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the euro-area natural rate of interest

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THE ECONOMETRICS OF THE EURO-AREA NATURAL RATE OF INTEREST

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ABSTRACT

The natural rate of interest (r^*) plays a central role in monetary policy analysis, but its estimation is inherently model dependent. This paper studies how alternative econometric specifications affect the estimates of r^* in the euro-area. We propose a Hybrid Common Trend Vector Autoregression that incorporates a large information set, stochastic volatility, and fat-tailed innovations. Using euro-area data spanning the last 25 years, we find that these features materially improve inference, reducing the width of 95% credible intervals for r^* by as much as 100 basis points relative to conventional specifications. Our results point to a persistent decline in the natural rate between 2000 and 2020, followed by a moderate post-pandemic recovery. The decomposition of r^* identifies the trend of labor productivity growth as the dominant driver of its long-run movements. The findings suggest that uncertainty surrounding r^* is partly econometric in nature and can be reduced through richer empirical specifications.

Keywords: Real interest rates, labor productivity growth trend, large Bayesian vector autoregressions, stochastic volatility, fat tails.

JEL classification: E43; E52; C11; C32;

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1 Introduction

The natural rate of interest, commonly denoted as r^* , is defined as the real short-term interest rate consistent with stable inflation and output at its potential level. As such, it plays a central role in modern monetary policy analysis and is widely used as a benchmark for assessing the stance of monetary policy. Deviations between observed real interest rates and r^* are commonly interpreted as indicators of accommodative or restrictive policy conditions (Laubach and Williams, 2003). In inflation-targeting regimes, such as that of the European Central Bank (ECB), estimates of r^* constitute an important input into policy deliberations and economic projections.¹

The practical importance—and inherent uncertainty—surrounding r^* was underscored by Philip Lane, Chief Economist of the ECB, who remarked in June 2026:

“...we look at a range of models of neutral, and the upper end of that range we think has crept up ...”

This statement reflects the fact that there is no single, directly observable measure of r^* . Instead, it must be inferred from econometric models, implying that assessments of the monetary policy stance depend critically on the modeling framework used to estimate it.

A large literature has proposed alternative approaches for estimating r^* . These include structural and semi-structural state-space models (Laubach and Williams, 2003; Holston et al., 2017; Cesa-Bianchi et al., 2022), time-series approaches (Lubik and Matthes, 2015; Del Negro et al., 2017, 2019), DSGE-based frameworks (Del Negro et al., 2017), term-structure models (Hördahl and Tristani, 2014; Bauer and Rudebusch, 2020; Brand et al., 2025), and survey-based measures Benigno et al. (2024). These methodologies, and numerous extensions thereof, have been applied extensively to the euro-area (see, e.g., Brand et al., 2018; Fiorentini et al., 2018; Brand and Mazelis, 2019; Carvalho, 2023; Christensen and Mouabbi, 2024; Brand et al., 2024). Although most studies document a substantial decline in r^* over recent decades, the resulting estimates often differ considerably across models, pointing to substantial model uncertainty (Brand et al., 2025).

¹See also Nagel (2025) for a discussion of the role of r^* in monetary policy deliberations.

The importance of model uncertainty has become even more pronounced in the aftermath of the global financial crisis and the COVID-19 pandemic. These episodes were characterized by heightened macroeconomic uncertainty, abrupt shifts in economic conditions, and unusually large shocks. Yet many empirical frameworks used to estimate r^* rely on linear Gaussian specifications with constant volatility and relatively limited information sets. Such assumptions may be restrictive during periods of elevated uncertainty and may affect both the estimated level of r^* and the precision with which it is measured. In particular, time-varying volatility and infrequent but economically significant shocks may distort inference when not explicitly incorporated into the econometric framework (Bobeica and Hartwig, 2023; Carriero et al., 2024). Consequently, estimates of r^* may depend not only on economic fundamentals but also on the treatment of volatility, extreme observations, and the information set employed.

Only a limited number of studies have explicitly examined these issues. For example, Holston et al. (2023) extend their original framework by allowing for time-varying shock variances in order to account for the effects of the COVID-19 pandemic and mitigate the influence of outlier observations. Similarly, Lubik and Matthes (2015) propose a flexible time-varying parameter vector autoregression with stochastic volatility, while Lubik and Matthes (2023) subsequently recalibrate the priors of this framework to constrain excessive parameter drift during the highly volatile post-pandemic period. In Del Negro et al. (2019), the authors acknowledge that their common-trend VAR model with constant volatility may be misspecified and suggest that stochastic volatility could provide a more suitable specification. Taken together, these contributions suggest that estimates of r^* may benefit from richer econometric frameworks that explicitly accommodate key features of macroeconomic data.

At the same time, understanding the structural determinants of r^* remains essential for explaining its long-run evolution and assessing its future trajectory. The secular decline in real interest rates observed across advanced economies over the four decades preceding the pandemic has been attributed to a variety of structural forces that increased desired saving and reduced desired investment (Benigno et al., 2024). Proposed explanations include slower productivity growth, demographic changes associated with population

aging, elevated global saving rates—particularly in emerging economies—and secular stagnation driven by persistently weak investment demand (Bernanke, 2005; Eichengreen, 2015; Gordon, 2015; Carvalho et al., 2016; Rachel and Summers, 2019; Gagnon et al., 2021; Cesa-Bianchi et al., 2022). In addition, a growing demand for safe assets has been identified as an important factor behind lower equilibrium interest rates, particularly following the global financial crisis (Caballero et al., 2017; Del Negro et al., 2017, 2019). Disentangling the relative importance of these factors remains one of the central challenges in the empirical literature on r^* .

Against this background, this paper examines how econometric specification affects the estimation of the euro-area natural rate of interest and its underlying drivers. To this end, we develop a Hybrid Common Trend Vector Autoregression (HCT-VAR) that combines four key features. First, following Del Negro et al. (2017), a subset of core macroeconomic variables is decomposed into permanent and transitory components, allowing r^* and its determinants to be identified from their long-run trends. Second, following Louzis (2026), the remaining variables are incorporated through a hybrid specification in which they fluctuate around constant or time-varying means. Third, stochastic volatility is introduced to account for time-varying macroeconomic uncertainty. Fourth, innovations are allowed to follow fat-tailed distributions to accommodate extreme observations and outliers. Combined with efficient band-matrix estimation algorithms (see, e.g., Chan and Jeliazkov, 2009), this framework permits the inclusion of a relatively large information set comprising more than 20 macroeconomic and financial variables. As a result, the proposed approach combines the economic interpretability of common-trend models with the information-rich environment of large Bayesian VARs (see, e.g., Chan, 2020b). To the best of our knowledge, this is the first study to estimate r^* within a unified framework that jointly incorporates common trends, a hybrid specification, stochastic volatility, fat-tailed innovations, and a large information set.

From an economic perspective, the richer information set allows for a more detailed analysis of the determinants of r^* . In particular, and following Antolin-Diaz et al. (2017), we explicitly incorporate labor productivity growth as a structural driver of the natural rate by decomposing long-run output and consumption growth into labor productivity

and labor input components. This approach differs from much of the existing literature, where productivity is typically proxied by aggregate output or consumption growth. Consequently, the proposed framework provides a unified setting for jointly studying the long-run determinants of economic growth and the natural rate of interest.

Using euro-area data spanning the period 2000–2025, we estimate a family of models that differ with respect to the information set, volatility specification, and distributional assumptions. Four main findings emerge. First, econometric specification matters for inference on r^* . Models incorporating stochastic volatility, fat-tailed innovations, and a broader information set generate substantially tighter credible intervals than conventional Gaussian specifications, reducing the width of 95% credible intervals by up to 100 basis points. Second, all specifications point to a pronounced decline in the euro-area natural rate between 2000 and 2020, followed by a moderate recovery thereafter. Third, labor productivity growth emerges as the dominant driver of the long-run decline in r^* , whereas labor input and financial factors play a comparatively smaller role. Fourth, differences in estimated values of r^* translate into economically meaningful differences in the assessment of the monetary policy stance, highlighting the importance of model uncertainty for policy analysis.

The contribution of this paper is therefore twofold. Methodologically, it demonstrates that richer econometric specifications can materially improve inference on r^* . Substantively, it provides new evidence on the evolution and determinants of the euro-area natural rate of interest. More broadly, the results suggest that uncertainty surrounding r^* is not solely an economic phenomenon but also an econometric one, and that part of this uncertainty can be reduced through more flexible empirical frameworks.

The remainder of the paper is organized as follows. Section 2 outlines the theoretical framework. Section 3 presents the econometric model and identification strategy. Section 4 discusses the empirical results and policy implications. Finally, section 5 concludes.

2 Theoretical framework

Our analysis on r^* starts from an Euler equation augmented with a convenience yield for a safe and liquid short-term government security (Del Negro et al., 2017):

$$1 = \mathbb{E}_t [(1 + r_t)(1 + CY_{t+1})M_{t+1}] \quad (1)$$

where r_t is the security's net real return, M_{t+1} is the stochastic discount factor (SDF) and CY_{t+1} denotes the convenience yield, i.e. a non-pecuniary premium investors accept for the superior safety and liquidity of Treasury securities over otherwise identical assets.² A key implication of (1) is that, *ceteris paribus*, an increase in convenience yield or the SDF must be accompanied by a decrease in the safe real interest rate, r_t , to maintain the equality, and vice versa.

