [1+(1-\tau_t^{\pi})\, i_t^{d}]j_{p,t-1} - [1+(1-\tau_t^{\pi})\, i_t^{z}]z_{p,t-1}\} \tag{11}$$

As in the above papers, it is assumed that after period t there is a probability $(1-\sigma)$ that a banker will exit the sector at $t+1$ transferring

[15] For a detailed presentation of this model, see Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013), while, a nice texbook presentation is in Walsh (2017, chapter 11.5.4). As already said above, Walsh (2017, chapter 11.5) reviews the main models in this literature.

his/her wealth to households and, at the same time, the same fraction of households will enter the banking sector transferring their money to this sector. Given that the bank pays dividends only when it exits, its objective at the end of t is to maximize its value, $V_{p,t}$, which is equal to the present discounted value of future dividends:

$$V_{p,t} = \max \sum_{j=1}^{\infty} (1-\sigma)\, \sigma^{j-1} \beta_{t,t+j} n_{p,t+j} \tag{12}$$

where the discount factor $\beta_{t,t+j}$ has been defined above.

Also, again as in the above papers, it is assumed that banks can divert a fraction, $0 \leq \vartheta \leq 1$, of their "divertable" net assets to their owners, namely, the households, and hence may go bankrupt. Given this possibility, for the bank to keep operating, its value, $V_{p,t}$, has to be equal to, or greater than, the amount it can divert. Hence, the bank faces the incentive constraint at each t:

$$V_{p,t} \geq \vartheta (L_{p,t} + N^b b_{p,t} + N^m m_{p,t} - N^z z_{p,t}) \tag{13}$$

where N^b, N^z, N^m are parameters associated respectively with the bank's loans to firms, bond holdings, reserves at the CB and loans obtained from the CB, so as to capture the idea that the ease of diverting different types of assets and liabilities differs across them; typically in this literature, $0 \leq N^b \leq 1$ meaning that it is easier to divert private loans than sovereign bonds. As the bank's first-order conditions will show, these parameters drive interest rate spreads or asset pricing wedges and will therefore be calibrated to give interest rate differentials as in the data.

Before we move on, it is worth reminding the qualitative implications of this model. As Walsh (2017, p. 552) points out, in this model, the key friction is the moral hazard problem, together with the assumption that the associated incentive constraint as in (13) is affected by the mix of assets and liabilities held by banks. When binding, this incentive constraint opens the door through which QE type policies by the CB can affect the credit policies of private banks. In particular, an increase in the CB's holdings of sovereign bonds, when translated to lower bond holdings by private banks, $b_{p,t}$, can - other things equal - raise the supply of loans to firms, $L_{p,t}$, and this can in turn ease the loan constraint faced by production firms. Note however that this is "other things equal"; here, the incentive constraint includes other items like reserves held at the CB, $m_{p,t}$, and loans obtained from the CB, $z_{p,t}$, so that, even when $b_{p,t}$ falls, this may not necessarily raise $L_{p,t}$ (this is further discussed in subsection 4.4.2 below).

Each p chooses $\{L_{p,t}, b_{p,t}, m_{p,t}, z_{p,t}\}_{t=0}^{\infty}$ to solve the above problem. The solution of the banks' problem in the case in which the CB also participates in the primary bond market is in Appendix A.4, while, the factual case in which only private banks do so is presented in subsection 2.7 below and solved in Appendix A.5.

2.5 The Treasury and fiscal policy instruments

The Treasury, or the fiscal branch of government, uses revenues from various taxes, the issuance of new bonds and a direct transfer from the CB to finance its spending activities. Its flow budget constraint written in per capita and real terms is:

$$g_t^c + g_t^g + g_t^t + (1 + i_t^b)\frac{p_{t-1}}{p_t}b_{t-1} = b_t + t_t^{cb} + t_t^{tax} \tag{14}$$

where g_t^c, g_t^g and g_t^t are spending on public consumption, public investment and transfer payments respectively, b_t is the end-of-period public debt, t_t^{cb} is a transfer from the CB to the Treasury,[16] and t_t^{tax} denotes per capita and real tax revenues defined as:

$$t_t^{tax} = \tau_t^c c_{h,t} + \tau_t^y w_t l_{h,t} + \tau_t^\pi (y_{i,t} - w_t l_{i,t}) +$$
$$+\tau_t^\pi \frac{p_{t-1}}{p_t}(i_t^l L_{p,t-1} + i_t^r m_{p,t-1} + i_t^b b_{p,t-1} - i_t^z z_{p,t-1} - i_t^d j_{p,t-1}) \tag{15}$$

Public investment, g_t^g, augments public capital whose motion is:

$$k_t^g = (1 - \delta^g)k_{t-1}^g + g_t^g \tag{16}$$

where $0 \leq \delta^g \leq 1$ is the public capital depreciation rate.

In our solutions, to maintain a closer link to the data, instead of working with the levels of public spending, we will work with their GDP shares, $0 < s_t^c,\ s_t^g,\ s_t^t < 1$, where $g_t^c = s_t^c y_{f,t}$, $g_t^g = s_t^g y_{f,t}$ and $g_t^t = s_t^t y_{f,t}$. One of the fiscal variables must follow residually to close the Treasury's budget constraint and, along the transition path, we will assume that this role is played by the end-of-period public debt, b_t, so that the rest of the fiscal policy variables, $e_t \equiv (s_t^c,\ s_t^g,\ s_t^t,\ \tau_t^c,\ \tau_t^y,\ \tau_t^\pi)$, can be set independently.

Following most of the related literature,[17] we will allow some of the independently set fiscal instruments, e_t, to follow feedback, or state-contingent, rules according to which, in addition to an exogenous $AR(1)$ process, they can also react to the beginning-of-period public debt to GDP ratio as deviation from its steady state value. Thus,

$$e_t = \rho_{t-1}^e e_{t-1} + (1 - \rho^e)\, e + \gamma^{e,b}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b}{y}\right) \tag{17}$$

where $\gamma^{e,b}$ s are feedback policy coefficients, $0 \leq \rho^e \leq 1$ are persistence parameters, and variables without time subscripts denote steady state values (see subsection 3.2 below for these values).

[16] As pointed out by Reis (2017), the charters of the ECB state that it must rebate its net profit to the national CBs of the ES every year and most of them, in turn, are required by national law to send them as dividends to their respective fiscal authorities.

[17] For a recent paper that also reviews the literature on state-contingent fiscal rules like in (17), see e.g. Malley and Philippopoulos (2023).

2.6 The Central Bank and monetary policy instruments

The assets of the CB include loans to private banks and government bonds, while, on the side of liabilities, we have interest-bearing reserves held by private banks; these are the largest items in the financial statements of the ES.[18]

As said above, for expositional convenience, we first model the case in which the CB participates in the primary market for government bonds (the secondary market is modeled in subsection 2.7 below). Then, the flow budget constraint of the CB linking changes in assets and liabilities written in real and per capita terms is:

$$b_{cb,t} + z_{p,t} + i_t^r \frac{p_{t-1}}{p_t} m_{p,t-1} + t_t^{cb} =$$

$$= (1 + i_t^b) \frac{p_{t-1}}{p_t} b_{cb,t-1} + (1 + i_t^z) \frac{p_{t-1}}{p_t} z_{p,t-1} + m_{p,t} - \frac{p_{t-1}}{p_t} m_{p,t-1} \tag{18}$$

where $b_{cb,t}$ is the amount of government bonds purchased by the CB at the end of period (all other variables have been defined above). Since, at this stage, we assume that the CB participates in the primary market of sovereign bonds, and since b_t is the total amount of these bonds (see equation (14) above), without loss of generality we denote $b_{cb,t} \equiv (1 - \Lambda_t) b_t$, where $(1 - \Lambda_t)$ is a monetary policy instrument.

Similarly to fiscal policy, we need to model the independently set monetary policy instruments, $(i_t^r,\ i_t^z,\ t_t^{cb},\ 1 - \Lambda_t)$. Starting with the nominal interest rates on reserves held by private banks at the CB and on CB loans obtained by private banks, i_t^r and i_t^z, we assume Taylor-type rules like:

$$\log(1 + i_t^z) = (1 - \rho^z) \log(1 + i^z) + \rho^z \log\left(1 + i_{t-1}^z\right) + \gamma^{z,\pi} \log\left(\frac{p_t}{p_{t-1}}\right) \tag{19a}$$

$$\log(1 + i_t^r) = (1 - \rho^r) \log(1 + i^r) + \rho^r \log\left(1 + i_{t-1}^r\right) + \gamma^{r,\pi} \log\left(\frac{p_t}{p_{t-1}}\right) \tag{19b}$$

where $\gamma^{z,\pi},\ \gamma^{r,\pi} \geq 0$ are feedback policy coefficients, $0 \leq \rho^z,\ \rho^r \leq 1$ are persistence parameters, and i^z, i^z denote steady state values (see subsection 3.2 below for these values). Note that since the policy rates never turn out to become negative in our solutions, we do not include an explicit zero lower

[18] On the side of assets, sequrities (most of them in the form of sovereign bonds) were around 64% of the total size of the ES's balance sheet at the end of 2022, with loans to financial institutions the second largest item around 17%. On the side of liabilities, reserves were the biggest item, around 50% of the total size at the end of 2022, being followed by banknotes or currency, around 20%. As said above, in our model, we assume away banknotes or currency held by households and firms since this does not alter our results. By contrast, loans to banks from the CB do play a role and this is why they are included (see subsection 6.2 below).

bound (ZLB) constraint on them. For the same reason, we do not explicitly add an inflation constraint; in all solutions below, inflation remains below 2% which is the official threshold of the ECB.

Regarding the transfer from the CB to the government, t_t^{cb}, following e.g. Reis (2017), Benigno and Nisticò (2020) and Sims and Wu (2021), we assume a policy rule like:

$$t_t^{cb} = (1+i_t^z)\frac{p_{t-1}}{p_t}z_{p,t-1} + \left(1+i_t^b\right)\frac{p_{t-1}}{p_t}b_{cb,t-1} - (1+i_t^r)\frac{p_{t-1}}{p_t}m_{p,t-1} \quad (20a)$$

But, as said above, since in practice the ES does not allow for the possibility of support from the national fiscal authorities, we rule out negative transfers so that in addition to (20a):[19]

$$t_t^{cb} \geq 0 \quad (20b)$$

Regarding the policy rule for the fraction of sovereign bonds held by the CB, $(1-\Lambda_t)$, since we want to explore the possibility that QE can work as a substitute for fiscal policy regarding public debt stabilization, we allow this rule to contain, in addition to an exogenous $AR(2)$ process,[20] a state-contingent component according to which the CB can react to the public debt to GDP gap. Thus,

$$(1-\Lambda_t) = \left(1-\rho_1^\Lambda-\rho_2^\Lambda\right)(1-\Lambda)+\rho_1^\Lambda(1-\Lambda_{t-1})+\rho_2^\Lambda(1-\Lambda_{t-2})+\gamma^{\Lambda,b}\left(\frac{b_{t-1}}{y_{t-1}}-\frac{b}{y}\right) \quad (21a)$$

where $\gamma^{\Lambda,b} \geq 0$ is a feedback policy coefficient, ρ_1^Λ and ρ_2^Λ are persistence parameters and $1-\Lambda$ denotes the steady state value of the fraction of bonds held by the CB (see subsection 3.2 below for these values).

But, as said above, according to the rules of the ES, this is subject to an upper limit so that in addition to (21a):

$$(1-\Lambda_t) \leq (1-\Lambda)^{\max} \quad (21b)$$

where the value of the policy threshold, $(1-\Lambda)^{\max}$, is specified in subsection 3.2 below.[21]

[19] Thus, in the code, we define an auxiliary variable $t\_temp_t^{cb}$ given by equation (20a) and then set $t_t^{cb} = \max(0, t\_temp_t^{cb})$.

[20] Papers by ECB researchers also use an $AR(2)$ process for the exogenous part of asset purchases (e.g. Coenen et al (2020, 2021) and Mazelis et al (2023)), so as to capture "the gradual build-up of overall asset holdings broady consistent with the patterrn of actual asset purchases carried out by central banks, and a gradual reduction thereafter as the purchased assets mature" (see Coenen et al (2020, p. 10)).

[21] Thus, in the code, we define an auxiliary variable $1-\Lambda\_temp_t$ given by equation (21a) and then set $1-\Lambda_t = \min((1-\Lambda)^{\max}, 1-\Lambda\_temp_t)$.

2.7 Adding a secondary market in sovereign bonds

In practice, CBs do not participate in the primary sovereign bond market. Instead, private banks can sell to the CB in the secondary market a fraction of the bonds they have previously purchased in the primary market. We now augment the above model to allow for this possibility; we will do so in a simple way.

We imagine that in the beginning of each period, each private bank, p, keeps a fraction, $0 \leq \Lambda_t \leq 1$, of the bonds, $b_{p,t-1}$, it purchased at $t-1$, and sells the rest, $0 \leq 1 - \Lambda_t \leq 1$, to the CB at a price Φ_t in the secondary market. In other words, for each bond it sells, the private bank receives Φ_t in exchange for $1 + (1 - \tau_t^{\pi}) i_t^b$, which is the net-of-tax return on these bonds if held to maturity. It is reasonable to assume that the private bank will exercise this exchange, or option, only if $\Phi_t \geq 1 + (1 - \tau_t^{\pi}) i_t^b$. In other words, to acquire bonds, the CB has to pay a premium to private banks. Actually, this is similar to Gertler and Kiyotaki (2010, section 3.3) who assume that the government or the CB have to pay a price above the market price to acquire bank equity; they call this premium a "gift" to private banks. Without loss of generality, we rewrite this inequality as an equality, $\Phi_t \equiv \kappa[1 + (1 - \tau_t^{\pi}) i_t^b]$, where the value of the parameter $\kappa \geq 1$ will be specified in subsection 3.2 below.

The rest of this subsection presents what changes relative to the model presented so far. We start with the Treasury.

2.7.1 The Treasury

The Treasury's budget constraint remains as in (14) except that now, in the primary sovereign bond market, bonds are purchased by private banks only. In other words, while the market-clearing condition was $b_t = b_{p,t}^T + b_{cb,t}$ when the CB was assumed to participate in the primary market, now it is $b_t = b_{p,t}^T$ (where $b_{p,t}^T \equiv N b_{p,t}$ denotes the total amount of bonds purchased by all private banks).