Since the r^* is the short-term real rate in a frictionless economy, it can be well approximated by the long-term trend of the observed real short-term rate. The latter captures the rate's secular movements in the long run when the economy operates at its potential. There is no reason to believe that the pricing mechanism in (1) ceases to hold over the long-run, where transitory shocks average out. We therefore posit that r^* is determined by the long-term trends in the convenience yield and the SDF. Taking logs yields:

$$r_t^* = \overline{m}_t - \overline{cy}_t \quad (2)$$

where $\overline{m}_t = -\log(\overline{M}_t)$, $\overline{cy}_t = \log(1 + \overline{CY}_t)$ and $r_t^* = \log(1 + \overline{r}_t)$.³

Equation (2) is the cornerstone of our empirical analysis of r^* . Because its components are unobservable, our modeling strategy faces two challenges: (i) selecting suitable variables to identify r^* and its drivers, and (ii) separating their trend and transitory components efficiently. Section 3 provides details on both issues.

²In the standard consumption model the SDF is typically defined as $M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$, which is the marginal rate of substitution between consumption today and consumption in the future, where $\beta \leq 1$ is a subjective discount factor.

³Throughout the following analysis, a variable with an overline, $\overline{x}_t$, represents the long-term trend of x_t . The standard logarithmic approximation $\log(1 + x) \approx x$ is applied where appropriate.

3 Empirical framework

This section presents the econometric framework used to identify and estimate r^* and its drivers. In particular, section 3.1 introduces the Hybrid Common Trend VAR (HCT-VAR) model, following Louzis (2026), that is robust to outliers and accommodates large information sets—a key feature of modern macroeconometrics—thereby reducing the risk of unstable parameter estimates compared to standard small-scale VARs with Gaussian innovations (see, e.g., Bobeica and Hartwig, 2023). Section 3.2 describes the decomposition of r^* into its drivers, with particular emphasis on decomposing $\bar{m}_t$ to isolate the contribution of long-run productivity.

3.1 Hybrid Common Trend VAR

HCT-VAR model allows the unconditional means of some variables to evolve over time while remaining constant for others. This parsimonious structure substantially reduces the number of parameters relative to models with fully time-varying trends (see, e.g., Louzis, 2019), facilitating the estimation of r^* in large datasets while mitigating concerns about omitted-variable bias and preserving low-frequency macroeconomic dynamics. The model further incorporates common stochastic volatility (CSV) and Student’s t -distributed errors in order to capture time-varying macroeconomic volatility and extreme observations, such as those observed during the COVID-19 period.⁴ The HCT-VAR is defined as:

$$\mathbf{y}_t = \boldsymbol{\tau}_t + \sum_{l=1}^p \mathbf{B}_l (\mathbf{y}_{t-l} - \boldsymbol{\tau}_{t-l}) + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \lambda_t \exp(h_t) \boldsymbol{\Sigma}) \quad (3)$$

$$\boldsymbol{\tau}_t = \mathbf{S}_\gamma \boldsymbol{\gamma} + \mathbf{S}_\mu \boldsymbol{\mu}_t \quad (4)$$

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\varepsilon}_t^\mu, \quad \boldsymbol{\varepsilon}_t^\mu \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \quad (5)$$

$$h_t = h_{t-1} + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \kappa^2) \quad (6)$$

$$\lambda_t | \nu \sim \mathcal{IG}(\nu/2, \nu/2) \quad (7)$$

⁴See (Chan, 2019, 2020a) for details on common stochastic volatility and fat-tailed error distributions in large VAR models.

where, $\mathbf{y}_t = (y_{1,t}, \dots, y_{n,t})'$ is an $n \times 1$ vector of endogenous variables and $\mathbf{B}_1, \dots, \mathbf{B}_p$ are autoregressive coefficient matrices. The factor h_t is a common log-volatility component evolving as a random walk in (F.4), while the inverse-gamma latent variable λ_t implies that $\boldsymbol{\varepsilon}_t$ is marginally Student's t -distributed with ν degrees of freedom.

The unconditional mean, $\mathbb{E}(\mathbf{y}_t) = \boldsymbol{\tau}_t$, is decomposed into a time-invariant component $\boldsymbol{\gamma} \in \mathbb{R}^{n_\gamma}$ and a time-varying component $\boldsymbol{\mu}_t \in \mathbb{R}^{n_\mu}$. The dimension n_γ determines the number of variables with constant means, while n_μ denotes the number of (common) stochastic trends. In general, $n_\mu \neq n - n_\gamma$ because multiple variables may load on the same trend. Selection matrices $\mathbf{S}_\gamma \in \mathbb{R}^{n \times n_\gamma}$ and $\mathbf{S}_\mu \in \mathbb{R}^{n \times n_\mu}$ determine which variables have constant or time-varying means and how variables load on the common trends. Because $\mathbf{S}_\mu$ identifies r^* and its drivers (see section 3.2), we also refer to it as the *identification matrix*.

The trend component $\boldsymbol{\mu}_t$ follows the driftless random walk in (F.3) with covariance matrix $\mathbf{Q}$. We further assume that the shocks in (F.1), (F.3), and (F.4) are mutually orthogonal. Under this assumption, $\mathbf{y}_t$ can be decomposed into a stochastic trend, $\boldsymbol{\tau}_t$, and a stationary (cyclical) component, $\tilde{\mathbf{y}}_t = \mathbf{y}_t - \boldsymbol{\tau}_t$ (see section 3.2 and Del Negro et al. (2017)).

3.1.1 Estimation

We estimate the model using Bayesian methods, relying on the band-matrix algorithms for the HCT-VAR model developed by Louzis (2026).⁵ This approach avoids the computational burden of the standard Kalman filtering framework and enables efficient estimation of large-scale systems with time-varying unconditional means.⁶ Full details of the prior specification and the Markov chain Monte Carlo (MCMC) algorithm are provided in section A of the Online Appendix.

3.2 Identifying r^* and its drivers in a data-rich environment

This study uses euro-area data divided into three categories. The first comprises a core set of variables with time-varying unconditional means that identify r^* and its fundamental

⁵Louzis (2026) derive the band-matrix algorithm for a closely related model that includes an additional measurement equation for inflation expectations.

⁶For an alternative estimation approach based on the Kalman filter, see Louzis (2019).

drivers. The remaining categories contain auxiliary variables with time-varying and constant unconditional means. Although not required for identification, these variables are included through Bayesian shrinkage to improve filtering and estimation precision. We focus here on the core variables.

To facilitate the discussion of variable selection, we first decompose each variable $y_{i,t}$ into trend and stationary components. We define:

- $\bar{y}_{it} = \tau_{i,t}$, where $\tau_{i,t}$ is the i -th element of $\boldsymbol{\tau}_t$. This is the slow-moving trend component, modeled as in (F.2) and (F.3) and
- $\tilde{y}_{i,t} = \sum_{l=1}^p \mathbf{b}_{i,l} (\mathbf{y}_{i,t-l} - \boldsymbol{\tau}_{i,t-l}) + \varepsilon_{i,t}$, where $\mathbf{b}_{i,l}$ the i -th row of the $\mathbf{B}_l$ coefficient matrix in (F.1), for $i = 1, \dots, n$. This represents the stationary cyclical component.

This decomposition allows us to rewrite each equation in (F.1) concisely as:

$$y_{i,t} = \bar{y}_{i,t} + \tilde{y}_{i,t} \quad (8)$$

3.2.1 Extracting r^* and its fundamental drivers: core variables

In principle, r^* can be identified as the slow-moving component of a safe and liquid short-term real rate. For the euro-area, we use the 3-month Euribor (r_t^{3m}) as the benchmark rate. Using the decomposition in (8), we write:

$$r_t^{3m} = \bar{r}_t^{3m} + \tilde{r}_t^{3m} \quad (9a)$$

$$= \underbrace{(r_t^* + \bar{\pi}_t)}_{\bar{r}_t^{3m}} + \tilde{r}_t^{3m} \quad (9b)$$

$$= \underbrace{[(\bar{m}_t - \bar{c}y_t)]}_{r_t^*} + \bar{\pi}_t + \tilde{r}_t^{3m} \quad (9c)$$

$$= \underbrace{[(\bar{l}p_t + \bar{h}r_t - \bar{c}y_t + \bar{r}o_t)]}_{r_t^*} + \bar{\pi}_t + \tilde{r}_t^{3m} \quad (9d)$$

The slow-moving component of the 3-month Euribor, $\bar{r}_t^{3m}$, is decomposed into the natural rate of interest, r_t^* , and trend inflation, $\bar{\pi}_t$, in (9b), assuming the Fisher equation holds in the long run. Equation (9c) further decomposes r_t^* into the trend components of the SDF,

$\overline{m}_t$, and the convenience yield, $\overline{cy}_t$, as implied by section 2. Unlike existing studies, (9d) further decomposes $\overline{m}_t$ into the long-run trends of labor productivity growth, $\overline{lp}_t$, and total hours worked, $\overline{hr}_t$. The residual component, $\overline{ro}_t$, captures any remaining determinants of r_t^* .