Also, since now only a fraction, Λ_t, of income from bonds is taxable, the definition for the Treasury's tax revenues changes from (15) to:

$$t_t^{tax} = \tau_t^c c_{h,t} + \tau_t^y w_t l_{h,t} + \tau_t^{\pi}(y_{i,t} - w_t l_{i,t}) +$$

$$+\tau_t^{\pi} \frac{p_{t-1}}{p_t} (i_t^l L_{p,t-1} + i_t^r m_{p,t-1} + i_t^b \Lambda_t b_{p,t-1} - i_t^z z_{p,t-1} - i_t^d j_{p,t-1}) \qquad (22)$$

2.7.2 Private banks

In the private banks' problem, net worth changes from (11) to:

$$n_{p,t} = \frac{p_{t-1}}{p_t} \{ [1 + (1 - \tau_t^{\pi}) i_t^l] L_{p,t-1} + \left[\Lambda_t [1 + (1 - \tau_t^{\pi}) i_t^b] + (1 - \Lambda_t) \Phi_t \right] b_{p,t-1} +$$

$$+[1+(1-\tau_t^{\pi})\, i_t^r]m_{p,t-1}-[1+(1-\tau_t^{\pi})\, i_t^d]j_{p,t-1}-[1+(1-\tau_t^{\pi})\, i_t^z]z_{p,t-1}\} \quad (23)$$

that is, net worth is affected by transactions in the secondary bond market.

Also note that although the banks' incentive constraint remains as in (13), we now have $b_{p,t} = b_{p,t}^T/N = b_t/N$ in equilibrium, since only private banks purchase bonds in the primary market, while, it was $b_{p,t} = b_{p,t}^T/N = \Lambda_t b_t/N$ when the CB was assumed to participate in the primary market.

A detailed solution of banks' new problem is in Appendix A.5.

2.7.3 The Central Bank

The CB's budget constraint and transfer to the government change from (18) and (20a) to (24) and (25) respectively:

$$\Phi_t(1-\Lambda_t)\frac{p_{t-1}}{p_t}b_{p,t-1} + z_{p,t} + (1+i_t^r)\,\frac{p_{t-1}}{p_t}m_{p,t-1} + t_t^{cb} =$$

$$= (1-\Lambda_t)(1+i_t^b)\frac{p_{t-1}}{p_t}b_{p,t-1} + (1+i_t^z)\frac{p_{t-1}}{p_t}z_{p,t-1} + m_{p,t} \quad (24)$$

$$t_t^{cb} = \left[\left(1+i_t^b\right) - \Phi_t\right](1-\Lambda_t)\frac{p_{t-1}}{p_t}b_{t-1} +$$

$$+ (1+i_t^z)\,\frac{p_{t-1}}{p_t}z_{p,t-1} - (1+i_t^r)\,\frac{p_{t-1}}{p_t}m_{p,t-1} \quad (25)$$

that is, now $b_{cb,t} \equiv (1-\Lambda_t)\, b_{p,t-1}^T$, where the monetary policy instrument, $(1-\Lambda_t)$, has been defined in (21a-b) above.

2.8 Macroeconomic system, monetary policy transmission and what comes next

Collecting equations, Appendix A.6 presents the macroeconomic system in the counter-factual case in which the CB purchases sovereign bonds in the primary market, while Appendix A.7 does the same when the CB purchases these bonds in the secondary market. While we focus on the latter, the former will be used for comparison. All this is given the paths of exogenous variables and policy instruments whose values will be set as in the EA data.

Before we move on, it is useful to clarify the channels through which monetary policy can have real effects in general equilibrium. Regarding interest rate policies, these policies can have real effects because of nominal rigidities as is common in the New Keynesian literature. Regarding quantitative monetary policies, they can have real effects through the moral hazard problem à la Gertler-Karadi-Kiyotaki and the working capital constraint faced by firms. In other words, the moral hazard problem opens the door through which quantitative monetary policies affect the credit policy of private banks and, in turn, the working capital constraint faced by firms opens the door through which private banks' credit policy can affect the production sector. Nevertheless, the general equilibrium effects of such policies on the real economy, as well as on public finances, are naturally a quantitative matter. As we shall see, they also depend on whether the CB participates in the secondary market for sovereign bonds.

In the next sections, we will parameterize the model and then solve it numerically under various policy scenaria. In particular, we will work as follows. After calibrating the model to EA data, we will get an initial "steady state" solution; this is in the next section 3. Then, in the remaining sections 4, 5 and 6, departing from this initial solution for 2022, we will shock the model and investigate which fiscal and/or monetary policies can ensure dynamic stability and determinacy and, if yes, under what conditions. In our solutions, we assume that all is common knowledge so that we solve the model under perfect foresight by using a non-linear Newton-type method implemented in Dynare.

3 Parameter values, policy variables and solution for the year 2022

This section first parameterizes the model using annual data of the EA over the period 2002-2022 (unless otherwise stated), then presents the values of the model's exogenous variables and, finally, solves for the model's "initial steady state" defined as a situation in which variables do not change and policy variables are set as in the most recent data. As we shall see, this solution can match reasonably well the recent key features of the EA and can thus serve as a reasonable departure point for the policy experiments in the next sections, 4, 5 and 6.

3.1 Parameter values

Parameter values, either calibrated or set, are listed in Table 1. Starting with preference parameters, private agents' time discount factor, β, is calibrated from the steady state version of the Euler equation for domestic deposits (equation (A.7.2.3) in Appendix A.7). We assume that the deposit rate equals the reserves rate set by the CB in September 2022, 2%, which in turn implies $\beta = 0.9804$. In the households' utility function, we set $\mu_1 = \mu_2 = 1$ (as in e.g. Davig and Leeper (2011)) and calibrate χ^l so as the steady state share of time spent at work 0.32 as in the data (this implies $\chi^l = 4.2$). We also set χ^g to 1 (the value of χ^g plays no role in positive results).

Continuing with technology parameters in the production function of goods, the exponent on labor, $1 - \alpha$, is calibrated from the expression $(1 - \alpha)(1 - \epsilon) = 0.471$, where 0.471 is the average labour income share in the data and ϵ measures the contribution of productivity-enhancing public goods/services in private production. Following e.g. the early paper by

Baxter and King (1993) but also more recent work of Ramey (2020) and Malley and Philippopoulos (2023), we set ϵ equal to 0.05.[22] This value for ϵ implies that α, which is the exponent on capital in the Cobb-Douglas production function, equals 0.454. The private and government capital depreciation rates, δ and δ^g respectively, are both set equal to 0.046 (see *Monthly Bulletin*, ECB, 2006). The steady state TFP parameter, A, is set at 1. Regarding the Dixit-Stiglitz parameter, θ, in equation (3) above, we use information from Eggertson et al (2014) who report that the gross markup is around 1.15 in EA countries; the latter implies a value of θ around 0.85. In turn, following Sims and Wolff (2017), the Rotemberg cost parameter, ξ^p, is calibrated by using the formula $\xi^p = \frac{\left(\frac{1}{1-\theta}-1\right)\times 0.75}{(1-0.75)(1-\beta\times 0.75)}$,[23] where the 0.75 value refers to the Calvo price stickiness parameter implying an average duration between price changes in the Calvo model of four quarters; this gives $\xi^p = 65$ in our case, which is a value within commonly used ranges (see e.g. Malley and Philippopoulos (2023)). Finally, we set the coefficient, ν^l, in the firms' financial constraint (8) at 0.3, which is as in e.g. Korinek and Mendoza (2014).

Continuing with the banking sector, we set the parameters in the banks' incentive constraint (13) so as to match some key features of the EA at the end of 2022. In particular, we calibrate ϑ and N^b using the steady state version of private banks' first-order conditions for loans and government bonds (equations (A.7.2.13) and (A.7.2.14) in Appendix A.7) so as to match the EA's lending and government bond rates at the end of 2022 ($i^l = 3.50\%$ and $i^b = 3.40\%$ where the data are from the site of the ECB); the resulting values are $\vartheta = 0.6$ and $N^b = 1.08$. To hit the above, we also need to set the parameter associated with loans provided by the CB, N^z, at 0.33, while, for simplicity, we set the parameter associated with banks' reserves at the CB, N^m, at 0.[24] Regarding the banks' survival rate, σ, and the proportional transfer of entering banks, γ, they are calibrated so as to match banks' reserves at the CB as a percentage of GDP at the end of 2022 ($m_p/y = 30\%$) and get a reasonable value of banks' deposits as share of GDP (around 60%) and total net worth as share of GDP (around 45%); the resulting values are $\sigma = 0.92$ and $\gamma = 0.015$.

[22] We report that our main results are robust to changes in the value of ϵ.

[23] According to this formula, Sims and Wolff (2017) show that a Rotemberg-type model is equivalent to a Calvo-type model at least to first-order.

[24] See also Sims and Wu (2021). We report that our results are not sensitive to the parameter value of N^m.

Table 1			
Baseline parameterization			
Parameter	Description	Value	
μ_1	inverse of intertemporal elasticity of sub.	1	set
μ_2	inverse of Frisch elasticity	1	set
χ^l	weight to labor effort	4.2	calibr
χ^g	weight to public consumption	1	set
β	time discount factor	0.9804	calibr
δ and δ^g	depreciation rates of priv and pub capital	0.046	calibr
A	TFP	1	set
α	share of capital in production	0.454	calibr
ε	contribution of public capital in production	0.05	set
θ	substitutability parameter of intermediate goods	0.85	calibr
ξ^p	Rotemberg price adjustment cost parameter	65	calibr
ϑ	parameter associated with loans to firms in banks' incentive	0.6	calibr
N^b	parameter associated with gov bonds in banks' incentive	0.65	calibr
N^z	parameter associated with loans from the CB in banks' incentive	0.2	set
N^m	parameter associated with reserves at the CB in banks' incentive	0	set
σ	bankers' survival rate	0.92	set
γ	proportional transfer to entering bankers	0.015	calibr
ν^l	coeff. in working capital constraint	0.3	set

3.2 Policy variables

Policy variables, as well as parameters and feedback coefficients included in the policy rules, are listed in Table 2.

Regarding fiscal policy, the recent data values of s_t^c, s_t^g, τ_t^c, τ_t^y and τ_t^π, namely, public spending on consumption and investment as shares of GDP, as well as the effective tax rates on consumption, personal income and corporate profits, are 0.22, 0.03, 0.165, 0.385 and 0.206 respectively.[25] Transfer payments as share of GDP, s_t^t, is set at 0.1 (in the data this share was 0.22 at 2022) to accommodate the debt-to-GDP ratio in the data at the end of 2022. Regarding the $AR(1)$ persistence parameters in the fiscal policy rules for the GDP share of public consumption and the income tax rate, s_t^c and τ_t^y,

[25] The source of the spending instruments is Eurostat while the tax rates are from *Taxation Trends in the EU* (European Commission (2022)). Note that the effective tax rates on consumption and labor income are at their 2020 values, while, the effective tax rate on corporate profits as well as government consumption and investment as shares to GDP are at their 2021 values.

which will be the two fiscal instruments used for public debt stabilization in what follows, their values are derived by running simple $AR(1)$ regressions using data for the period 2001-2022; our estimated $AR(1)$ coefficients are around $\rho^{s,c} = \rho^{\tau,y} = 0.9$. The rest of fiscal policy instruments, s_t^g, τ_t^c and τ_t^π, will simply be kept constant at their most recent data values (except otherwise stated).

Regarding monetary policy, we set the nominal interest rates on reserves held at the CB, i_t^r, at 2%, which was its value at the end 2022, while, the nominal interest rate at which banks borrow from the CB, i_t^z, is set at 2.5% which equals the rate on the Main Refinancing Operations (MROs) in the ES.[26] In our baseline solutions, the starting value of the fraction of cumulated euro area government debt held by the ES, $1 - \Lambda$, will be set at 21%, which was the fraction held by the ES under PSPP in the end of 2022,[27] while, its upper issue limit, $(1 - \Lambda)^{\max}$, will be set at 33%, which is the official threshold of PSPP (but we will consider a wide range of robustness checks in subsection 4.3 below). The parameter κ, which quantifies the premium paid by the CB to private banks when purchasing bonds in the secondary market, is calibrated at 1.0065, which is the value that captures the difference between the yield in the primary and the yield in the secondary market for German sovereign bonds (Bunds) over 2001-23.[28] The $AR(1)$ persistence parameters in the Taylor rules for the two policy interest rate, ρ^z and ρ^r, are both set equal to the estimated value, around 0.9, reported in Coenen et al (2018, 2020) for the ES. Regarding the autoregressive parameters in the $AR(2)$ process for $(1 - \Lambda_t)$, their values are set as in studies by ECB researchers (see e.g. Coenen et al (2020)), namely

[26] The two primary lending policies of the ECB are its refinancing operations (MROs, LTROs) and the marginal lending facility used for overnight liquildity. See the site of the ECB for details.

[27] There are two active asset purchase programs today. The Asset Purchase Programme (APP) that started in late 2014 and whose biggest item has been the PSPP, and the PEPP that was a response to the covid-19 pandemic. The sum of PSPP and PEPP stocks relative to all member-countries national debts in the end of 2022, was 31.8%, 21% under PSPP and 10.8% under PEPP. In June 2023, the ECB decided to discontinue reinvestments under APP. Regarding the PEPP, since March 2022, the ECB has discontinued net asset purchases under this program but the maturing principal payments will be reinvested until the end of 2024 at least.

[28] Recall that the pricing function is $\Phi_t \equiv \kappa[1 + (1 - \tau_t^\pi)\, i_t^b]$. Thus, κ is calibrated by this function when we use data on interest rates in the primary market for i_t^b and data on interest rates in the secondary market for Φ_t. The data used here are on the returns to German government bonds and are publicly available at https://www.deutsche-finanzagentur.de/en/federal-securities/issuances/issuance-results). Details are available upon request. It is worth pointing out two things here. First, this is a model for the EA and not for any individual EA member countries; hence, EA supranational institutions enjoy ratings similar to those enjoyed by the German government. Second, the parameter κ captures the difference between the yield levels in primary and secondary sovereign markets. This difference is small in general. What differs across EA member-countries is their levels, not their κ's.

$\rho_1^\Lambda = 1.5$ and $\rho_2^\Lambda = -0.54$.

Finally, regarding feedback policy coefficients, the reaction to inflation in the Taylor rules for the two policy rates are both set at 1.01 so that the Taylor principle is satisfied ($\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$), except otherwise said. The values of the feedback policy coefficients on public debt imbalances will be specified below in each case studied, since the minimum value required for stability differs across policy experiments. Thus, following usual practice, in each case, they will be set at the minimum value required to guarantee that public debt remains on a stable path so there is a unique determinate equilibrium.

Table 2. Policy variables

Parameter	Description	Value	
s^g	gov investment to GDP	3%	data
s^c	gov consumption to GDP	22%	data
s^t	gov transfers to GDP	9.81%	set
τ^c	consumption tax rate	16.5%	data
τ^y	personal income tax rate	38.5%	data
τ^π	corporate tax rate	20.6%	data
i^r	interest rate on reserves	2.00%	data
i^z	interest rate on CB's loans to banks	2.50%	data
$1-\Lambda$	CB's gov bonds' holdings	21%	data
$(1-\Lambda)^{\max}$	CB's gov bonds' holdings threshold	33%	set
κ	parameter in pricing function of bonds in secondary market	1.0065	calibrated
$\rho^{s,c}$	persistence of gov consumption	0.9	estimated
$\rho^{\tau,y}$	persistence of income tax rate	0.9	estimated
ρ^r	persistence of reserves rate	0.9	set
ρ^z	persistence of CB's lending rate	0.9	set
ρ_1^Λ	persistence of CB's bond holdings	1.5	set
ρ_2^Λ	persistence of CB's bond holdings	-0.54	set
$\gamma^{r,\pi}$	coefficient on inflation in Taylor rule for reserves rate	1.01	set
$\gamma^{z,\pi}$	coefficient on inflation in Taylor rule for lending rate	1.01	set

3.3 Solution for the year 2022 (initial year)

Table 3 reports the values of the main endogenous variables produced by the model's solution when we use the parameter values in Table 1 and the policy instruments and coefficients in Table 2. In this solution, variables do not change so this is what we call the "initial steady state". As can be seen, the model's solution can mimic reasonably well the situation in the EA in 2022 and can therefore serve as a departure point from what will follow next.[29] Notice that for this initial steady state solution, the GDP share of government transfers, s_t^t, plays the role of the residually determined public financing instrument that closes the government budget constraint with the public debt to GDP ratio being set at its data value (91.6%); this gives $s_t^t = 9.81\%$ which is much lower than in the data (22%); this provides a first indication that some kind of fiscal correction will be unavoidable sooner or later[30] and this will be confirmed below when we shock the model and study transition paths.

Table 3
Model's solution for key endogenous variables in 2022

Variable	Description	Model	Data
b/y	public debt to GDP	91.6%	91.6%
c/y	private consumption to GDP	55%	52%
inv/y	private investment to GDP	20%	22%
k/y	private capital to output	4.41	NA
L/y	private banks' loans to GDP	13%	37%
j/y	households' deposits to GDP	61%	60%
m_p/y	private banks' reserves to GDP	30%	30%
i^l	interest rate on bank loans	3.5%	3.5%
i^d	interest rate on bank deposits	2%	1.45%
i^b	interest rate on government bonds	3.4%	3.4%
l	work hours	0.32	0.32

4 Main results

Departing from the solution in subsection 3.3. above, transition dynamics are the result of an exogenous adverse TFP shock. The latter is such that there is a sudden rise in the public debt to GDP ratio by around 15% relative to its initial value.[31] Such a rise is similar to that observed in the data when

[29] Regarding bank loans to firms L/y, in 2022 the outstanding amount of loans to non-financial corporations to GDP was around 37%, while the amount of new loans to GDP was around 6% (the data are from the ECB's website https://data.ecb.europa.eu/publications/financecial-markets-and-interest-rates/3030664). In our set up, with one period loans, these two amounts coincide, hence the difference between our solution and the data.

[30] D'Erasmo et al (2016) call this "the classic debt sustainability analysis". Since it focuses only on the long run implications of fiscal policies, its main flaw is that it cannot guarantee that the inherited public debt is sustainable.

[31] This rise is generated by an adverse 9.5% TFP shock at the initial steady state. With a persistence parameter equal to 0.5 in the deterministic $AR(1)$ process for the TFP, the effects of this shock vanish within approximately 10 periods. In subsection 4.3 below, we will consider a rather wide range of robustness checks regarding the size and persistence of this shock. Note that similar shocks have been used in the literature (e.g. Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Sims and Wu (2021));

the global financial crisis and the pandemic crisis erupted.[32]

At this early stage, only the Taylor rules, according to which the policy interest rates react to inflation, are switched on (see equations (19a)-(19b) above). This is consistent with the ECB's policy mandate of price stability since early 2022. Then, our experiments imply that when none of the other policy instruments (namely, tax-spending instruments and quantitative monetary policy instruments) react to public debt, the model is dynamically unstable and cannot produce a transition solution. We will therefore investigate which policies can restore dynamic stability and determinacy. Before we proceed, it is worth reporting that we have experimented with various types of shocks (in addition to TFP), and of various signs and sizes, and the above qualitative result remains the same; namely, if a shock hits the EA economy, and if there is no some kind of systematic policy reaction to public debt, the path of the latter is explosive and hence a solution does not exist over the transition.[33]

Recall that we focus on the case in which the CB purchases sovereign bonds in the secondary market; but in subsection 6.1 below we will also report what changes in the counter-factual case of primary market participation.

4.1 The conventional policy assignment

We start with the conventional case in which the fiscal authorities do their job. Our simulations show that dynamic stability and determinacy are restored, when at least one of the fiscal (tax-spending) instruments, $e_t \equiv (s_t^c$, s_t^g, s_t^t, τ_t^c, τ_t^y, $\tau_t^\pi)$, reacts systematically to the public debt gap by following (17), while QE monetary policy remains exogenous and as it was at the end of 2022. To save on space, we will present results for the public consumption share, s_t^c, and the income tax rare, τ_t^y, only (results for the other fiscal instruments are similar).

Specifically, in this set of simulations, we set the feedback fiscal policy coefficient on the public debt gap, $\gamma^{e,b}$, where $e_t \equiv (s_t^c, \tau_t^y)$, at 0.01 in (17), which is approximately the minimum value that ensures stability of debt across this set of policy experiments, while, at the same time, we set $\gamma^{\Lambda,b} = 0$ in the rule for QE monetary policy in (21a). With this policy mix, we get stability and determinacy and, at the same time, t_t^{cb} and, by construction,

although different authors use different shocks, a common feature is that they generate an economic downturn like in our paper.

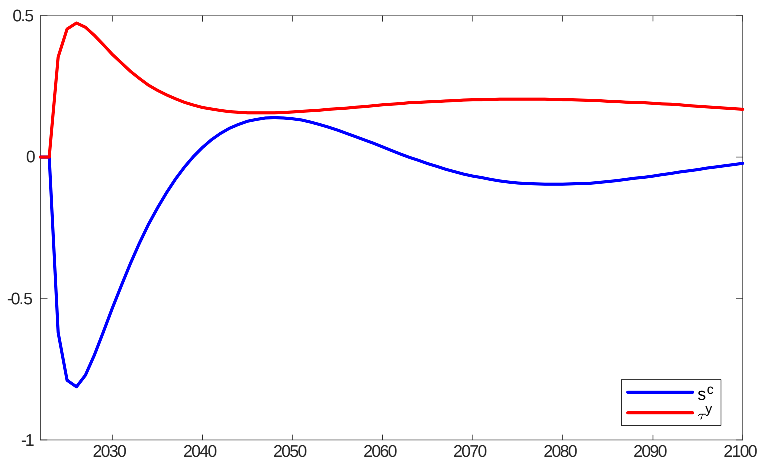
[32] Schucknecht (2022) discusses how various risks can impact on a rise in public debt and threaten fiscal sustainability in the current situation in the EU.

[33] It is worth pointing out that such instability arises even in the case of positive public investment shocks; in other words, the usual claim by politicians that if pubic spending is on productive activities, it can be self-financing - in the sense that no spending cuts and/or tax rises will be necessary in the future - is not supported by our model (see also Malley and Philippopoulos (2023) for the recent US infrastructure stimulus).

$1-\Lambda_t$ remain within their ES ranges as defined in equations (20b) and (21b). In other words, a stable ES can be guaranteed when policy interest rates react to inflation and at least one of the tax-spending instruments reacts to public debt imbalances. This is in accordance with Leeper's (1991, 2016) policy mix of passive fiscal policy and active monetary policy in the sense that when fiscal policy stabilizes the public debt, interest rate policy should react to inflation. Regarding interest rate policy under this regime, as said above, we have set $\gamma^{r,\pi}$ and $\gamma^{z,\pi}$ just above 1 for the Taylor principle to be satisfied, although, we report that our results do not change even when the Taylor principle is not satisfied and even when we set $\gamma^{z,\pi}=0$ in the policy rule for the interest rate on loans to private banks; in particular, they hold for $\gamma^{r,\pi} \geq 0.2$ in the policy rule for the interest rate earned by reserves at the CB.

Graph 1 plots the time-paths of s_t^c and τ_t^y expressed as percentage deviations from their departure 2022 values. As expected, the spending share has to be reduced, while the tax rate has to rise, to restore debt and macro stability, and all this lasts for several years.

Graph 1
Public spending to GDP and tax rates used for debt stabilization (percentage deviation from 2022)



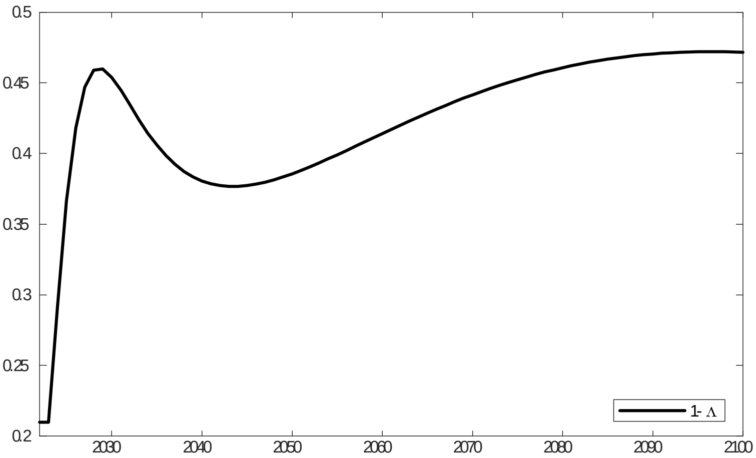
4.2 Can quantitative monetary policy do the unpleasant job?

We now switch off any kind of fiscal policy reaction to public debt and instead investigate what happens when this task is assigned to the CB. Bond purchases by the latter in the secondary market, $1-\Lambda_t$, are now assumed to be contingent on the public debt gap as in the feedback policy rule (21a), that is, now $\gamma^{\Lambda,b} > 0$, while, at the same time, we set $\gamma^{e,b} = 0$ in (17).

We start by assuming away the ES-type restrictions on the conduct of quantitative monetary policy. In other words, we start by assuming that, over time, there is no upper limit to the fraction of bonds, $1 - \Lambda_t$, that the CB can hold, to the extent of course that this fraction does not exceed 1, and that the CB's transfer to the Treasury, t_t^{cb}, is free to also take negative values if this is needed. In this unrestricted case, that does not differ from the conduct of monetary policy in the US, our simulations imply that stability and determinacy can be restored when the feedback policy coefficient on the public debt gap, $\gamma^{\Lambda,b}$, is set at a relatively high value, at 0.6, which naturally implies that $1 - \Lambda_t$ rises a lot in some time periods, sometimes as high as around 47% in this set of experiments (recall that the starting value is 21% in these baseline solutions). Regarding interest rate policy in this regime, to get stability both policy rates should react to inflation which differs from the conventional regime above where $\gamma^{z,\pi}$ could be set to zero without implications for dynamic stability; we thus set $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$ as in the baseline parameterization. In other words, strong QE-type policies should be accompanied by control of inflation via a relatively strong interest rate management.

Graph 2 shows the time-path of $1 - \Lambda_t$ under this policy scenario. As can be seen, the CB needs to increase its QE a lot and for many periods to ensure public debt and macroeconomic stability. That is, stability with QE requires that the CB becomes a long-lasting creditor of the government.

Graph 2
QE used for debt stabilization
(in levels)



It then naturally follows that if, other things equal, we impose the ES-type restrictions, namely, that $(1 - \Lambda_t) \leq 0.33$ and $t_t^{cb} \geq 0$, we cannot get a transition solution.[34] Actually, only the restriction, $(1 - \Lambda_t) \leq 0.33$ is violated in this particular set of experiments, which should not come as a surprise: 21% is rather close to the threshold of 33% so that, in the presence of a new shock, there will be no much space left for a further significant increase in $1 - \Lambda_t$. In other words, when we start with 21%, which was the fraction of euro public debt held by the ES under PSPP at the end of 2022, and the upper limit is 33%, which is the official upper limit of PSPP, the ES restrictions need to be violated for QE to be able to do the unpleasant job. The next subsection examines the robustness of this result, considering different initial conditions, different upper limits and different properties of the adverse shock.

4.3 How general are the above results?

To check the sensitivity of the above results, we investigate whether debt-contingent QE policy could restore stability on its own and still respect the rules of the ES, if conditions were different. In particular, if the CB faced more favorable initial conditions, different upper limits to bond holdings, or if the economy were hit by a smaller and/or less persistent shock.

For example, imagine that, other things equal, the initial fraction of bonds in the hands of the CB is 31.8%, which was the sum of bond holdings under both PSPP and PEPP in the data at the end of 2022 ($21+10.8 = 31.8$), while, at the same time, the upper limit is rather loose, say 50%; then, we report that we get the same result as above, namely, the ES restrictions need to be violated for QE to be able to do the unpleasant job on its own. On the other hand, if we start with 21% so that, as in the baseline solution, we ignore holdings under PEPP, and the upper limit is set at 50%, then, resolving the model, QE can restore stability on its own and this can happen without violating the ES rules. In other words, as expected, the initial stock of bonds in the hands of the CB, in combination with its upper limit, does matter for the effectiveness of QE policies and this can perhaps contribute to explaining why the ECB's intervention during the global financial crisis of the previous decade was successful; at that time, the ECB had much more

[34] We report that this result is robust to changes in the parameterization of private banks.

space to manoeuvre the economy. Table 4a summarizes these results.

Table 4a		
Different initial positions and upper limits		
Initial share	Upper limit	
21%	33%	Instability
21%	50%	Stability
31.8%	50%	Instability

We have also experimented with different TFP shocks from the one assumed so far (as said at the very beginning of this section, we have assumed a 9.5% adverse shock with a 0.5 $AR(1)$ parameter). We report that our qualitative results do not change for any adverse shock equal to, or higher than, 5% other things equal. Only when we assume that the adverse shock is smaller than 5%, QE can do the job without violating the ES restrictions. All this is with $\rho^A = 0.5$. As ρ^A rises, which means that the adverse shock lasts longer, instability arises even with a relatively small shock; for instance, when $\rho^A = 0.8$, a shock equal to, or higher than, 3% results in instability. And vice versa: low persistence allows for stability even with a relatively strong shock; for instance, when $\rho^A = 0.2$, we need a shock stronger than 7% for instability. These results are summarized in Table 4b.