Our objective is to estimate (9d) and conduct inference on r_t^* and its drivers. To this end, we first select observable variables to identify its latent components.

Identification of $\overline{lp}_t$ and $\overline{hr}_t$. Under standard forms of utility functions (e.g. logarithmic or power utility), the SDF in (1) is proportional to consumption growth and by extension, so is its long-term counterpart, $\overline{m}_t$. We could, therefore, use data on consumption growth (c_t) and identify its slow-moving component ($\overline{c}_t$) as one of the key determinants of $\overline{m}_t$ and accordingly r_t^* (Del Negro et al., 2017, 2019).

This study proposes decomposing $\overline{c}_t$ into long-term labor productivity and labor input growth as in Antolin-Diaz et al. (2017). To this end, we use growth data on real GDP (gdp_t), consumption (c_t) and total hours worked (hr_t). The corresponding equations are:

$$gdp_t = \overline{lp}_t + \overline{hr}_t + \widetilde{gdp}_t \quad (10)$$

$$c_t = \overline{lp}_t + \overline{hr}_t + \overline{co}_t + \widetilde{c}_t \quad (11)$$

$$hr_t = \overline{hr}_t + \widetilde{hr}_t \quad (12)$$

Defining labor productivity growth as the growth on GDP per hour worked, it follows that long-term GDP growth is the sum of long-term labor productivity and hours growth ($\overline{gdp}_t = \overline{lp}_t + \overline{hr}_t$), as specified in (10).⁷ We assume that the long-term trend in consumption growth is also driven by these same two factors, thus sharing $\overline{lp}_t + \overline{hr}_t$ with gdp_t . The residual component, $\overline{co}_t$, captures all other slow-moving factors that influence $\overline{c}_t$. Finally, we use (12) to identify the slow-moving component of labor input, $\overline{hr}_t$.

⁷Using total hours worked allows us to identify and estimate the long-run trend in labor productivity, which in this framework captures both technological and non-technological components. A more detailed decomposition of productivity—distinguishing, for instance, between total factor productivity (TFP) and other drivers—would require augmenting the model with additional variables (see, e.g., (Antolin-Diaz et al., 2017, Appendix K)). Such an analysis is beyond the scope of this paper and is left for future research.

Identification of $\bar{\pi}_t$ and $\bar{cy}_t$. We use HICP inflation (π_t) and long-term inflation expectations (π_t^e) to identify $\bar{\pi}_t$.⁸ To improve the identification of r_t^* we also include the 10-year government bond yield (r_t^{10y}). The corresponding equations are:

$$\pi_t = \bar{\pi}_t + \tilde{\pi}_t \quad (13)$$

$$\pi_t^e = \bar{\pi}_t + \tilde{\pi}_t^e \quad (14)$$

$$r_t^{10y} = \underbrace{(\bar{lp}_t + \bar{hr}_t - \bar{cy}_t + \bar{ro}_t)}_{r_t^*} + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_t^{10y} \quad (15)$$

In this setup, inflation and inflation expectations share the common trend, $\bar{\pi}_t$. The decomposition of the long-term trend of the 10-year bond yield in (15) mirrors that of the short-term rate but includes an additional term, $\bar{tp}_t$, representing the slow-moving trend of the term premium.⁹

We further identify $\bar{cy}_t$ by using the 10-year BBB corporate bond yield (r_t^{3b}) (see also Krishnamurthy and Vissing-Jorgensen, 2012), which is considered less safe and liquid than the benchmark 10-year government bond, thereby positing a zero convenience yield for it. This translate to:

$$r_t^{3b} = \bar{lp}_t + \bar{hr}_t + \bar{ro}_t + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_t^{3b} \quad (16)$$

Consequently, the slow-moving component of r_t^{3b} is identical to that of r_t^{10y} in (15), except for the absence of the convenience yield.¹⁰ implying that $\bar{r}_t^{3b} - \bar{r}_t^{10y} = \bar{cy}_t$. Thus, a key identification assumption is that the secular changes in the spread between the long-term trends of these two yields are primarily driven by changes in the convenience yield.¹¹

HCT-VAR representation. The vector of variables is $\mathbf{y}_t = (gdp_t, c_t, hr_t, \pi_t, \pi_t^e, r_t^{10y}, r_t^{3b}, r_t^{3m})'$ and that of the trends is $\boldsymbol{\mu}_t = (\bar{lp}_t, \bar{hr}_t, \bar{\pi}_t, \bar{ro}_t, \bar{tp}_t, \bar{co}_t, \bar{cy}_t)'$. In this small-scale speci-

⁸This section follows closely Del Negro et al. (2017).

⁹The term premium compensates investors for the risk of holding long-term assets. This specification implies a rejection of the expectations hypothesis, which assumes a zero or constant term premium. See Del Negro et al. (2017) and references therein.

¹⁰If BBB bonds carry a small, non-zero convenience yield, equation (16) could be rewritten as $r_t^{3b} = \bar{lp}_t + \bar{hr}_t - \lambda \bar{cy}_t + \bar{ro}_t + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_t^{3b}$ with $0 < \lambda < 1$. This study does not examine this case.

¹¹As noted in (Del Negro et al., 2017, see also references therein), the *level* of $\bar{r}_t^{3b} - \bar{r}_t^{10y}$ spread may be influenced by factors other than safety and liquidity, such as the probability of default.

fication, we do not use any additional variables with constant unconditional means, so we set $\mathbf{S}_\gamma = \mathbf{0}$, while $\mathbf{S}_\mu$ is defined as:

$$\mathbf{S}_\mu = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

3.2.2 Large-scale specifications: additional variables

We augment the small-scale specification by expanding $\mathbf{y}_t$ to include a broader set of macroeconomic variables. The objective is to assess whether the additional information improves the estimation and identification of r^* and its underlying drivers. The resulting large-scale specification includes all 22 variables reported in Table 1, preserving the same ordering and spanning domestic activity, inflation, labor market conditions, financial variables, and global factors.

Among these variables, the unemployment rate, ur_t , the GDP deflator, π_t^{gdp} , and the private consumption deflator, π_t^c , are assumed to feature time-varying trends (category 2), while the remaining variables are characterized by constant unconditional means (category 3). For π_t^{gdp} and π_t^c , we further impose that they share a common trend component with π_t and π_t^e , such that:

$$\pi_t^{gdp} = \bar{\pi}_t + \bar{\pi}_t^d + \tilde{\pi}_t^{gdp}, \quad (17)$$

$$\pi_t^c = \bar{\pi}_t + \bar{\pi}_t^d + \tilde{\pi}_t^c, \quad (18)$$

$$ur_t = \bar{ur}_t + \tilde{ur}_t. \quad (19)$$

Here, $\bar{\pi}_t^d$ captures deviations of the long-run trends in π_t^{gdp} and π_t^c from the common inflation trend $\bar{\pi}_t$.

HCT-VAR representation. Equations (9d)–(19) imply the following (9×1) vector of stochastic trends: $\boldsymbol{\mu}_t = (\bar{l}p_t, \bar{h}r_t, \bar{\pi}_t, \bar{\pi}_t^d, \bar{r}o_t, \bar{t}p_t, \bar{c}o_t, \bar{c}y_t, \bar{u}r_t)'$. Thus, in the large-scale specification the number of stochastic trends is $n_\mu = 9$, which is substantially smaller than

the total number of variables, $n = 22$.

The selection matrix $\mathbf{S}_\gamma$ is defined as $\mathbf{S}_\gamma = \mathbf{I}_{22 \times 11}^-$, obtained from a 22×22 identity matrix by removing the columns corresponding to variables with time-varying trends. The identification matrix $\mathbf{S}_\mu$ for the large-scale specification is given by:

$$\mathbf{S}_\mu = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ & & & & \mathbf{0}_{4 \times 9} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ & & & & \mathbf{0}_{6 \times 9} & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The large-scale specification therefore preserves the identification structure of the small-scale model (Section 3.2.1) while exploiting the richer information set provided by a larger cross-section of variables.

4 Empirical results

4.1 Data and competing models

This section presents the estimation results of the proposed models using a sample period that covers the last 25 years of the euro-area economy, i.e. from 2000:Q1 to 2025:Q2. We use annualized growth rates of the variables outlined in Table 1 except for the interest rates, inflation expectations and unemployment rate for which we use variables in levels.