Table 4b		
Size and persistence of adverse shock		
Shock to A	Persistence of A	
$\leq 7\%$	0.2	Stability
$< 5\%$	0.5	Stability
$< 3\%$	0.8	Stability

Summing up, as is perhaps expected, if the starting situation were more favorable than that at the end of 2022, or if the shocks triggering dynamics were relatively mild, it would be possible for the CB to stabilize the economy on its own via debt-contingent QE and still respect the numerical rules of the ES. Nevertheless, the main result does not change. Namely, given the current situation, if a relatively big shock hits the European economy and fiscal policy remains active, meaning that it does not adjust to address public debt imbalances, there is no room left for further quasi-fiscal actions by the ECB except if the ES's rules are revised (see also the discussion in the closing section 7 below).

4.4 Macroeconomic implications of the above policies

In this subsection, we show how the above two polar cases (public debt stabilization via fiscal adjustment, or via QE without the ES's restrictions) affect public finances and some key macroeconomic variables.

4.4.1 Implications for public finances

To understand how different policies work to restore public debt stability, we compute the public finance implications of the three policies studied above: the case in which public debt stabilization is achieved by adjustments in a public spending instrument like public consumption, s_t^c, the case in which this is achieved by adjustments in a tax instrument like the income tax rate, τ_t^y, and the case in which this is achieved by (free-of-ES-restrictions) adjustments in the share of sovereign bonds purchased by the CB in the secondary market, $1 - \Lambda_t$.[35] For each policy, we compute the resulting paths of the real gross interest rate on sovereign bonds, $(1 + i_t^b)\frac{p_{t-1}}{p_t}$, the transfer from the CB to the Treasury, t_t^{cb}, and the primary fiscal surplus, $t_t^{tax} - \left(g_t^c + g_t^g + g_t^t\right)$; these three endogenous variables shape the dynamics of public debt in the Treasury's budget constraint (14).[36] The corresponding graphs are Graphs 3a, 3b, 3c, while Graph 3d shows the resulting paths of the debt-to-GDP ratio. Again, the departure values are those of the year 2022.

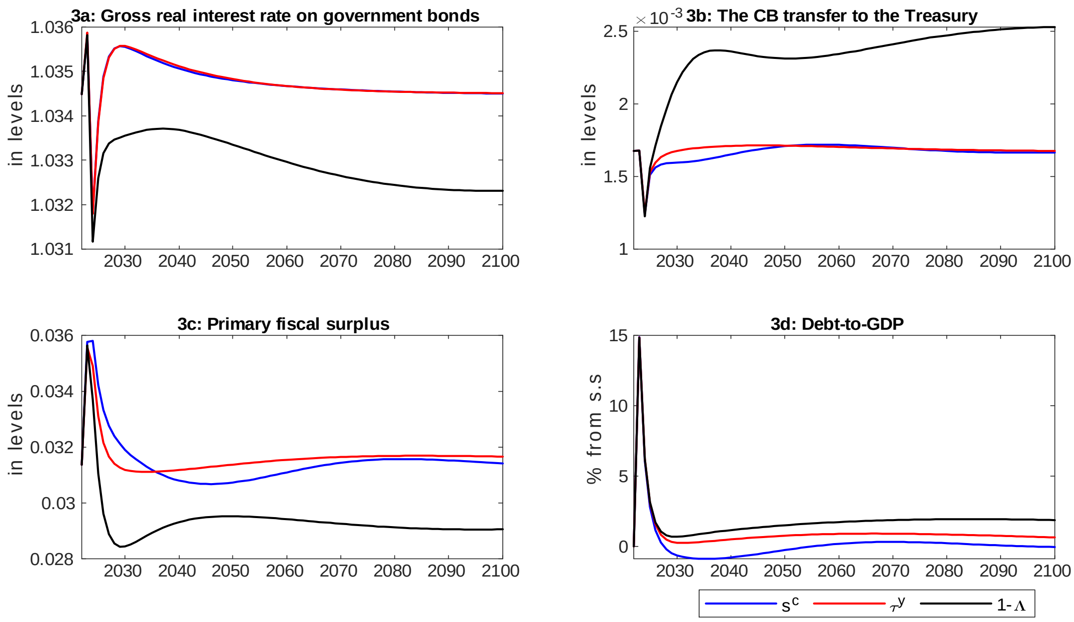
Inspection of Graphs 3a, 3b and 3c implies that the use of QE policies through the endogenous adjustment of $1 - \Lambda_t$ to the public debt gap is superior to fiscal policies in terms of reducing the real interest rate on sovereign bonds (see Graph 3a) which is the coefficient on inherited public debt in the difference equation (14), as well as in terms of the transfer from the CB to the Treasury (see Graph 3b),[37] while, it is inferior to spending cuts or tax rises in terms of the primary fiscal surplus (see Graph 3c). These are intuitive results and consistent with the general belief that the main benefit of QE has been to reduce nominal and real sovereign yields and calm financial markets rather than to generate extra resources for the fiscal authorities (or, quoting Reis (2017), rather than to "alleviate fiscal burdens"). Since it is developments in the primary fiscal balance that dominate, we observe a slower public debt convergence when it is QE that reacts to the debt gap; reversing the argument, when we use cuts in government spending or tax rises, the debt ratio falls relatively fast (see Graph 3d).

[35] In this set of experiments, we set $\gamma^{\Lambda,b} = 0.6$ when it is QE that reacts to debt, while, we set $\gamma^{e,b} = 0.01$ when it is fiscal policy instruments that do the job. Regarding interest rate policy, we set $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$.

[36] That is, in general equilibrium models, where the sovereign real interest rate, public spending, tax revenues, etc, are all endogenous variables depending, among other things, on the outstanding public debt, dynamic stability is a more complex issue than in simple debt arithmetic calculations where debt stability depends only on the differerence between the exogenous real interest rate and the exogenous growth rate, i.e. the so-called $r - g$ differential. See the discussion in Economides and Philippopoulos (2023).

[37] This is mainly because of the gain that the CB has when it lends to private bank at i^z and borrows from them at i^r. Overall, however, transfers are small as is also the case in the data.

Graph 3: Public finance implications



4.4.2 Implications for macroeconomic variables

We now compute the implications of the three policies studied above - namely, the case in which debt stabilization is restored by adjustments in s_t^c, the case in which this is achieved by adjustments in τ_t^y and, finally, the case in which this is achieved by unrestricted adjustments in $1 - \Lambda_t$ - for some selective macro variables. We start with the path of cumulative discounted output, as difference from its departure value in 2022. For each policy,[38] we compute the value of φ_t defined as:

$$\varphi_t \equiv \sum_{s=0}^{t} (y_s - y)$$

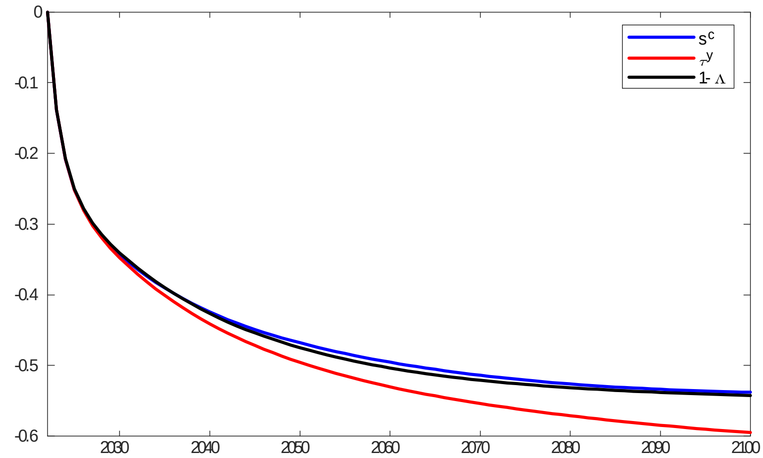
where y is the value of output in the initial steady state.[39]

[38] For this set of experiments, we set $\gamma^{\Lambda,b} = 0.6$ when it is QE that reacts to debt, while, we set $\gamma^{e,b} = 0.01$ when it is fiscal policy instruments that do the job. Regarding interest rate policy, again we set $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$.

[39] We report that our results are qualitatively the same if we compute the PDV of the output gap by using the formula $\varphi_t \equiv \sum_{s=0}^{t} \frac{y_s - y}{\left(1+i^b\right)^s}$ where i^b is the steady state value of the interest rate on sovereign bonds. We prefer however to use the non-discounted sum simply because differences in the effects of different policy experiments show up more in the medium-run rather than in the short-term and these differences get a small or negligible weight when discounted.

The three paths of φ_t are illustrated in Graph 4a. As can be seen, the fall in output - triggered by the adverse TFP shock - is bigger over time when it is the income tax rate that reacts to stabilize the public debt trajectory than when this is achieved by cuts in public consumption spending or by purchases of sovereign bonds by the CB in the secondary market. More specifically, our simulations imply that by the year, say, 2100, the cumulative output loss will be around 10% bigger if we use income taxes relative to spending cuts or QE. It is well established that public debt stabilization is more recessionary when it is tax-based. What perhaps looks a bit surprising, at least at first sight, is that the real effects are almost identical when we use spending cuts, s_t^c, or higher QE, $1-\Lambda_t$. Recall however that, in this class of models, public consumption spending, s_t^c, provides utility-enhancing services only, so the cut in s_t^c is not damaging the supply side of the economy.

Graph 4a
Output gap under alternative debt stabilization policies



It is useful to put these results in the context of the literature. The literature on balance sheet monetary policies in general, and sovereign bond purchases (QE) in particular, has highlighted the beneficial effects that such policies can have on interest rates and asset prices and in turn, although this is rather model-specific, on aggregate demand (AD) and the real economy (see e.g. Walsh (2017, p. 538) for a summary). These effects can work through various channels. First, QE lowers sovereign spreads (see also our Graph 3a above). On the other hand, in the presence of financial frictions and asset price wedges like those discussed above (see subsection 2.4), lower sovereign interest rates may not be translated to lower yields in other markets too and this weakens the transmissiocn of QE policies to the rest of the economy. Second, and related to the first point, QE frees up private banks' financial capital since, by construction, their holdings of sovereign bonds decreases. However, with a rich menu of assets and liabilities in the banks' portfolios, as is also the case in the data, this extra financial capital may not be automatically used to finance more bank loans to production firms. For example, in our model, the CB's purchases of bonds in the secondary market leads to several portfolio rebalancing effects, like an increase in interest-bearing reserves held by banks at the CB, and again this weakens the transmission of QE policies to the rest of the economy.[40] Third, as also shown in Graphs (3a-c) above, different debt stabilization policies have mixed implications for public finances and hence for AD. For instance, debt stabilization via QE leads to higher primary fiscal deficits over time than via spending cuts and tax rises which stimulates AD other things equal but, at the same time, by lowering the interest rate on sovereign bonds, QE reduces the real wealth of private banks from the bonds kept and this tempers the increase in AD. Putting all this together, different debt stabilization policies generate general equilibrium effects working in several directions so that, at the end of the day, differences in the real economy may be relatively small in size, although, in our case, it is a matter of interpretation whether the above reported 10% difference in the cumulative output gap can be considered as small or large.[41]

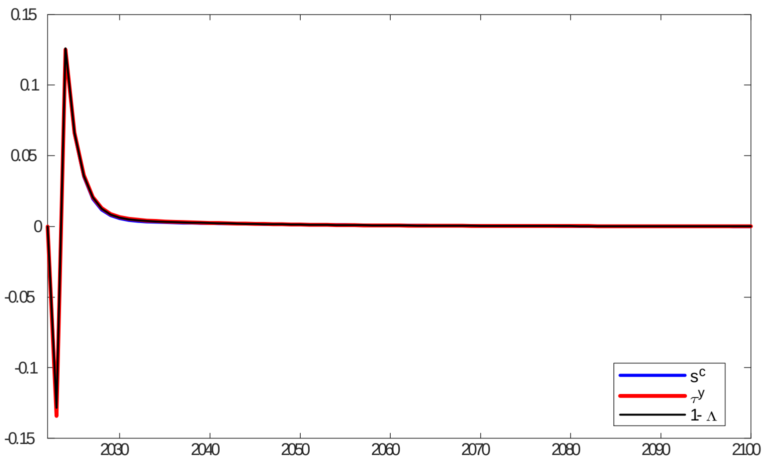
The same economic reasoning can help us to understand the path of inflation and, in particular, the negligible inflation differences between alternative debt stabilization policies (see Graph 4b). That is, inflation is mainly driven by the adverse TFP shock and what happens to output. It is less affected by which policy instrument is used for debt stability. This happens because, as argued above, different debt stabilization policies have mixed effects on AD that more or less seem to cancel each other out in equilibrium. Besides, recall that, under QE, stability requires both policy rates to react to inflation; in other words, under QE, stability requires monetary policy to respond to inflation more strongly than under fiscal policy and this

[40] This can be compared to e.g. Gertler and Karadi (2013), where banks' assets consist of sovereign bonds and loans to firms only, and where liabilities consist of households' deposits only, so that an increase in the CB's bold holdings via QE, meaning a decrease in the bonds held by private banks given total public debt, is directly translated to an increase in the supply of credit to the private economy and this supports the real economy (see also Walsh (2017, p. 551) for a review of this model, Sims and Wu (2021) for an application to the US economy, and Benigno et al (2022) for a more general discussion of portfolio rebalancing effects).

[41] We report that if we assume that the CB participates in the primary sovereign bond market, that the path of public debt is exogenous (so as there are no stability issues like in our paper) and that there are no loans from the CB to private banks, then the real effects of QE policies become even stronger (see Appendix A.8 for this scenario). This rather counter-factual set of assumptions is employed by e.g. Sims and Wu (2021) for the US, Coenen et al (2018, 2020) in their model for the ES, as well as by Gertler and Karadi (2013).

can also contribute to explaining why inflation is not higher under QE.[42] Finally, note that inflation does not exceed the ES's critical threshold of 2% in all these experiments.

Graph 4b
Inflation dynamics under alternative debt stabilization policies (% deviation from 2022)



5 Policy mixes

So far we have studied two polar cases. We have seen that stability and determinacy can be restored either by conventional fiscal corrections, or by debt-contingent QE monetary policy, although, in the latter case, the ES's rules have to be violated. In this section, we search for policy mixes in the sense that monetary and fiscal policies work together and, at the same time, the ES's rules are respected. Since there can be many possibilities, we will be selective focusing on some policy scenaria currently practiced or debated in the EU. In particular, we will study what happens when fiscal inaction is only temporary (subsection 5.1), the implications of an increase in public investment like that under the Recovery and Resilience Facility (RRF) of NextGenerationEU (NGEU) (subsection 5.2), and whether a policy reversal, like QT, will come at a cost (subsection 5.3). Dynamics are again triggered by the adverse TFP shock as defined in the beginning of section 4.

[42] On the other hand, if we are willing to employ the same set of counter-factual assumptions as listed in the previous footnote, there are more distinct differences in inflation (see Appendix A.8).

5.1 Temporary fiscal inaction but eventually the government does its job

Since big shocks keep hitting the European economy since the start of 2020, it is natural to ask ourselves a question, which is similar to that asked by Leeper et al (2010) in their study for the sustainability of public debt in the US economy. In particular, if so far we cannot observe any systematic fiscal reaction to public debt imbalances and, at the same time, the space for further QE monetary policy has been exhausted given the self-imposed ES's restrictions, then, quoting Leeper and his co-authors, a natural question to ask ourselves is "Why do forward-looking agents continue to purchase bonds with relatively low interest rates and bond prices don't plummet?". As Leeper and his co-authors argue, a natural answer to this - to the extent that we want to maintain the assumption of rationality - could be that private agents believe that the current fiscal inaction is only temporary and it will be replaced by necessary fiscal corrections of some kind in the future. In other words, the belief is that the necessary fiscal reaction has been just backloaded.

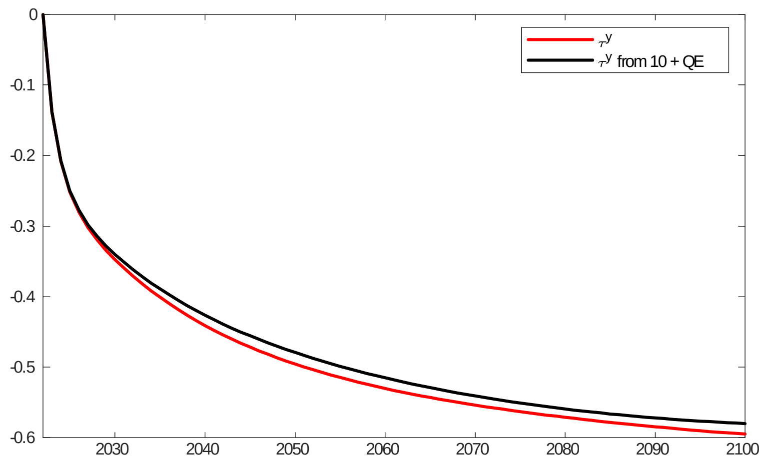
To address this possible scenario, we now allow for fiscal reaction to public debt after, say, 10 periods, complemented by mild QE reaction in the sense that $1 - \Lambda_t$ also helps by reacting to public debt, say, from the very beginning. Specifically, using, for example, the income tax rate as the debt-contingent fiscal instrument, we set $\gamma^{\tau^y,b} = 0.01$ after 10 periods and zero before, while, regarding monetary policy, we set $\gamma^{\Lambda,b}$ and $\gamma^{r,\pi}$, $\gamma^{z,\pi}$ as in subsection 4.2 above.[43] Our simulations show that now we do get stability and determinacy and, in addition, quantitative monetary policy respects the rules of the ES. Therefore, although QE policy cannot on its own restore stability and determinacy and at the same time respect the rules of the ES, it can do so if there is the anticipation of fiscal reaction to public debt in the near future and this anticipation proves to be credible.

Graph 5 illustrates the path of the output gap, φ_t, under this policy mix. This graph also includes, for comparison, the path of φ_t in the case in which public debt is stabilized by fiscal policy only and from the very beginning, as it was the case in subsection 4.1. As can be seen, the recession is smaller in the former case in which QE policy complements the backloaded fiscal policy. In other words, the adverse real effects of the negative TFP shock are mitigated when the CB gives the Treasury a hand through debt-contingent QE policy even if the latter is a relatively mild one. Fiscal and

[43] That is, at $t \geq 0$, $\gamma^{\Lambda,b} = 0.6$ and $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$. We report that now we can get stability even if $\gamma^{\Lambda,b}$ is lower than 0.6. That is, to the extent that the fiscal authorities will eventually do their job, we get stability with a milder QE than in subsection 4.2. We set $\gamma^{\Lambda,b} = 0.6$ simply for comparability. It is worth pointing out however that, even if the feedback coefficient is set at the same value ($\gamma^{\Lambda,b} = 0.6$), the resulting increase in $1 - \Lambda_t$ is smaller because the simultaneous fiscal reaction keeps the debt ratio lower than in subsection 4.2.

monetary policy reinforce each other by creating space for each other (for similar synergies between fiscal and monetary policies in the EA, see also e.g. Bankowski et al (2021) although in a model without quantitative monetary policy).

Graph 5
Output gap with a delayed fiscal reaction and a policy mix



5.2 Public investment spending backing unconventional monetary policy

A big part of EU policy since the eruption of the pandemic crisis in early 2020 has been the still ongoing RRF of NGEU. This implies a significant increase in public investment spending during 2021-2027 being financed by a temporary increase in public borrowing at the EU level.[44]

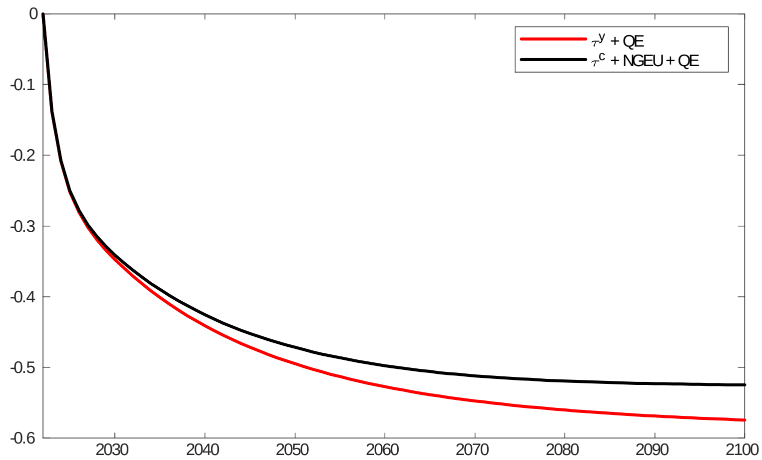
To quantify the implications of a policy like this within our setup, we now increase the share of public investment to output from 0.03 to 0.032 in each of the first 5 periods after the departure year of 2022 (so that the cumulative increase is 1pp from 0.03 to 0.04) and, at the same time, in order to see the implications of using a less distorting fiscal instrument than the income tax rate used so far, we allow the consumption tax rate to react to public debt imbalances instead of the income tax rate, $\gamma^{\tau^c,b} = 0.01$. This fiscal mix backs the mild QE policy as defined in the previous subsection.[45]

[44] See Malley and Philippopoulos (2023) for the recent increase in public infrastructure spending in the US.

[45] In particular, the $AR(1)$ parameter in the rule for the consumption tax rate is set at $\rho^{\tau,c} = 0.9$, while, as above, $\gamma^{\Lambda,b} = 0.6$ and $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$. This is at $t \geq 0$.

Results for the output gap, φ_t, are shown in Graph 6. This graph also includes, for comparison, the path of φ_t in the case in which, other things equal, we switch off the rise in public investment and assume that it is the income tax rate that reacts to public debt.[46] As can be seen, the output loss is systematically smaller in the former case. More specifically, our simulations imply that by the year, say, 2100, the cumulative output loss will be around 9% smaller in the former case. In other words, a mix of higher public investment being accompanied by debt-contingent QE and consumption taxes mitigates the recessionary effects of the adverse supply shock.

Graph 6
Output gap with NGEU fiscal policy and a policy mix



5.3 Unwinding QE

The literature has stressed several downsides to using large-scale asset purchase programmes (see e.g. the discussion in Benigno et al (2022)), with the most important being the implications of an unavoidable (sooner or later) policy reversal or what is known as quantitative tightening (QT).

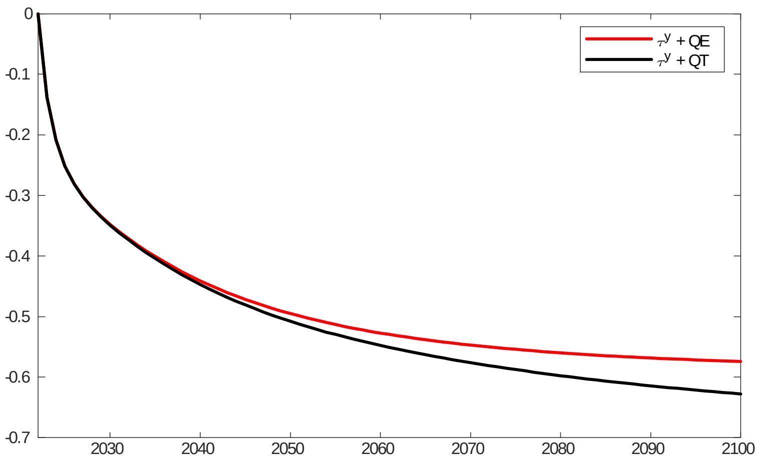
In this subsection, we study the case of quantitative gradual tightening, in the sense that now, instead of being accommodative, $1 - \Lambda_t$ exogenously and gradually decreases over time from 21% (initial steady state) to, say, 5% (new, terminal steady state), and, at the same time, the income tax rate reacts to debt imbalances from the very beginning.[47] The path of the

[46] That is, in this benchmark case, at $t \geq 0$, $\gamma^{\tau^y,b} = 0.01$, $\gamma^{\Lambda,b} = 0.6$ and $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$.

[47] That is, at $t \geq 0$, $\gamma^{\tau^y,b} = 0.01$ and $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$.

output gap is shown in Graph 7. This graph also includes, for comparison, the path of φ_t in the case in which, instead of QT, we have the mild QE as defined above.[48] As can be seen, the former case is clearly more recessionary than the latter (more specifically, our simulations imply that by the year, say, 2100, the cumulative output loss will be around 9% bigger in the former case), thus confirming the fear that QT will not come without real costs.

Graph 7
Output gap with QE and with QT



6 The importance of two commonly made assumptions

In this section, we check out the importance of two popular, although counter-factual assumptions, usually made by the related literature. We start with the assumption that the CB participates in the primary sovereign bond market (although this does not happen in practice) and in turn we will evaluate the assumption that there are no loans from the CB to private banks (although these loans have been an important item in the ES's financial statements; they were around 17% of the total size of the ES's balance sheet at the end of 2022, which was the second biggest item after securities whose share was 64%). The natural benchmark used for comparison in this set of experiments will be the baseline case studied in subsection 4.2 above; namely, the case in which debt stabilization is conducted by QE only but this violates the ES's rules.

[48] That is, at $t \geq 0$, $\gamma^{\tau^y,b} = 0.01$, $\gamma^{\Lambda,b} = 0.6$ and $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$.

6.1 The CB participates in the primary market for sovereign bonds

Here we solve the model for the case in which the CB is assumed to participate in the primary sovereign bond market like private banks do (as said, modelling details are in Appendices A.4 and A.6 which can be compared to Appendices A.5 and A.7 for the secondary market).

We report that our simulations imply the following (results are available upon request). The main qualitative result is that now, other things equal which means relative to subsection 4.2 above, QE monetary policy is a substitute for fiscal policy regarding debt stabilization and, at the same time, respects the rules of the ES in (20b) and (21b), i.e. now $t_t^{cb} \geq 0$ and $1 - \Lambda_t \leq 0.33$. The latter happens because stability is now achieved by a lower value of $\gamma^{\Lambda,b}$ than in the case in which this is done in the secondary market (for example, now $\gamma^{\Lambda,b} = 0.2$, while we had to set $\gamma^{\Lambda} = 0.6$ in subsection 4.2);[49] this makes the resulting increase in $1 - \Lambda_t$ smaller so that the latter remains within its ES range.

Generally speaking, the CB has a more direct control over public finances and public debt stability when it purchases sovereign bonds in the primary market. More specifically, balance sheet monetary policies can affect public finances and public debt stability through several channels, direct and indirect.[50] Direct effects are most obviously manifested in the composition of total public debt, b_t, in the Treasury's budget constraint (see equation (14)). As said above, when the CB is assumed to participate in the primary market, $b_t = b_{p,t}^T + b_{cb,t}$, while, when the CB participates in the secondary market, $b_t = b_{p,t}^T$ and only in turn a fraction of $b_{p,t}^T$ is sold to the CB as $b_{cb,t}$. Hence, although in both cases $b_{p,t}^T$ is affected by monetary policies and $b_{cb,t}$ is debt-contingent, the effect of the latter on the path of b_t is obviously more direct in the case in which the CB acts in the primary market (see also subsection 2.7.3 above). This is why stability is restored by $\gamma^{\Lambda,b} = 0.2$ and hence a relatively small rise in $1 - \Lambda_t$ and in turn in $b_{cb,t}$ in the primary case.

Therefore, the usual assumption that the CB purchases government bonds in the primary market is not innocent when the issue is public debt and macroeconomic stability.

[49] Fiscal policy reaction to debt and interest rate reaction to inflation are as in subsection 4.2 above. Namely, $\gamma^{e,b} = 0$ and $\gamma^{r,\pi} = \gamma^{z,\pi} = 1.01$. We also report that, to match the data with the new model specification, we have: (a) changed the parameter γ included in households' transfer to entering bankers so as to better match reserves and CB loans as shares of GDP, (b) slightly decreased ξ^b and (c) adjusted government transfers as share of GDP so as to match the debt-to-GDP ratio.

[50] As inspection of equation (14) reveals, indirect channels work through inflation, the interest rate on sovereign bonds, tax revenues and the primary fiscal balance, etc. Direct channels have to do with the composition of b_t, the CB's transfer, t_t^{cb}, etc. See Reis (2017) for a review paper.

6.2 The role of loans from the CB to private banks

We now assume away loans from the CB to private banks. That is, we set $z_{p,t}$ and its respective parameter ξ^z in in the private banks' incentive constraint, at zero, other things equal.