Table 1: Variables used in the empirical analysis

Description	Code	Symbol	Transf.	Category
Real GDP	YER	gdp_t	✓	1
Real household consumption	PCR	c_t	✓	1
Gross fixed capital formation	ITR	i_t	✓	3
Exports of goods and services	XTR	ex_t	✓	3
Imports of goods and services	MTR	im_t	✓	3
Total employment	LNN	em_t	✓	3
Unemployment rate	URX	ur_t	-	2
Hours worked	LHW	hr_t	✓	1
HICP overall index	HICPSYA	π_t	✓	1
10-year ahead HICP forecasts (SPF ECB)	-	π_t^e	-	1
GDP deflator	YED	π_t^{gdp}	✓	2
Private consumption deflator	PCD	π_t^c	✓	2
Commodity prices	COMPR	cp_t	✓	3
Oil prices	POILU	oil_t	✓	3
Unit labor costs	ULC	ulc_t	✓	3
Compensation of employees	WIN	ce_t	✓	3
World GDP	YWR	wg_t	✓	3
World demand	YWRX	wd_t	✓	3
Nominal long-term interest rate	LTN	r_t^{10y}	-	1
Nominal yield BBB corporate bond index	MLEC8JL	r_t^{3b}	-	1
Nominal short-term interest rate	STN	r_t^{3m}	-	1
Nominal effective exchange rate	EEN	exr_t	✓	3

Notes: (a) All variables are obtained from the Area-Wide Model Database (AWMD) of İpek and Kısacıkoglu (2025), with the exception of the BBB corporate bond index and the 10-year-ahead HICP forecasts. The former corresponds to the ICE Bank of America (BofA) 7–10 Year BBB Euro Corporate Index and is sourced from Datastream, while the latter is obtained from the European Central Bank (ECB) Statistical Data Warehouse (<https://data.ecb.europa.eu/data/datasets/SPF/SPF.Q.U2.HICP.POINT.LT.Q.AVG>).

(b) ✓ indicates that the variable is transformed into an annualized growth rate, defined as $\tilde{x}_t = 400 \times \ln(x_t/x_{t-1})$.

(c) Category 1 variables are those required for the identification of r^* and its drivers. Categories 2 and 3 include additional variables with time-varying and time-invariant trends, respectively (see section 3.2).

(d) All real variables are expressed in per capita terms using the euro-area population.

Given that one of the main objectives of this study is to investigate how the structure of the error terms and the size of the cross-section of the endogenous variables in the system affect the estimation of r^* and its determinants, we compare eight (8) different specifications summarized in Table 2.

Table 2: List of alternative models

Model	Short description
Baseline	8-variable HCT-VAR (Category 1 variables) with constant volatility and Gaussian errors
SM-sv	Baseline with common stochastic volatility SV
SM-t	Baseline with Student’s t errors
SM-t-sv	Baseline with common SV and Student’s t errors
LSM	22-variable HCT-VAR (all variables in Table 1) with constant volatility and Gaussian errors
LSM-sv	LSM with common stochastic volatility (SV)
LSM-t	LSM with Student’s t errors
LSM-t-sv	LSM with common SV and Student’s t errors

Note: The error specification in the Baseline model follows Del Negro et al. (2017, 2019).

In particular, these models comprise small (BM) and large (LSM) cross-sections of variables combined with various error structures as reported in Table 2. The Baseline model (BM) is a small-scale 8-variable HCT-VAR model with standard error structure, i.e. constant volatility and Gaussian errors, as it is the common approach in the related literature (Laubach and Williams, 2003; Holston et al., 2017; Del Negro et al., 2017, 2019). Thereby, this model serves as a well-established benchmark model allowing for comparison to the proposed specifications.

All models are estimated using the MCMC algorithm presented in section A of the Online Appendix. Specifically, we run 30,000 iterations, discard the first 5,000 as burn-in, and retain every fifth draw, yielding 5,000 posterior draws for inference.

4.1.1 Specification of priors

For the specification of the priors, we follow the standard practice in the literature (Chan, 2020a; Del Negro et al., 2017); full details are provided in section B of the Online Appendix. Here, we focus on the specification of the innovation covariance matrix, $\mathbf{Q}$, which governs the evolution of the trends, $\boldsymbol{\mu}_t$, and plays a crucial role in the identification of r^* , its evolution and its underlying drivers. Particularly, we assume an inverse Wishart prior:

$$\mathbf{Q} \sim \mathcal{IW}(w_{\varepsilon_\mu}, (w_{\varepsilon_\mu} + n_\mu + 1)(\underline{\mathbf{Q}})). \quad (20)$$

The existing literature typically adopts highly informative, i.e. high degrees-of-freedom (w_{ε_μ}) and conservative, i.e. small scale parameter ($\underline{\mathbf{Q}}$), to avoid implausible trend dynamics and excessive high-frequency variation in $\boldsymbol{\mu}_t$, which would otherwise lead to noisy trend

estimates and unreliable identification of r^* . Such prior specifications generally imply that the standard deviation of the trend innovation corresponds to a drift of approximately 0.5–1 percentage point per century and the w_{ε_μ} is set to 100 (see, e.g., Del Negro et al., 2017, 2019; Maffei-Faccioli, 2025).

Our baseline specification follows this convention. However, to assess the robustness of our findings, we also conduct a comprehensive prior sensitivity analysis in Sections D–F of the Online Appendix. Specifically, we consider a wide range of both more and less informative and conservative priors for $\mathbf{Q}$ in order to evaluate the extent to which our estimates of r^* and its drivers depend on the prior specification.

4.2 r^* point estimates under different specifications

Panel (a) of Figure 1 presents the posterior means of r^* obtained from the eight specifications reported in Table 2, together with the real short-term interest rate i.e. $r_t^{3m} - \pi_t^e$. Across all specifications, the estimated r^* exhibits a clear downward trend from the beginning of the sample (2000:Q1) until the onset of the COVID-19 pandemic in 2020. On average, r^* declines by approximately 100–150 basis points (bp), falling from around 1.5% to between 0.5% and 0% over the first two decades of the sample. This common pattern suggests that the secular decline in r^* is robust to model specification.¹²

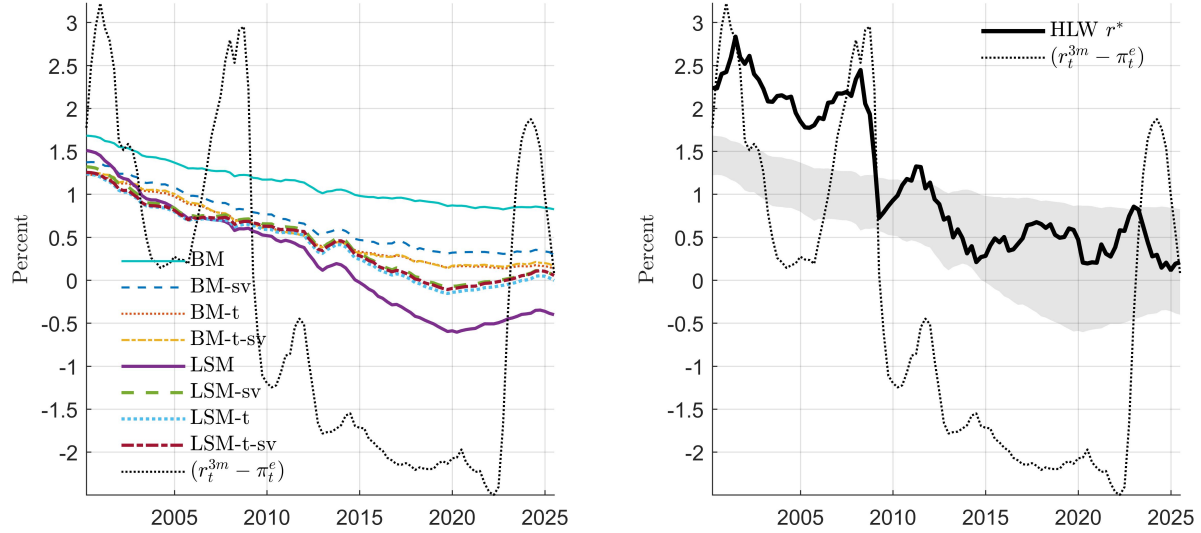
Differences across models become more pronounced during the 2020–2025:Q2 period. In particular, small-scale specifications imply a broadly stable r^* following the pandemic, whereas large-scale models indicate a moderate increase that begins to reverse slightly after 2024:Q4. Overall, these results are consistent with the prevailing empirical evidence on the evolution of the euro-area r^* and broadly align with recent studies (see, e.g., Christensen and Mouabbi, 2024; Brand et al., 2025; Beyer and Brandao-Marques, 2026).

Panel (b) of Figure 1 compares the estimates from Panel (a) with those obtained from the well established model of Holston et al. (2017) (hereafter HLW).¹³ Following the global financial crisis (GFC), the HLW estimates lie within the grey-shaded area, which represents

¹²In section D of the Online Appendix, we show that this secular decline is broadly robust to the prior specification of $\mathbf{Q}$.

¹³The HLW estimates are publicly available from the New York Fed: <https://www.newyorkfed.org/research/policy/rstar>. We use the real-time r^* series corresponding to the 2025:Q2 vintage.

the minimum–maximum range of the estimates shown in Panel (a). Before the GFC, however, the HLW estimates are, on average, about 100 bp higher than those implied by our models. Although the HLW estimates display greater variation over time, possibly reflecting differences in model structure, estimation assumptions and prior beliefs, they support the same broad narrative: a persistent decline in r^* followed by a partial recovery after the COVID-19 pandemic.



(a) Posterior mean estimates of r_t^* under alternative specifications. The black dotted line shows the short-term real rate $r_t^{3m} - \pi_t^e$.

(b) The gray-shaded area shows the min–max range of posterior mean estimates of r^* across alternative models, and the black line shows the HLW r^* estimate.

Figure 1: Posterior mean estimates of r^* under alternative specifications

Nonetheless, the most striking feature of Figure 1 is the substantial cross-model dispersion in the estimates of r^* reported in Panel (a). In 2025:Q2, for example, the estimated value of r^* ranges from approximately -0.4% in the LSM specification to 0.8% in the BM model. By contrast, specifications that incorporate stochastic volatility and/or fat-tailed innovations produce considerably less dispersion, with estimates clustered between 0% and 0.3% , a range that is also broadly consistent with the HLW estimates.

Since r_t^* is unobservable, the accuracy of individual point estimates cannot be directly assessed. Nevertheless, the reduction in cross-model dispersion may be economically relevant from a monetary policy perspective. First, the implied monetary policy stance can

differ substantially across alternative estimates of r_t^* . Second, lower model uncertainty may contribute to more reliable policy assessments and, ultimately, better-informed monetary policy decisions.

4.2.1 Sensitivity to the prior specification of $\mathbf{Q}$

The prior sensitivity analysis presented in section D of the Online Appendix shows that our main empirical findings are broadly robust to the choice of prior for $\mathbf{Q}$. In particular, the secular decline in r^* over the 2000–2020 period is present under all but the most conservative prior specification, namely when a very small scale matrix, $\underline{\mathbf{Q}}$, is used. This outcome is expected, as the implied innovation variances approach zero, forcing the trends to evolve only gradually and become nearly flat.

The modest recovery in r^* after 2020 is more pronounced under less conservative prior specifications, that is, for smaller values of w_{ε_μ} or larger values of $\underline{\mathbf{Q}}$, which allow greater trend variation. Models incorporating stochastic volatility and fat-tailed measurement errors continue to produce estimates that exhibit less dispersion, although this difference becomes less pronounced as the prior on $\mathbf{Q}$ is relaxed.

Finally, the LSM-t-sv model exhibits considerably greater robustness to alternative prior specifications than the baseline (BM) model, particularly before 2015 and after 2023. During the 2015–2023 period, however, the estimated path of r^* becomes noticeably more sensitive to the prior choice under less conservative specifications, reflecting the greater flexibility allowed in the evolution of the latent trends.

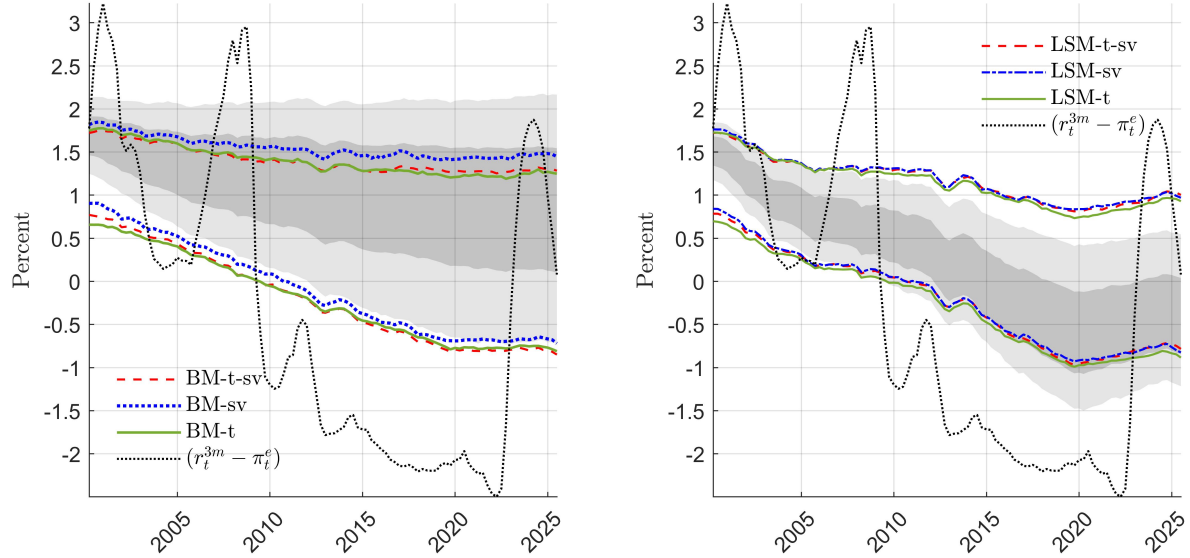
4.3 r^* estimation uncertainty under alternative specifications

The results presented so far suggest that a more flexible error structure can reduce model uncertainty in the HCT-VAR framework. In this section, we examine whether the alternative proposed specifications also reduce estimation uncertainty, as measured by posterior credible intervals (CrIs).

4.3.1 The effect of stochastic volatility and fat-tailed innovations

Panel (a) of Figure 2 compares the 95% CrIs of the BM specification with those of the alternative small-scale specifications that allow for stochastic volatility and/or fat-tailed innovations. These extensions produce systematically narrower credible intervals, by as much as 70 basis points, particularly towards the end of the sample. Among the non-Gaussian and stochastic-volatility specifications (BM-sv, BM-t, and BM-t-sv), differences in interval width are negligible.

By contrast, the effect of a more flexible error structure on estimation uncertainty is less pronounced in the large-scale models. As shown in Panel (b), the benchmark LSM specification generates credible intervals that are at least as narrow as those of the alternative large-scale models. The only exception occurs during the COVID-19 period, when the LSM-t, LSM-sv, and LSM-t-sv specifications produce slightly narrower credible intervals, by approximately 15 basis points.



(a) 95% credible intervals of r_t^* from the small-scale models. Gray (light-gray) shading indicates the 68% (95%) credible interval of the baseline model (BM).

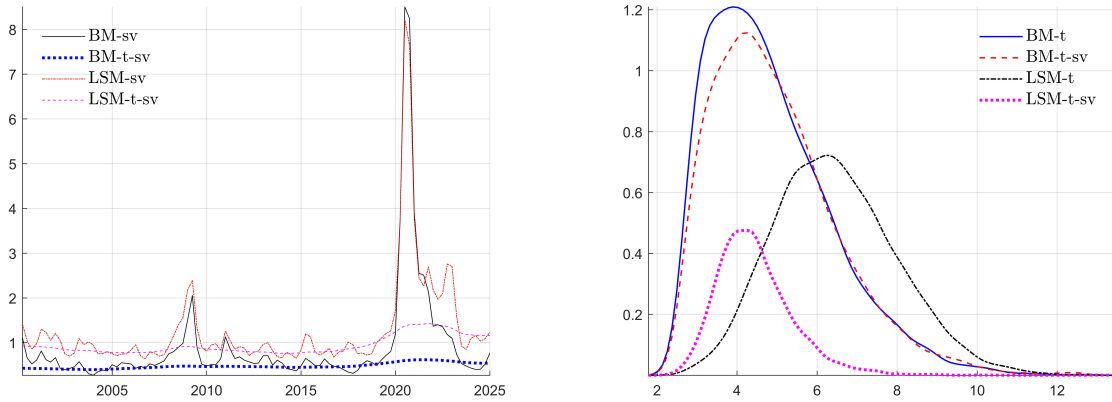
(b) 95% credible intervals of r_t^* from the large-scale models. Gray (light-gray) shading indicates the 68% (95%) credible interval of the LSM model.

Figure 2: Posterior 95% credible intervals for r_t^* under alternative specifications.

Figure 3 provide further evidence on the empirical relevance of stochastic volatility and

fat tails. Panel (a) displays the posterior mean of $\exp(h_t/2)$, for the four specifications that allow for time-varying volatility. These models, especially those with Gaussian error terms, capture pronounced increases in macroeconomic uncertainty during the euro-area sovereign debt crisis (2008–2010) and the Covid-19 pandemic (2020–2021). Panel (b) depicts the posterior density of ν in the fat-tailed specifications. Posterior means of ν between 4 and 5 indicate substantial departures from Gaussianity, implying that extreme observations are frequent enough to invalidate the normality assumption. Consistent with Chan (2020a), Gaussian models tend to attribute large shocks entirely to volatility spikes, whereas fat-tailed models significantly absorb such episodes through heavy-tailed innovations.

These empirical findings suggest that neglecting stochastic volatility and fat tails may induce likelihood misspecification, which in turn can produce less precise and potentially misleading inference regarding r^* and its determinants especially in small-scale specifications. Large-scale models, which incorporate the information content of a large number of variables filtered via Bayesian shrinkage, are shown to generate more precise estimates irrespective of their error structure.