We report that our simulations imply the following (results are available upon request). Without $z_{p,t}$, it is natural that the overall size of the banking sector shrinks. Regarding public debt (in)stability, the elimination of $z_{p,t}$ has two effects on the path of public debt in the government budget constraint in (14). There is a direct effect that works mainly through the demand for sovereign bonds and their interest rate. A fall in borrowing, namely less $z_{p,t}$, means that banks have less financial capital, so their demand for assets falls and this includes demand for sovereign bonds, $b_{p,t}$. Lower demand means lower asset prices and equivalently higher yields. The increase in nominal and real interest rates increases the coefficient on b_{t-1} in (14) making stability harder. But there is also an indirect effect that works through the transfer from the CB to the government. Without $z_{p,t}$, this transfer does not include the CB's interest income from loans to private banks. Hence, the transfer to the Treasury is smaller. In sum, both effects work in the same direction making debt stability harder in the absence of $z_{p,t}$. In the case where the task of debt stabilization is assigned to fiscal policy, the exclusion of $z_{p,t}$ does not have any qualitative implications since tax rises or spending cuts directly enter the government budget constraint and guarantee stability. But, in the case this task is assigned to QE monetary policy, our simulations imply that stability is not feasible even for very high values of γ^{Λ} in the feedback rule.

Therefore, combining this evidence with the previous results, debt stabilization via QE also requires that the banking sector has enough liquidity thanks to loans from the CB. These loans, also recorded in the ES's data, allow private banks to finance, among other things, their purchases of sovereign bonds. Note that this condition of excess liquidity is in addition to the conditions identified so far, mainly that the CB continues to be a long-lasting holder of sovereign bonds purchased in the secondary market.

7 The need for a new monetary framework and possible extensions

In this paper, we investigated whether debt-contingent QE policies can substitute spending cuts and/or tax rises for public debt stabilization in an otherwise unstable model. Our answer is a qualified "yes". The ES seems to have exhausted much of its room for further fiscal-type manoeuvre given its self-imposed upper limit on sovereign bond holdings and the non-allowance of fiscal support.

Since the main results have already been listed in some detail in the Introduction, here we wish to deliver a general policy message. If national fiscal authorities continue to be unable (because of new shocks and challenges hitting the European economy) or reluctant (because of political economy reasons) to adjust their tax-spending policies to public debt imbalances, and, at the same time, the growth momentum is not strong enough to allow member-countries to grow their way out of public debt, so that there is political pressure on the ES to continue playing its fiscal role, there is little space for further quasi-fiscal manoeuvre given the self-imposed restrictions. A natural implication of our paper is that more pragmatism is required and this can only enhance the credibility of the ES and the effectiveness of the ECB's policies (see also the column by Ricco et al (2021)). Just renaming things (for instance, that the upper limit is 1/3 but purchases via the PEPP do not count because they are "exceptional") and not being able to communicate openly that it can play a fiscal role if this is needed, undermine the credibility and effectiveness of monetary policies, especially when these policies are, by construction, complex and have to balance different needs. Hence, together with the current debate over the EU's new fiscal framework and the agreement on the new fiscal rules of the Stability and Growth Pact (SGP), an amendment to the existing legal framework of the ECB is perhaps unavoidable if the ES is once more called upon to step in as a deux ex machina in the case a new crisis erupts. A wait-and-see approach and resort to "exceptional" policies ex post will only add to uncertainty. But of course this is a decision that needs to be taken at the European Union (EU) level and is not a criticism to the ECB. The latter has been doing a very good job if we take into account that, paraphrasing the title of the paper by Bassetto and Caracciolo (2021), operates subject to 42 budget constraints.

We close with possible extensions of our work. Here, we have used an aggregate model for the EA. Such a model masks differences and asymmetries across member countries and the relevance of those differences and asymmetries to the general equilibrium effects of quantitative monetary policies. Our plan is to study these issues first in the context of an open economy being a member of the EA and in turn in the context of a two-region (core and periphery) model of the EA.

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Appendices

A.1 Solution of households' problem

The first-order conditions for $c_{h,t}$, $l_{h,t}$, $j_{h,t}$ are respectively:

$$\frac{1}{c_{h,t}^{\mu_1}} = \lambda_{h,t}(1+\tau_t^c) \tag{A.1a}$$

$$\chi^l l_{h,t}^{\mu_2} = \lambda_{h,t}(1-\tau_t^y)w_t \tag{A.1b}$$

$$\lambda_{h,t} = \beta\lambda_{h,t+1}(1+i_{t+1}^d)\frac{p_t}{p_{t+1}} \tag{A.1c}$$

where $\lambda_{h,t}$ is the Lagrangean multiplier associated with h's budget constraint.

A.2 Solution of final good firms' problem

Final good firms act competitively. The first-order condition for $y_{f,i,t}$, and since $y_{f,i,t} = \frac{y_{i,t}}{N}$, gives the standard demand function:

$$p_{i,t} = p_t\left(\frac{y_{i,t}}{y_{f,t}}\right)^{\theta-1} \tag{A.2}$$

That is, in a symmetric equilibrium, we simply have $y_{f,t} = y_{i,t}$, $p_t = p_{i,t}$ and $\pi_{f,t} = 0$.

A.3 Solution of intermediate goods firms' problem

The first-order conditions for $l_{i,t}$, $k_{i,t}$ and $L_{i,t}$ are respectively:

$$(1-\tau_t^\pi)w_t + N_{i,t}\nu^l w_t = [(1-\tau_t^\pi)\theta\left(\frac{y_{i,t}}{y_{f,t}}\right)^{\theta-1} - \xi^p\left(\frac{p_{i,t}}{p_{i,t-1}}-1\right)\frac{p_t}{p_{i,t-1}}(\theta-1)\left(\frac{y_{i,t}}{y_{f,t}}\right)^{\theta-1}\frac{\overline{y}_{i,t}}{y_{i,t}} +$$

$$+\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{i,t+1}}{p_{i,t}}-1\right)\frac{p_{i,t+1}}{p_{i,t}}p_t(\theta-1)\left(\frac{y_{i,t}}{y_{f,t}}\right)^{\theta-1}\frac{\overline{y}_{i,t+1}}{y_{i,t}}]\frac{\partial y_{i,t}}{\partial l_{i,t}} \tag{A.3a}$$

$$1 = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}[1-\delta+(1-\tau_{t+1}^\pi)\theta\left(\frac{y_{i,t}}{y_{f,t}}\right)^{\theta-1}\frac{\partial y_{i,t+1}}{\partial k_{i,t}}] -$$

$$-\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{i,t+1}}{p_{i,t}}-1\right)\frac{p_{t+1}}{p_{i,t}}(\theta-1)\frac{\overline{y}_{i,t+1}}{y_{i,t+1}}\left(\frac{y_{i,t+1}}{y_{f,t+1}}\right)^{\theta-1}\frac{\partial y_{i,t+1}}{\partial k_{i,t}} +$$

$$+\frac{\beta^2\lambda_{h,t+2}}{\lambda_{h,t}}\xi^p\left(\frac{p_{i,t+2}}{p_{i,t+1}}-1\right)\frac{p_{i,t+2}}{p_{i,t+1}}p_{t+1}(\theta-1)\frac{\overline{y}_{i,t+2}}{y_{i,t+1}}\left(\frac{y_{i,t+1}}{y_{f,t+1}}\right)^{\theta-1}\frac{\partial y_{i,t+1}}{\partial k_{i,t}} \tag{A.3b}$$

$$1 + N_{i,t} = \frac{\beta \lambda_{h,t+1}}{\lambda_{h,t}} \left(1 + i^{l}_{t+1}\right) \frac{p_t}{p_{t+1}} \tag{A.3c}$$

where $N_{i,t}$ is the multiplier associated with i's working capital constraint.and we also have:

$$N_{i,t}(L_{i,t} - \nu^{l} w_t l_{i,t}) = 0 \tag{A.3d}$$

Finally, TFP, A_t, is assumed to follow an $AR(1)$ process of the form:

$$A_t = A_t^{\rho^A} A^{1-\rho^A} + \varepsilon_t^A \tag{A.3e}$$

where A denotes the steady state value, $0 < \rho^A < 1$ is the persistence parameter and ε_t^A is a shock term.

A.4 Solution of private banks' problem

We solve private banks' maximization problem following Sims and Wu (2021). Each p's value function satisfies the Bellman:

$$V_{p,t} = \max \left(1 - \sigma\right) \beta_{t,t+1} n_{p,t+1} + \sigma \beta_{t,t+1} V_{p,t+1} \tag{A.4a}$$

Using the bank's balance sheet in (10) to substitute out $j_{p,t}$, we can rewrite the bank's net worth in (11) as:

$$n_{p,t} = \frac{p_{t-1}}{p_t} \{ (1 - \tau_t^{\pi}) (i_t^{l} - i_t^{d}) L_{p,t-1} + (1 - \tau_t^{\pi}) \left(i_t^{b} - i_t^{d}\right) b_{p,t-1} +$$

$$+ (1 - \tau_t^{\pi}) (i_t^{r} - i_t^{d}) m_{p,t-1} - (1 - \tau_t^{\pi}) (i_t^{z} - i_t^{d}) z_{p,t-1} +$$

$$+ \left[1 + (1 - \tau_t^{\pi}) i_t^{d}\right] n_{p,t-1} \} \tag{A.4b}$$

so that (A.4a) becomes:

$$V_{p,t} = \max \left(1 - \sigma\right) \beta_{t,t+1} \frac{p_t}{p_{t+1}} \{ \left(1 - \tau_{t+1}^{\pi}\right) (i_{t+1}^{l} - i_{t+1}^{d}) L_{p,t} +$$

$$+ \left(1 - \tau_{t+1}^{\pi}\right) \left(i_{t+1}^{b} - i_{t+1}^{d}\right) b_{p,t} + \left(1 - \tau_{t+1}^{\pi}\right) (i_{t+1}^{r} - i_{t+1}^{d}) m_{p,t} -$$

$$- \left(1 - \tau_{t+1}^{\pi}\right) (i_{t+1}^{z} - i_{t+1}^{d}) z_{p,t} + \left[1 + \left(1 - \tau_{t+1}^{\pi}\right) i_{t+1}^{d}\right] n_{p,t} \} + \sigma \beta_{t,t+1} V_{p,t+1} \tag{A.4c}$$

which is like equation (A.10) in Sims and Wu (2021).

In what follows, since θ is a constant, for notational simplicity we rewrite the bank's incentive constraint in (13) as:

$$V_{p,t} \geq \vartheta(L_{p,t} + N^b b_{p,t} + N^m m_{p,t} - N^z z_{p,t}) \tag{A.4d}$$

The Lagrangean of this problem, including the constraint (A.4d), is:

$$\mathcal{L}_{p,t} \equiv (1+\zeta_t)\left\{(1-\sigma)\,\beta_{t,t+1}\frac{p_t}{p_{t+1}}\left\{\left(1-\tau^{\pi}_{t+1}\right)(i^l_{t+1}-i^d_{t+1})L_{p,t}+\right.\right.$$

$$+\left(1-\tau^{\pi}_{t+1}\right)\left(i^b_{t+1}-i^d_{t+1}\right)b_{p,t}+\left(1-\tau^{\pi}_{t+1}\right)(i^r_{t+1}-i^d_{t+1})m_{p,t}-$$

$$-\left(1-\tau^{\pi}_{t+1}\right)(i^z_{t+1}-i^d_{t+1})z_{p,t}+\left[1+\left(1-\tau^{\pi}_{t+1}\right)i^d_{t+1}\right]n_{p,t}\}$$

$$\left.+\sigma\beta_{t,t+1}V_{p,t+1}\right\}-\zeta_t\vartheta\left(L_{p,t}+N^b b_{p,t}+N^m m_{p,t}-N^z z_{p,t}\right)$$

where ζ_t is the multiplier associated with (A.4d).

Each p's first-order conditions for $L_{p,t}$, $b_{p,t}$, $z_{p,t}$, $m_{p,t}$ are respectively:

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^{\pi}_{t+1}\right)(i^l_{t+i}-i^d_{t+i}) = \frac{\zeta_t}{(1+\zeta_t)}\vartheta \tag{A.4e}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^{\pi}_{t+1}\right)(i^b_{t+i}-i^d_{t+i}) = \frac{\zeta_t}{(1+\zeta_t)}\vartheta N^b \tag{A.4f}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^{\pi}_{t+1}\right)\left(i^z_{t+1}-i^d_{t+1}\right) = \frac{\zeta_t}{(1+\zeta_t)}\vartheta N^z \tag{A.4g}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^{\pi}_{t+1}\right)\left(i^r_{t+1}-i^d_{t+1}\right) = \frac{\zeta_t}{(1+\zeta_t)}\vartheta N^m \tag{A.4h}$$

where Ω_{t+1} is defined below and $\beta_{t,t+1}$ equals the household's marginal rate of substitution between consumption at t and $t+1$, i.e. $\beta_{t,t+1} \equiv \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}$.

To derive an expression for Ω_t, since the underlying problem is linear, we guess that the value function is linear in net worth:

$$V_{p,t} = \phi_t n_{p,t} \tag{A.4i}$$

so that Ω_{t+1} is:

$$\Omega_{t+1} \equiv 1-\sigma+\sigma\phi_{t+1} \tag{A.4j}$$

Using (A.4i) and (A.4j), we rewrite (A.4a) as:

$$\phi_t n_{p,t} = (1-\sigma)\,\beta_{t,t+1} n_{p,t+1} + \sigma \beta_{t,t+1} \phi_{t+1} n_{p,t+1} =$$

$$= \beta_{t,t+1} n_{p,t+1} \left(1 - \sigma + \sigma \phi_{t+1}\right) =$$

$$= \beta_{t,t+1} n_{p,t+1} \Omega_{t+1} \tag{A.4k}$$

To generate the RHS of (A.4k), we move (A.4b) one period forward and multiply by $\beta_{t,t+1}\Omega_{t+1}$. Then, using the first-order conditions above, we get:

$$\beta_{t,t+1}\Omega_{t+1} n_{p,t+1} = \frac{\zeta_t}{(1+\zeta_t)} \vartheta (L_{p,t} + N^b b_{p,t} + N^m m_{p,t} - N^z z_{p,t}) +$$

$$+ \beta_{t,t+1}\Omega_{t+1} \frac{p_t}{p_{t+1}} \left[1 + \left(1 - \tau^{\pi}_{t+1}\right) i^d_{t+1}\right] n_{p,t} \tag{A.4l}$$

which holds when the incentive constraint binds.