(a) Posterior mean of common stochastic volatility, $\exp(h_t/2)$, using the BM-sv, BM-t-sv, LSM-sv, LSM-t-sv models

(b) Posterior density of the degrees of freedom parameter ν using the BM-t, BM-t-sv, LSM-t, LSM-t-sv models

Figure 3: Common stochastic volatility and Student's t degrees-of-freedom estimates

4.3.2 The effect of model size

Figure J.2 compares small- and large-scale models for all alternative error structures outlined in this study. The empirical evidence shows that the large-scale model produces consistently tighter credible intervals for r^* throughout the sample.

This apparent gain in precision may reflect two mechanisms. First, the inclusion of additional macroeconomic indicators expands the information set and improves the identification of common trends relevant for r^* . Second, by mitigating omitted-variable bias, the large-scale specification reduces through Bayesian shrinkage systematic distortions in the model's dynamics, which would otherwise contaminate the extraction of r^* .

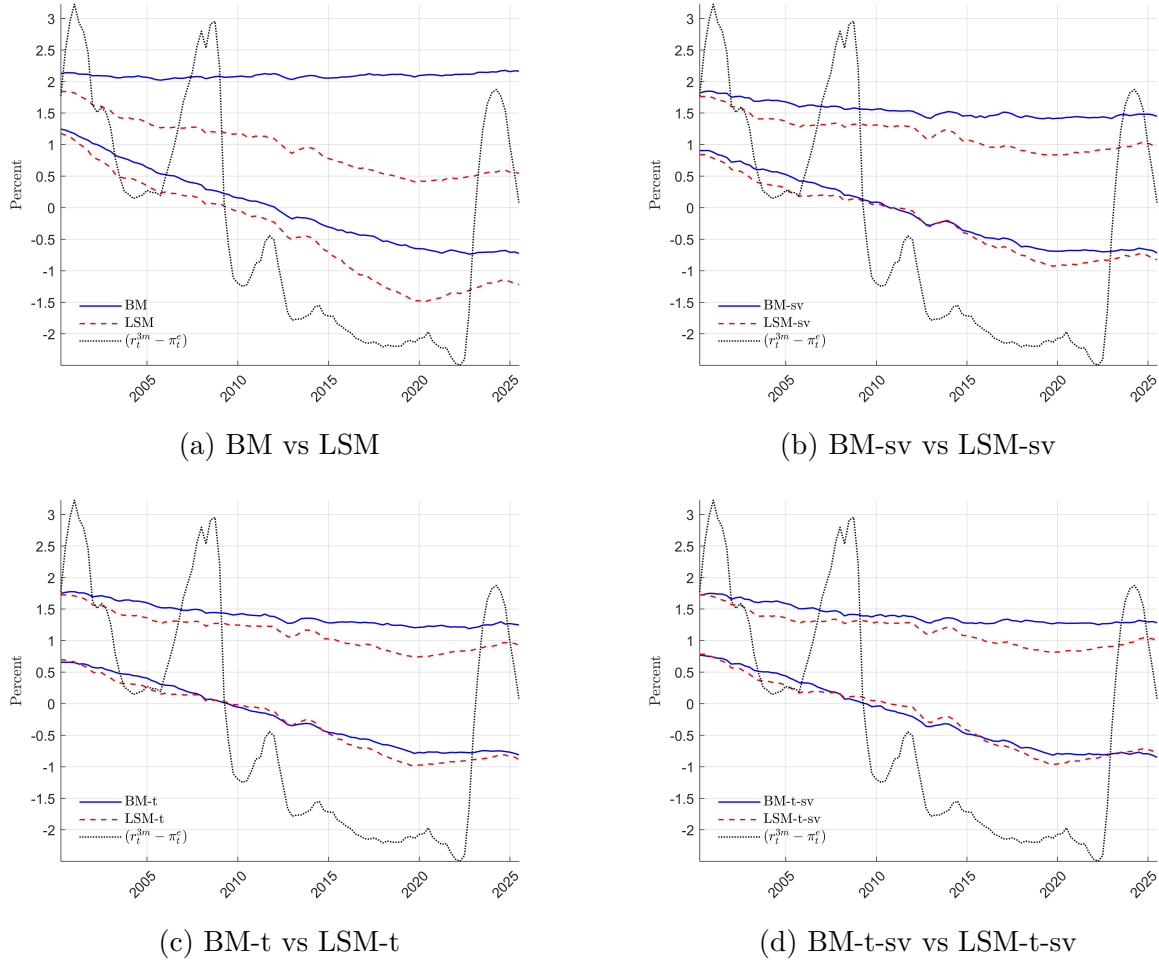


Figure 4: Posterior 95% credible intervals for r_t^* : Small- vs large-scale specifications

Figure 5 summarizes the results of this section by reporting the width of the 95%

credible intervals for all models. Two main findings emerge. First, the large-scale models produce the most precise estimates of r^* overall, with credible intervals that are over 100 basis points narrower than those of the baseline model, corresponding to an increase in estimation precision of more than 35%. Second, a more flexible error structure substantially improves estimation precision in the small-scale models, reducing credible interval widths by up to 70 basis points (around 25%) relative to the BM. In contrast, its effect is much less pronounced in the large-scale models and is largely confined to the COVID-19 pandemic.¹⁴

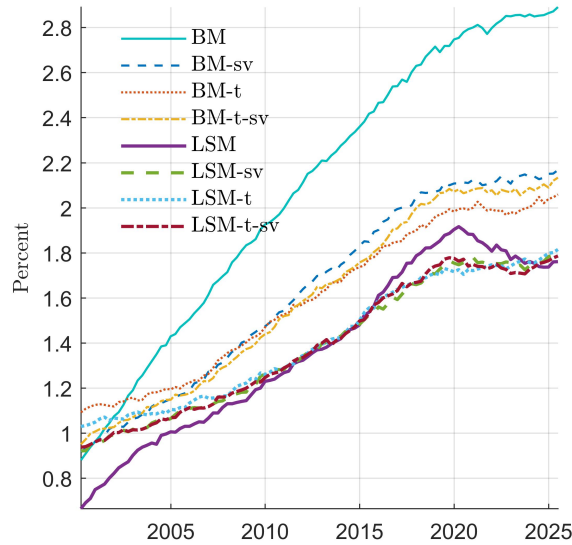


Figure 5: Width of the 95% credible intervals of r^* under alternative specifications

4.4 Implications for the assessment of monetary policy

The differences in estimated values of r^* documented in Section 4.2 have direct implications for the assessment of the monetary policy stance. Figure 6 reports the real interest rate gap, i.e. $(r_t^{3m} - \pi_t^e) - r_{s,t}^*$, computed using the alternative model specifications for the 2022:Q3 - 2025:Q2 period.¹⁵ A positive value indicates that the real policy rate exceeds the natural rate and therefore points to a restrictive monetary policy stance.

¹⁴These results are also robust to alternative prior specifications of $\mathbf{Q}$ as shown in section E of the Online Appendix

¹⁵ $s \in (\text{BM}, \text{BM-sv}, \text{BM-t}, \text{BM-t-sv}, \text{LSM}, \text{LSM-sv}, \text{LSM-t}, \text{LSM-t-sv})$

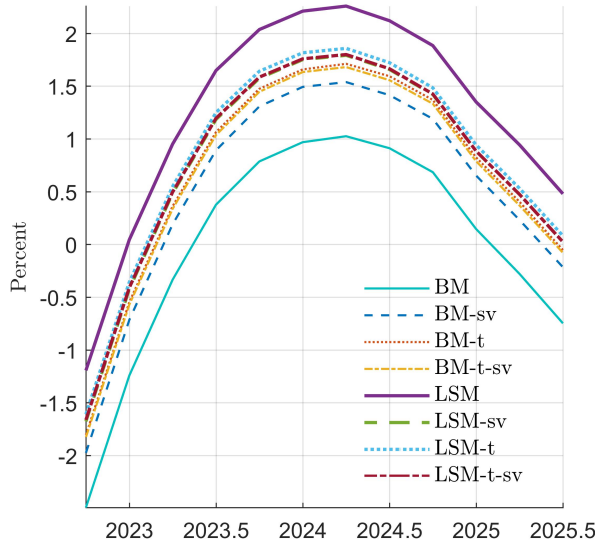


Figure 6: Real interest rate gap calculated as $(r_t^{3m} - \pi_t^e) - r_t^*$ under alternative models

All specifications deliver broadly the same qualitative conclusion. The stance of monetary policy tightened rapidly during 2023, reached a peak around 2024, and subsequently eased as inflation declined and policy rates began to normalize. This common pattern suggests that the broad evolution of the monetary policy stance is robust to the econometric specification employed.

However, the estimated degree of restrictiveness differs substantially across models. Around the peak of the tightening cycle, the estimated policy gap ranges from approximately 1 percentage point in the large-scale model to more than 2 percentage points in the baseline specification. While all models therefore indicate a restrictive policy stance, they imply markedly different assessments of its intensity.

More importantly, the divergence across models becomes economically consequential at the end of the sample. In 2025:Q2, the estimated policy gap ranges from approximately -0.7 percentage points under the baseline model to about 0.5 percentage points under the large-scale model, while the remaining specifications imply values close to zero. Consequently, the same observed policy rate can be interpreted as accommodative, restrictive, or broadly neutral depending on the model used to estimate r^* . This finding highlights that uncertainty regarding the natural rate translates directly into uncertainty regarding

the assessment of the monetary policy stance.

Overall, the results suggest that policymakers should place greater emphasis on the range of plausible estimates implied by alternative models rather than relying on a single estimate of r^* .¹⁶ While the direction of policy is generally robust across specifications, conclusions regarding the degree of policy restrictiveness—and, at times, even whether policy is restrictive or accommodative—remain sensitive to the econometric framework employed.

4.5 The driving forces of r^*

Table 3 reports the change in r^* and its structural determinants between 2000:Q1 and 2019:Q4 across the eight specifications described in Table 2. This period captures nearly the full secular decline in the natural rate of interest in the euro-area and is therefore ideal for examining the underlying sources of this decline.

Table 3: Changes between 2000:Q1 and 2019:Q4 for the r^* and its determinants

Trends	BM (baeline)	BM-t	BM-sv	BM-t-sv	LSM-t-sv	LSM-t	LSM-sv	LSM
r_t^*	-0.81 (-1.89, 0.24)	-1.06* (-1.83, -0.13)	-1.05* (-1.84, -0.08)	-1.07* (-1.87, -0.11)	-1.35* (-2.09, -0.44)	-1.38* (-2.12, -0.52)	-1.39* (-2.13, -0.49)	-2.10* (-2.87, -1.12)
$\bar{c}_t$	-0.56 (-1.29, 0.21)	-0.81* (-1.40, -0.12)	-0.77* (-1.35, -0.08)	-0.83* (-1.41, -0.12)	-0.75* (-1.25, -0.15)	-0.75* (-1.25, -0.15)	-0.75* (-1.26, -0.17)	-1.00* (-1.50, -0.36)
$\bar{l}p_t$	-0.37 (-0.86, 0.15)	-0.52* (-0.93, -0.04)	-0.50 (-0.91, 0.00)	-0.53* (-0.94, -0.04)	-0.56* (-0.98, -0.07)	-0.57* (-0.99, -0.09)	-0.56* (-0.98, -0.08)	-0.71* (-1.12, -0.21)
$\bar{h}r_t$	-0.19 (-0.63, 0.29)	-0.30 (-0.69, 0.16)	-0.28 (-0.67, 0.18)	-0.30 (-0.69, 0.17)	-0.19 (-0.58, 0.28)	-0.18 (-0.56, 0.28)	-0.19 (-0.59, 0.28)	-0.29 (-0.69, 0.21)
$-\bar{c}y_t$	-0.18 (-0.68, 0.23)	-0.14 (-0.60, 0.23)	-0.17 (-0.63, 0.23)	-0.14 (-0.60, 0.24)	-0.39 (-0.90, 0.05)	-0.40 (-0.92, 0.02)	-0.43 (-0.96, 0.02)	-0.73* (-1.28, -0.28)
$\bar{r}o_t$	-0.08 (-0.38, 0.26)	-0.10 (-0.41, 0.24)	-0.11 (-0.40, 0.24)	-0.11 (-0.41, 0.26)	-0.21 (-0.55, 0.19)	-0.22 (-0.57, 0.18)	-0.21 (-0.55, 0.18)	-0.37 (-0.73, 0.04)

Notes: The table reports the median changes (in percentage points) over the period 2000:Q1 to 2019:Q4 for the different models described in Table 2. r_t^* is the natural rate of interest; $\bar{c}_t$ is consumption trend; $\bar{l}p_t$ is labor productivity trend; $\bar{h}r_t$ is total hours worked trend; $\bar{c}y_t$ is convenience yield trend and $\bar{r}o_t$ is the trend of all other factors. 95% posterior confidence intervals are reported in parentheses while asterisks (*) denote that zero is not included in this interval.

Two main results stand out. First, the decline in r^* is statistically significant in all specifications that allow for a larger data set or stochastic volatility and/or fat-tailed shocks. This implies that posterior credible intervals exclude zero in every model except for the BM (baseline) specification. Estimated r^* declines range from -1.05 to -1.39 percentage points in the small- and large-scale models with flexible shock processes, and reach -2.10

¹⁶Philip Lane’s statement mentioned in the introduction shows that the ECB has already adopted such a strategy.

percentage points in the LSM model. By contrast, the Baseline model—imposing Gaussian innovations and constant volatility—implies a smaller and statistically insignificant decline (−0.81 percentage points). Hence, inference about low-frequency movements in r^* is materially affected by the assumed shock structure and the information set employed in the model.

Second, the decomposition of r^* into its main drivers is stable across specifications. The consumption trend, $\bar{c}_t$, accounts for the largest share of the decline in r^* in all but the BM model. Its reduction explains roughly 70% of the total decline in small-scale models and about 50% in large-scale models. The magnitude of this contribution is economically large and statistically robust.

Decomposing the consumption channel further, labor productivity, $\bar{p}_t$, emerges as the primary underlying force. The slowdown in productivity growth accounts for approximately 45% of the overall decline in r^* —closer to 50% in small-scale models and about 40% in large-scale models—and is statistically significant across specifications except for the BM model.¹⁷ In contrast, changes in total hours worked are small and statistically indistinguishable from zero.

Financial factors play a secondary role. The convenience yield contributes meaningfully only in large-scale specifications, where $-\bar{cy}_t$ accounts for up to 35% of the decline (LSM model). However, its statistical significance is not robust across models, suggesting that its quantitative importance is more specification-dependent than that of productivity.

Overall, the evidence indicates that the secular decline in r^* over 2000–2020 primarily reflects weaker trend consumption growth, itself largely driven by slower labor productivity growth. Small-scale specifications that abstract from stochastic volatility and non-Gaussian shocks understate both the magnitude and the statistical robustness of this decline.

4.5.1 Empirical results based on the LM-t-sv specification

In this subsection, we focus on the LSM-t-sv specification, as it incorporates all methodological innovations considered in this study, namely a large information set, stochastic volatility, and fat-tailed innovations. Moreover, the alternative specifications of Table 2

¹⁷The BM-sv model yields marginal statistical insignificance.

yield qualitatively similar results with respect to the drivers of r^* and are presented in section C of the Online Appendix.

Figure H.25 reports the estimated natural rate in panel (a) together with the long-run trends of the stochastic discount factor (SDF), $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, shown in panels (b) and (c), respectively.¹⁸

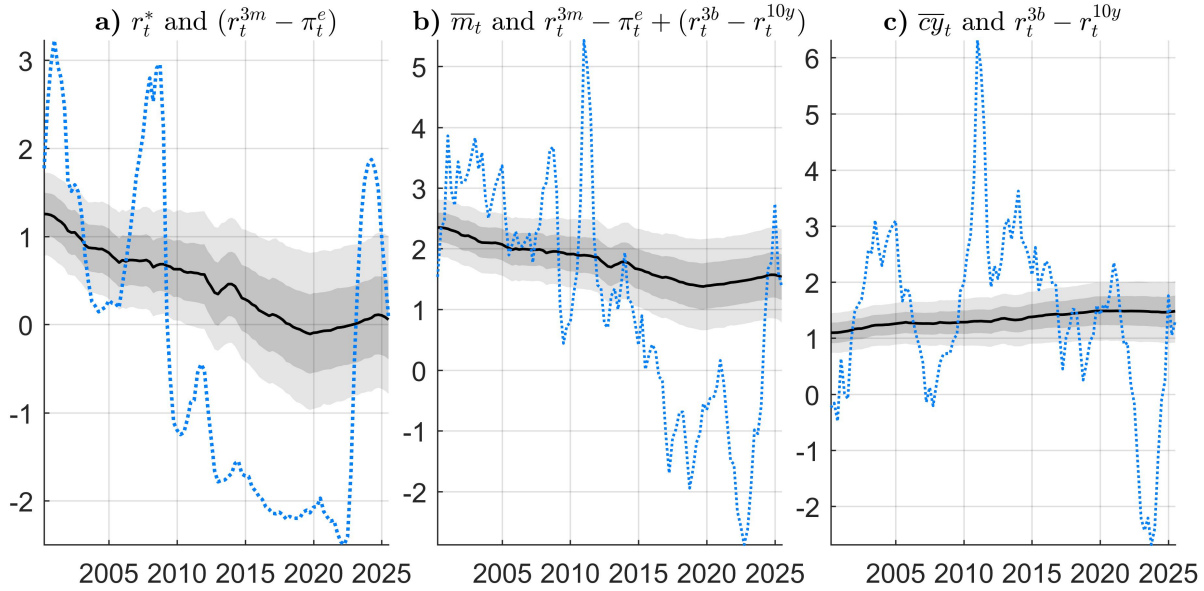


Figure 7: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior coverage intervals.

Both components exhibit broadly similar patterns until the onset of the COVID-19 pandemic. Since the convenience yield enters the Euler equation (2) with a negative sign, the joint decline in $\bar{m}_t$ and $-\bar{c}y_t$ contributes to the persistent fall in r^* up to 2020. After the pandemic, however, the convenience yield stabilizes while the SDF trend increases, indicating that the recent rise in r^* is primarily driven by developments in $\bar{m}_t$.