Then, if we combine $(A.4i)$, $(A.4k)$ and $(A.4l)$, we get:

$$V_{p,t} = \phi_t n_{p,t} =$$

$$= \beta_{t,t+1} n_{p,t+1} \Omega_{t+1} =$$

$$= \frac{\zeta_t}{(1+\zeta_t)} \vartheta (L_{p,t} + N^b b_{p,t} + N^m m_{p,t} - N^z z_{p,t}) + \beta_{t,t+1}\Omega_{t+1} \frac{p_t}{p_{t+1}} \left[1 + \left(1 - \tau^{\pi}_{t+1}\right) i^d_{t+1}\right] n_{p,t} =$$

$$= \frac{\zeta_t}{(1+\zeta_t)} V_{p,t} + \beta_{t,t+1}\Omega_{t+1} \frac{p_t}{p_{t+1}} \left[1 + \left(1 - \tau^{\pi}_{t+1}\right) i^d_{t+1}\right] n_{p,t} =$$

$$= \frac{\zeta_t}{(1+\zeta_t)} \phi_t n_{p,t} + \beta_{t,t+1}\Omega_{t+1} \frac{p_t}{p_{t+1}} \left[1 + \left(1 - \tau^{\pi}_{t+1}\right) i^d_{t+1}\right] n_{p,t}$$

And, after some calculations, we get for ϕ_t:

$$\phi_t = (1+\zeta_t)\, \beta_{t,t+1}\Omega_{t+1} \frac{p_t}{p_{t+1}} \left[1 + \left(1 - \tau^{\pi}_{t+1}\right) i^d_{t+1}\right] \tag{A.4m}$$

which is similar to equation (2.15) in Sims and Wu (2021).

Aggregation: Aggregate the balance sheet condition of private banks in (10):

$$L^T_{p,t} + b^T_{p,t} + m^T_{p,t} = j^T_{p,t} + z^T_{p,t} + N^T_{p,t} \tag{A.4n}$$

where $N^T_{p,t}$ is the total net worth of private banks in the beginning of t. We can derive an equation of motion for $N^T_{p,t}$, by first recognizing that it is the sum of the net worth of "surviving" bankers and the net worth of "entering" bankers. The latter is equal to the "start up" funds provided by households, $\gamma(L_{p,t-1}+b_{p,t-1}+m_{p,t-1})$, where γ is a parameter (see also Gertler and Karadi (2011)). Thus, we have:

$$N^T_{p,t}=\sigma n^T_{p,t}+\gamma\frac{p_{t-1}}{p_t}\left\{L^T_{p,t-1}+b^T_{p,t-1}+m^T_{p,t-1}\right\} \tag{A.4o}$$

where the first term on the RHS is the net worth of banks that stay in the market and the second term is households' transfers to new bankers.

Aggregating $(A.4b)$, the net worth of banks that remain in the market, $\eta^T_{p,t}$, is given by:

$$n^T_{p,t}=\frac{p_{t-1}}{p_t}\{(1-\tau^\pi_t)\,(i^l_t-i^d_t)L^T_{p,t-1}+(1-\tau^\pi_t)\left(i^b_t-i^d_t\right)b^T_{p,t-1}+$$

$$+(1-\tau^\pi_t)\,(i^r_t-i^d_t)m^T_{p,t-1}-(1-\tau^\pi_t)\,(i^z_t-i^d_t)z^T_{p,t-1}+$$

$$+(1+(1-\tau^\pi_t)i^d_t)n^T_{p,t-1}\} \tag{A.4p}$$

Banks' profits transferred to households are:

$$\pi^T_{p,t}=(1-\sigma)\,n^T_{p,t} \tag{A.4q}$$

which is the wealth of exiting banks.

Aggregating $(A.4h)$, we have:

$$V^T_{p,t}=\phi_t N^T_{p,t} \tag{A.4r}$$

Aggregating $(A.4d)$, we have with a binding incentive constraint:

$$V^T_{p,t}=\vartheta(L^T_{p,t}+N^b b^T_{p,t}+N^m m^T_{p,t}-N^z z^T_{p,t}) \tag{A.4s}$$

Therefore, in this block of the model, we have 12 variables, $V^T_{p,t}$, $L^T_{p,t}$, $b^T_{p,t}$, $m^T_{p,t}$, $z^T_{p,t}$, $j^T_{p,t}$, ζ_t, $n^T_{p,t}$, $N^T_{p,t}$, $\pi^T_{p,t}$, ϕ_t, Ω_t, in 12 equations, $(A.4e)$-$(A.4h)$, $(A.4j)$ and $(A.4m)$-$(A.4s)$.

A.5 Solution of private banks' problem when they sell bonds to the CB

In this appendix, we present the banks' problem when they can sell bonds to the CB in the secondary market. We will present what changes relative Appendix A.4.

The equation for net worth is now:

$$n_{p,t}=\frac{p_{t-1}}{p_t}\{(1-\tau^\pi_t)\,(i^l_t-i^d_t)L_{p,t-1}+(1-\tau^\pi_t)\,(i^r_t-i^d_t)m_{p,t-1}+$$

$$+\left[\Lambda_t\left(1+(1-\tau_t^\pi)\,i_t^b\right)+(1-\Lambda_t)\,\Phi_t-\left(1+(1-\tau_t^\pi)\,i_t^d\right)\right]b_{p,t-1}-$$

$$-\,(1-\tau_t^\pi)\,(i_t^z-i_t^d)z_{p,t-1}+\left(1+(1-\tau_t^\pi)\,i_t^d\right)n_{p,t-1}\} \tag{A.5a}$$

Working as in Appendix A.4, the four optimality conditions for $L_{p,t}$, $b_{p,t}$, $z_{p,t}$, $m_{p,t}$, are given by:

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^\pi\right)(i_{t+i}^l-i_{t+i}^d)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta \tag{A.5b}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[\Lambda_{t+1}(1+\left(1-\tau_{t+1}^\pi\right)i_{t+1}^b)+\Phi_{t+1}\left(1-\Lambda_{t+1}\right)-(1+\left(1-\tau_{t+1}^\pi\right)i_{t+1}^d)\right]=$$

$$=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^b \tag{A.5c}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^\pi\right)\left(i_{t+1}^z-i_{t+1}^d\right)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^z \tag{A.5d}$$

$$\beta_{t,t+1}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^\pi\right)\left(i_{t+1}^r-i_{t+1}^d\right)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^m \tag{A.5e}$$

where $\Omega_{t+1}=1-\sigma+\sigma\phi_{t+1}$ and $\Phi_t\equiv\kappa[1+(1-\tau_t^\pi)\,i_t^b]$.

Aggregation: The total net worth of private banks in the beginning of period t is now given by:

$$N_{p,t}^T=\sigma n_{p,t}^T+\gamma\frac{p_{t-1}}{p_t}\left\{L_{p,t-1}^T+\Lambda_t b_{p,t-1}^T+m_{p,t-1}^T\right\} \tag{A.5f}$$

where the net worth of banks that stay in the market, $\eta_{p,t}^T$, is:

$$n_{p,t}^T=\frac{p_{t-1}}{p_t}\{(1-\tau_t^\pi)\,(i_t^l-i_t^d)L_{p,t-1}^T+(1-\tau_t^\pi)\,(i_t^r-i_t^d)m_{p,t-1}^T+$$

$$+\left[\Lambda_t\left(1+(1-\tau_t^\pi)\,i_t^b\right)+(1-\Lambda_t)\,\Phi_t-\left(1+(1-\tau_t^\pi)\,i_t^d\right)\right]b_{p,t-1}^T-$$

$$-(1-\tau_t^\pi)(i_t^z-i_t^d)z_{p,t-1}^T+(1+(1-\tau_t^\pi)i_t^d)n_{p,t-1}^T\} \tag{A.5g}$$

The rest of equations are as in Appendix A.4.

A.6 Macroeconomic system (when the CB participates in the primary bond market)

A.6.1 Market-clearing conditions

In the market for dividends:

$$\pi_{h,t}=\pi_{i,t}+\pi_{p,t}-\gamma(L_{p,t-1}+b_{p,t-1}+m_{p,t-1}) \tag{A.6.1.1}$$

In the labor market:

$$l_{h,t}=l_{i,t}=l_t \tag{A.6.1.2}$$

In the market for bank deposits:

$$j_{h,t} = j_{p,t}^{T} = j_t \tag{A.6.1.3}$$

In the market for bank loans:

$$L_{i,t} = L_{p,t}^{T} = L_t \tag{A.6.1.4}$$

In the primary bond market:

$$b_{p,t}^{T} + b_{cb,t} = b_t \tag{A.6.1.5}$$

where $b_{cb,t} = (1 - \Lambda_t)\, b_t$.

A.6.2 Equations and unknowns

Collecting equations, the macroeconomic system that we solve numerically consists of the following equations:

Households

$$\frac{1}{c_{h,t}^{\mu_1}} = \lambda_{h,t}(1 + \tau_t^c) \tag{A.6.2.1}$$

$$\chi^l l_t^{\mu_2} = \lambda_{h,t}(1 - \tau_t^y) w_t \tag{A.6.2.2}$$

$$\lambda_{h,t} = \beta \lambda_{h,t+1}(1 + i_{t+1}^d) \frac{p_t}{p_{t+1}} \tag{A.6.2.3}$$

$$(1 + \tau_t^c) c_{h,t} + j_t \equiv$$

$$\equiv (1 - \tau_t^y) w_t l_t + (1 + i_t^d) \frac{p_{t-1}}{p_t} j_{t-1} + \pi_{h,t} + g_t^t \tag{A.6.2.4}$$

Firms In a symmetric equilibrium, $y_{f,t} = y_{i,t} \equiv y_t$, $k_{i,t} \equiv k_t$ and $p_{i,t} = p_t$. Thus,

$$\pi_{i,t} = (1 - \tau_t^{\pi})(y_t - w_t l_t) - x_t - \frac{\xi^p}{2} \left(\frac{p_t}{p_{t-1}} - 1 \right)^2 y_t + \left(L_t - \left(1 + i_t^l\right) \frac{p_{t-1}}{p_t} L_{t-1} \right) \tag{A.6.2.5}$$

$$k_t = x_t + (1 - \delta)\, k_{t-1} \tag{A.6.2.6}$$

$$y_t = A \left(k_{t-1}^g\right)^{\varepsilon} (k_{t-1}^{\alpha} l_t^{1-\alpha})^{1-\varepsilon} \tag{A.6.2.7}$$

$$(1-\tau_t^{\pi})w_t + N_{i,t}\nu^l w_t = [(1-\tau_t^{\pi})\theta - \xi^p\left(\frac{p_t}{p_{t-1}}-1\right)\frac{p_t}{p_{t-1}}(\theta-1)+$$

$$+\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t}-1\right)\frac{p_{t+1}}{p_t}\frac{(\theta-1)y_{t+1}}{y_t}]\frac{\partial y_t}{\partial l_t} \tag{A.6.2.8}$$

$$1=\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}[1-\delta+(1-\tau_{t+1}^{\pi})\theta\frac{\partial y_{t+1}}{\partial k_t}]-$$

$$-\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t}-1\right)\frac{p_{t+1}}{p_t}(\theta-1)\frac{\partial y_{t+1}}{\partial k_t}+$$

$$+\frac{\beta^2\lambda_{h,t+2}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+2}}{p_{t+1}}-1\right)\frac{p_{t+2}}{p_{t+1}}(\theta-1)\frac{y_{t+2}}{y_{t+1}}\frac{\partial y_{t+1}}{\partial k_t} \tag{A.6.2.9}$$

$$1+N_{i,t}=\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\left(1+i_{t+1}^l\right)\frac{p_t}{p_{t+1}} \tag{A.6.2.10}$$

$$N_{i,t}\left(L_t-\nu^l w_t l_t)\right)=0 \tag{A.6.2.11}$$

Private banks

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\left(1-\tau_{t+1}^{\pi}\right)\frac{p_t}{p_{t+1}}(i_{t+i}^l - i_{t+i}^d)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta \tag{A.6.2.12}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\left(1-\tau_{t+1}^{\pi}\right)\frac{p_t}{p_{t+1}}(i_{t+i}^b - i_{t+i}^d)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^b \tag{A.6.2.13}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^z-i_{t+1}^d\right)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^z \tag{A.6.2.14}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau_{t+1}^{\pi}\right)\left(i_{t+1}^r-i_{t+1}^d\right)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^m \tag{A.6.2.15}$$

$$V_{p,t}^T=\phi_t N_{p,t}^T \tag{A.6.2.16}$$

$$V_{p,t}^T=\vartheta(L_t+N^b\Lambda_t b_t+N^m m_{p,t}^T-N^z z_{p,t}^T) \tag{A.6.2.17}$$

$$n_{p,t}^T=\frac{p_{t-1}}{p_t}\{(1-\tau_t^{\pi})(i_t^l-i_t^d)L_{t-1}+(1-\tau_t^{\pi})\left(i_t^b-i_t^d\right)\Lambda_{t-1}b_{t-1}+$$

$$+(1-\tau_t^{\pi})\,(i_t^r - i_t^d)m_{p,t-1}^T - (1-\tau_t^{\pi})\,(i_t^z - i_t^d)z_{p,t-1}^T +$$

$$+\left[1+(1-\tau_t^{\pi})i_t^d\right] n_{p,t-1}^T\} \tag{A.6.2.18}$$

$$N_{p,t}^T = \sigma n_{p,t}^T + \gamma \frac{p_{t-1}}{p_t}\left\{L_{t-1} + \Lambda_{t-1}b_{t-1} + m_{p,t-1}^T\right\} \tag{A.6.2.19}$$

$$L_t + \Lambda_t b_t + m_{p,t}^T = j_t + z_{p,t}^T + N_{p,t}^T \tag{A.6.2.20}$$

$$\pi_{p,t}^T = (1-\sigma)\, n_{p,t}^T \tag{A.6.2.21}$$

$$\phi_t = (1+\zeta_t)\,\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[1+\left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^d\right] \tag{A.6.2.22}$$

$$\Omega_t = 1-\sigma+\sigma\phi_t \tag{A.6.2.23}$$

Treasury

$$g_t^c + g_t^g + g_t^t + (1+i_t^b)\frac{p_{t-1}}{p_t}b_{t-1} = b_t + t_t^{cb} + t_t^{tax} \tag{A.6.2.24}$$

$$t_t^{tax} \equiv \tau_t^c c_{h,t} + \tau_t^y w_t l_t + \tau_t^{\pi}(y_t - w_t l_t) +$$

$$+\tau_t^{\pi}\frac{p_{t-1}}{p_t}(i_t^l L_{t-1} + i_t^r m_{p,t-1}^T + i_t^b \Lambda_{t-1} b_{t-1} -$$

$$-i_t^z z_{p,t-1}^T - i_t^d j_{t-1}) \tag{A.6.2.25}$$

$$k_t^g = (1-\delta^g)k_{t-1}^g + g_t^g \tag{A.6.2.26}$$

Central Bank

$$(1-\Lambda_t)b_t + z_{p,t}^T + (1+i_t^r)\,\frac{p_{t-1}}{p_t}m_{p,t-1}^T + t_t^{cb} \equiv$$

$$\equiv (1+i_t^b)(1-\Lambda_{t-1})\frac{p_{t-1}}{p_t}b_{t-1} + (1+i_t^z)\frac{p_{t-1}}{p_t}z_{p,t-1}^T + m_{p,t}^T \tag{A.6.2.27}$$

Dividends

$$\pi_{h,t} = \pi_{i,t} + \pi_{p,t} - \gamma \frac{p_{t-1}}{p_t} \left\{ L_{t-1} + \Lambda_{t-1} b_{t-1} + m_{p,t-1}^T \right\} \qquad \text{(A.6.2.28)}$$

Endogenous and exogenous variables This is a dynamic system of 28 equations in 28 variables which are $\{c_{h,t},\, j_t,\, l_t,\, \pi_{h,t}\}_{t=0}^{\infty}$, $\{\lambda_{h,t},\, N_{i,t}\}_{t=0}^{\infty}$, $\{\pi_{i,t},\, y_t,\, x_t,\, k_t,\, L_t\}_{t=0}^{\infty}$, $\{\pi_{p,t}^T,\, z_{p,t}^T,\, m_{p,t}^T,\, V_{p,t}^T,\, \zeta_t,\, n_{p,t}^T,\, N_{p,t}^T,\, \phi_t,\, \Omega_t\}_{t=0}^{\infty}$, $\{t_t^{tax}\}_{t=0}^{\infty}$, $\{b_t\}_{t=0}^{\infty}$, $\{k_{g,t}^g\}_{t=0}^{\infty}$, $\{p_t/p_{t-1},\, i_t^b,\, i_t^d,\, i_t^l,\, w_t\}_{t=0}^{\infty}$. This is given the paths/rules of fiscal policy instruments, $\{\tau_t^c,\, \tau_t^y,\, \tau_t^{\pi},\, s_t^c,\, s_t^g,\, s_t^t,\}_{t=0}^{\infty}$ and monetary policy instruments, $\{i_t^z,\, i_t^r,\, t_t^{cb},\, (1-\Lambda_t)\}_{t=0}^{\infty}$. In the steady state only, b_t and s_t^t change places.

A.7 Macroeconomic system (when the CB participates in the secondary bond market)

A.7.1 Market-clearing conditions

The only market clearing that changes relative to above is the one referring to government bonds in the primary market, which now is:

$$b_{p,t}^T \equiv b_t \qquad \text{(A.7.1.1)}$$

A.7.2 Equations and unknowns

Collecting equations, the macroeconomic system that we solve numerically consists of the following equations:

Households

$$\frac{1}{c_{h,t}^{\mu_1}} = \lambda_{h,t}(1+\tau_t^c) \qquad \text{(A.7.2.1)}$$

$$\chi^l l_t^{\mu_2} = \lambda_{h,t}(1-\tau_t^y) w_t \qquad \text{(A.7.2.2)}$$

$$\lambda_{h,t} = \beta \lambda_{h,t+1}(1+i_{t+1}^d)\frac{p_t}{p_{t+1}} \qquad \text{(A.7.2.3)}$$

$$(1+\tau_t^c)c_{h,t} + j_t \equiv$$

$$\equiv (1-\tau_t^y) w_t l_t + (1+i_t^d)\frac{p_{t-1}}{p_t} j_{t-1} + \pi_{h,t} + g_t^t \qquad \text{(A.7.2.4)}$$

Firms

$$\pi_{i,t} = (1-\tau_t^{\pi})(y_t - w_t l_t) - x_t - \frac{\xi^p}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 y_t + \left(L_t - \left(1 + i_t^l\right)\frac{p_{t-1}}{p_t} L_{t-1}\right) \tag{A.7.2.5}$$

$$k_t = x_t + (1-\delta)\, k_{t-1} \tag{A.7.2.6}$$

$$y_t = A\left(k_{t-1}^g\right)^{\varepsilon}(k_{t-1}^{\alpha} l_t^{1-\alpha})^{1-\varepsilon} \tag{A.7.2.7}$$

$$(1-\tau_t^{\pi})w_t + N_{i,t}\nu^l w_t = [(1-\tau_t^{\pi})\theta - \xi^p\left(\frac{p_t}{p_{t-1}} - 1\right)\frac{p_t}{p_{t-1}}(\theta-1) + \\ + \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t}\frac{(\theta-1)y_{t+1}}{y_t}]\frac{\partial y_t}{\partial l_t} \tag{A.7.2.8}$$

$$1 = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}[1-\delta+(1-\tau_{t+1}^{\pi})\theta\frac{\partial y_{t+1}}{\partial k_t}] - \\ - \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+1}}{p_t} - 1\right)\frac{p_{t+1}}{p_t}(\theta-1)\frac{\partial y_{t+1}}{\partial k_t} + \\ + \frac{\beta^2\lambda_{h,t+2}}{\lambda_{h,t}}\xi^p\left(\frac{p_{t+2}}{p_{t+1}} - 1\right)\frac{p_{t+2}}{p_{t+1}}(\theta-1)\frac{y_{t+2}}{y_{t+1}}\frac{\partial y_{t+1}}{\partial k_t} \tag{A.7.2.9}$$

$$1 + N_{i,t} = \frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\left(1 + i_{t+1}^l\right)\frac{p_t}{p_{t+1}} \tag{A.7.2.10}$$

$$N_{i,t}\left(L_t - \nu^l w_t l_t)\right) = 0 \tag{A.7.2.11}$$

Private banks

$$\Phi_t = \kappa\left[1 + (1-\tau_t^{\pi})i_t^b\right] \tag{A.7.2.12}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\left(1-\tau_{t+1}^{\pi}\right)\frac{p_t}{p_{t+1}}(i_{t+i}^l - i_{t+i}^d) = \frac{\zeta_t}{(1+\zeta_t)}\vartheta \tag{A.7.2.13}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left[\Lambda_{t+1}(1 + \left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^b) + \Phi_{t+1}\left(1-\Lambda_{t+1}\right) - (1 + \left(1-\tau_{t+1}^{\pi}\right)i_{t+1}^d)\right] = \frac{\zeta_t}{(1+\zeta_t)}\vartheta N^b \tag{A.7.2.14}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^{\pi}_{t+1}\right)\left(i^{z}_{t+1}-i^{d}_{t+1}\right)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^{z} \tag{A.7.2.15}$$

$$\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}\left(1-\tau^{\pi}_{t+1}\right)\left(i^{r}_{t+1}-i^{d}_{t+1}\right)=\frac{\zeta_t}{(1+\zeta_t)}\vartheta N^{m} \tag{A.7.2.16}$$

$$V^{T}_{p,t}=\phi_t N^{T}_{p,t} \tag{A.7.2.17}$$

$$V^{T}_{p,t}=\vartheta(L_t+N^{b}b_t+N^{m}m^{T}_{p,t}-N^{z}z^{T}_{p,t}) \tag{A.7.2.18}$$

$$n^{T}_{p,t}=\frac{p_{t-1}}{p_t}\{(1-\tau^{\pi}_t)(i^{l}_t-i^{d}_t)L_{t-1}+(1-\tau^{\pi}_t)(i^{r}_t-i^{d}_t)m^{T}_{p,t-1}+$$

$$+\left[\Lambda_t\left(1+(1-\tau^{\pi}_t)\,i^{b}_t\right)+(1-\Lambda_t)\,\Phi_t-\left(1+(1-\tau^{\pi}_t)\,i^{d}_t\right)\right]b_{t-1}-$$

$$-(1-\tau^{\pi}_t)(i^{z}_t-i^{d}_t)z^{T}_{p,t-1}+\left(1+(1-\tau^{\pi}_t)\,i^{d}_t\right)n^{T}_{p,t-1}\} \tag{A.7.2.19}$$

$$N^{T}_{p,t}=\sigma n^{T}_{p,t}+\gamma\frac{p_{t-1}}{p_t}\left\{L_{t-1}+\Lambda_t b_{t-1}+m^{T}_{p,t-1}\right\} \tag{A.7.2.20}$$

$$L_t+b_t+m^{T}_{p,t}=j_t+z^{T}_{p,t}+N^{T}_{p,t} \tag{A.7.2.21}$$

$$\pi^{T}_{p,t}=(1-\sigma)\,n^{T}_{p,t} \tag{A.7.2.22}$$

$$\phi_t=(1+\zeta_t)\,\frac{\beta\lambda_{h,t+1}}{\lambda_{h,t}}\Omega_{t+1}\frac{p_t}{p_{t+1}}(1+\left(1-\tau^{\pi}_{t+1}\right)i^{d}_{t+1}) \tag{A.7.2.23}$$

$$\Omega_t=1-\sigma+\sigma\phi_t \tag{A.7.2.24}$$

Treasury

$$g^{c}_t+g^{g}_t+g^{t}_t+(1+i^{b}_t)\frac{p_{t-1}}{p_t}b_{t-1}=b_t+t^{cb}_t+t^{tax}_t \tag{A.7.2.25}$$

$$t^{tax}_t\equiv\tau^{c}_t c_{h,t}+\tau^{y}_t w_t l_t+\tau^{\pi}_t(y_t-w_t l_t)+$$

$$+\tau^{\pi}_t\frac{p_{t-1}}{p_t}(i^{l}_t L_{t-1}+i^{r}_t m^{T}_{p,t-1}+\Lambda_t i^{b}_t b_{t-1}-$$

$$-i^{z}_t z^{T}_{p,t-1}-i^{d}_t j_{t-1}) \tag{A.7.2.26}$$

$$k^{g}_t=(1-\delta^{g})k^{g}_{t-1}+g^{g}_t \tag{A.7.2.27}$$

Central Bank

$$\Phi_t(1-\Lambda_t)\frac{p_{t-1}}{p_t}b_{t-1}+z^T_{p,t}+(1+i^r_t)\,\frac{p_{t-1}}{p_t}m^T_{p,t-1}+t^{cb}_t\equiv$$

$$\equiv(1-\Lambda_t)(1+i^b_t)\frac{p_{t-1}}{p_t}b_{t-1}+(1+i^z_t)\frac{p_{t-1}}{p_t}z^T_{p,t-1}+m^T_{p,t} \tag{A.7.2.28}$$

Dividends

$$\pi_{h,t}=\pi_{i,t}+\pi_{p,t}-\gamma\frac{p_{t-1}}{p_t}\left\{L_{t-1}+\Lambda_t b_{t-1}+m^T_{p,t-1}\right\} \tag{A.7.2.29}$$

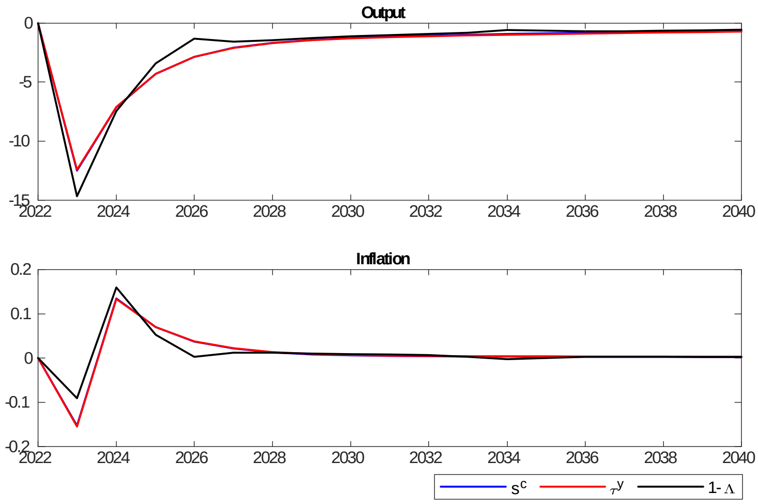
Endogenous and exogenous variables We therefore have a dynamic system of 29 equations in 29 variables which are $\{c_{h,t},\ j_t,\ l_t,\ \pi_{h,t}\}_{t=0}^{\infty}$, $\{\lambda_{k,t},\ N_{i,t}\}_{t=0}^{\infty}$, $\{\pi_{i,t},\ y_t,\ x_t,\ k_t,\ L_t\}_{t=0}^{\infty}$, $\{\pi^T_{p,t},\ z^T_{p,t},\ m^T_{p,t},\ V^T_{p,t},\ \zeta_t,\ \eta^T_{p,t},\ N^T_{p,t},\ \phi_t,\ \Omega_t\}_{t=0}^{\infty}$, $\{t^{tax}_t\}_{t=0}^{\infty}$, $\{b_t\}_{t=0}^{\infty}$, $\left\{k^g_{g,t}\right\}_{t=0}^{\infty}$, $\{p_t/p_{t-1},\ i^b_t,\ i^d_t,\ i^l_t,\ w_t,\ \Phi_t\}_{t=0}^{\infty}$. This is given the paths/rules of fiscal policy instruments, $\{\tau^c_t,\ \tau^y_t,\ \tau^\pi_t,\ s^c_t,\ s^g_t,\ s^t_t,\}_{t=0}^{\infty}$ and monetary policy instruments, $\{i^z_t,\ i^r_t,\ t^{cb}_t,\ (1-\Lambda_t)\}_{t=0}^{\infty}$. In the steady state only, b_t and s^t_t change places.

A.8 A counterfactual set of assumptions

If we assume that there are no loans from the CB to private banks, that the path of public debt is exogenous, and that the CB participates in the primary sovereign bond market (this is the set of assumptions employed by Sims and Wu (2021) for the US and Coenen et al (2018) for the ES), then the real effects of QE policies become stronger and also different policy instruments affect the path of inflation differently. The impulse response functions are shown in Graph A.8.[51] Notice that now inflation is higher under QE in the short-term, although this is reversed in the medium run as the economy returns to its initial steasdy state.

[51] Here, $\gamma^{\tau^y,b}=0.01$, $\gamma^{\Lambda,b}=0.3$ and $\gamma^{r,\pi}=\gamma^{z,\pi}=1.01$, at all $t\geq 0$.

Graph A.8
Inflation and output under alternative debt stabilzation policies
(% deviation from steady state)



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