To further investigate these dynamics, Figure H.26 decomposes the SDF trend into its labor productivity, $\bar{l}p_t$, and hours worked, $\bar{h}r_t$, components. Following Del Negro et al.

¹⁸The trend of the SDF is defined as $\bar{m}_t = \bar{l}p_t + \bar{h}r_t$; see equation (9d). In section C of the Online Appendix we also provide estimates for μ_t and τ_t for all models employed in this study.

(2017), all drivers of r^* defined in equation (9d) are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$. This normalization facilitates a visual comparison of the relative contribution of each factor to the evolution of r^* .

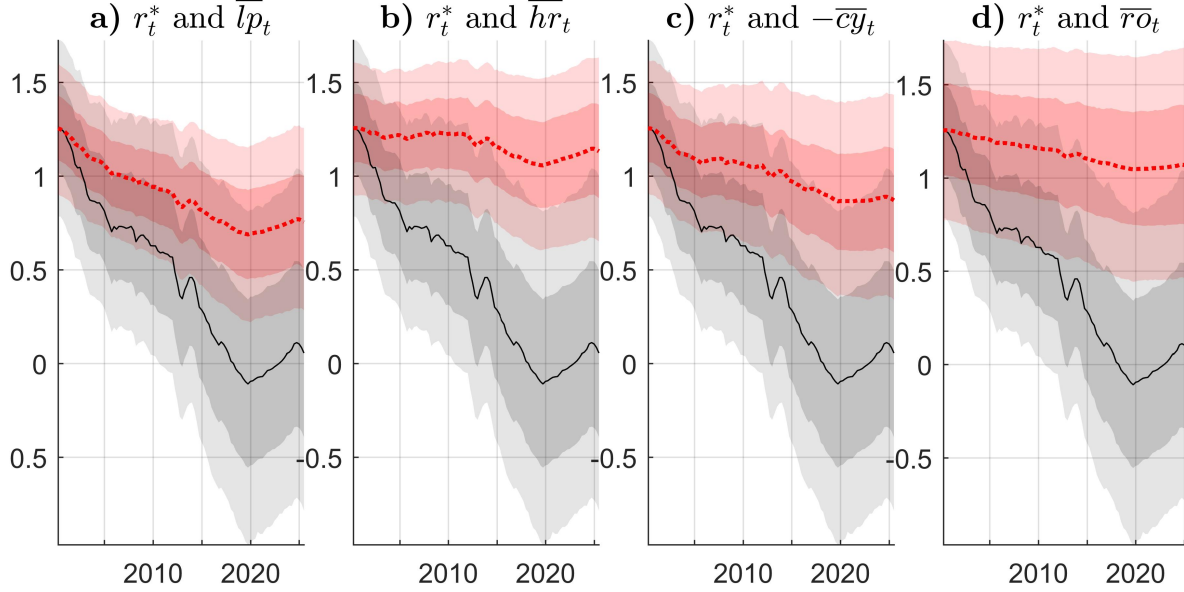


Figure 8: Decomposition of the natural interest rate, r_t^* , based on the LM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior coverage intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

The figure confirms the evidence reported in Table 3. The long-run trend in labor productivity, $\bar{l}p_t$ (Panel a), is the dominant driver of r^* , exhibiting the largest decline prior to 2020. The convenience yield (Panel c) also contributes to the secular decline, though to a lesser extent. In contrast, the trend in hours worked, $\bar{h}r_t$ (panel b), remains broadly stable until around 2015 and declines only modestly thereafter, implying a limited role in the long-run evolution of r^* . The remaining determinants summarized by $\bar{r}o_t$ (panel d) follow a gradual downward path but account for only a small share of the overall movement in the natural rate. The post-pandemic period displays a different pattern. Both $\bar{l}p_t$ and $\bar{h}r_t$ contribute to the observed increase in r^* , whereas the contributions of $\bar{c}y_t$ and $\bar{r}o_t$ remain negligible.

Sensitivity to the prior specification of $\mathbf{Q}$. Section F of the Online Appendix reports empirical evidence regarding the sensitivity of the estimated drivers of r^* to the prior specification of $\mathbf{Q}$. Specifically, we present results for the BM and LSM-t-sv models under a wide range of alternative values for w_{ε_μ} and $\underline{\mathbf{Q}}$ in (F.12). As expected, less informative (smaller w_{ε_μ}) and less conservative (larger $\underline{\mathbf{Q}}$) prior specifications allow greater variation in the latent trends, resulting in more pronounced fluctuations in the estimated drivers of r^* . Nevertheless, the main conclusions remain unchanged. Across the vast majority of prior specifications and for both models, labor productivity is the dominant force shaping the long-run evolution of the euro-area natural rate of interest, followed by the convenience yield.

Robustness check: using alternative variables to identify the drivers of r^* . We assess the robustness of our results to alternative measures used to identify labor productivity and the convenience yield. Throughout, we focus again on the LM-t-sv specification, which incorporates all methodological extensions considered in this paper. We first replace total hours worked, hr_t , with employment, em_t , measured as the number of employed persons, as the labor input used to identify labor productivity. Under this alternative specification, equations (10) and (11) become:

$$gdp_t = \bar{l}p_t + \bar{e}m_t + \widetilde{gdp}_t \quad (21)$$

$$c_t = \bar{l}p_t + \bar{e}m_t + \bar{c}o_t + \widetilde{c}_t \quad (22)$$

$$em_t = \bar{e}m_t + \widetilde{e}m_t \quad (23)$$

Here, labor productivity growth is defined as the growth rate of GDP per employed person, while the trend component of hr_t is assumed to be constant.

Figure I.6 shows that the decomposition of r^* is largely unaffected by this alternative identification scheme. In particular, labor productivity remains the dominant contributor to the secular decline in r^* over the period 2000–2020, indicating that our results are robust to the choice of labor input measure.

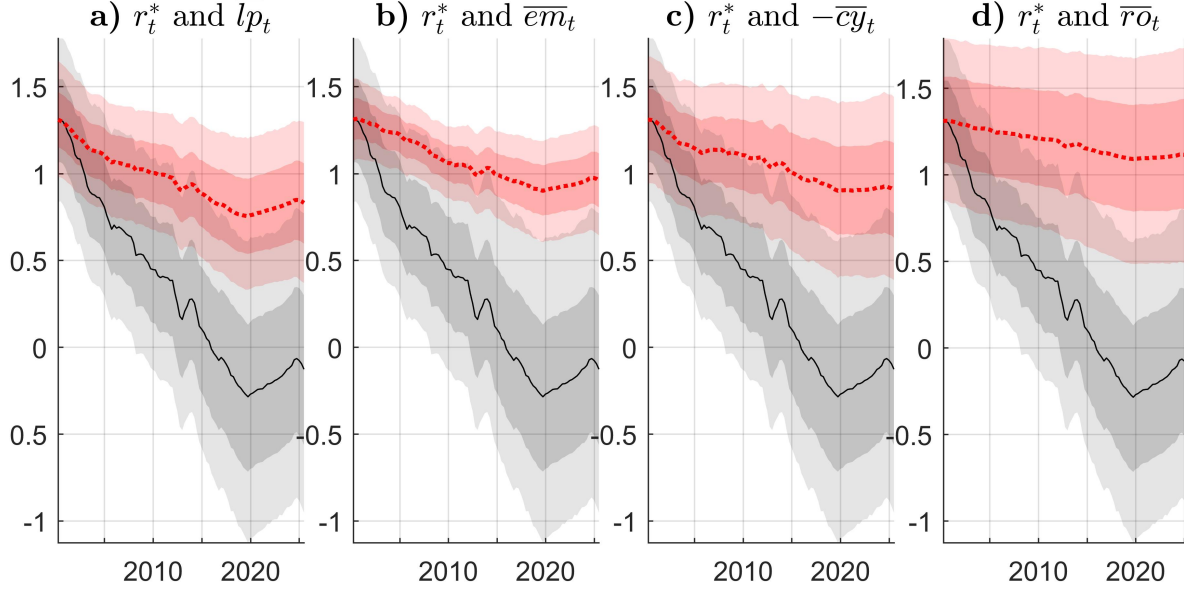


Figure 9: Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. This model uses employment, em_t instead of hours worked, hr_t , for the identification of the labor productivity. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{em}_t$, $-\bar{cy}_t$, and $\bar{ro}_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{hr}_t$, $-\bar{cy}_t$, and $\bar{ro}_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

Figure 10 reports the results obtained when the ICE BofA BBB 10+ Year corporate bond index is used in place of the BBB (7–10 Year) index to identify the convenience-yield component. The results remain qualitatively similar to those of the benchmark specification. In particular, $\bar{l}p_t$ continues to account for the largest share of the decline in r^* between 2000 and 2020, followed by the convenience-yield component, $-\bar{cy}_t$. Relative to the benchmark case, however, the alternative measure implies a somewhat larger contribution of the convenience yield to the recovery in r^* after 2020.

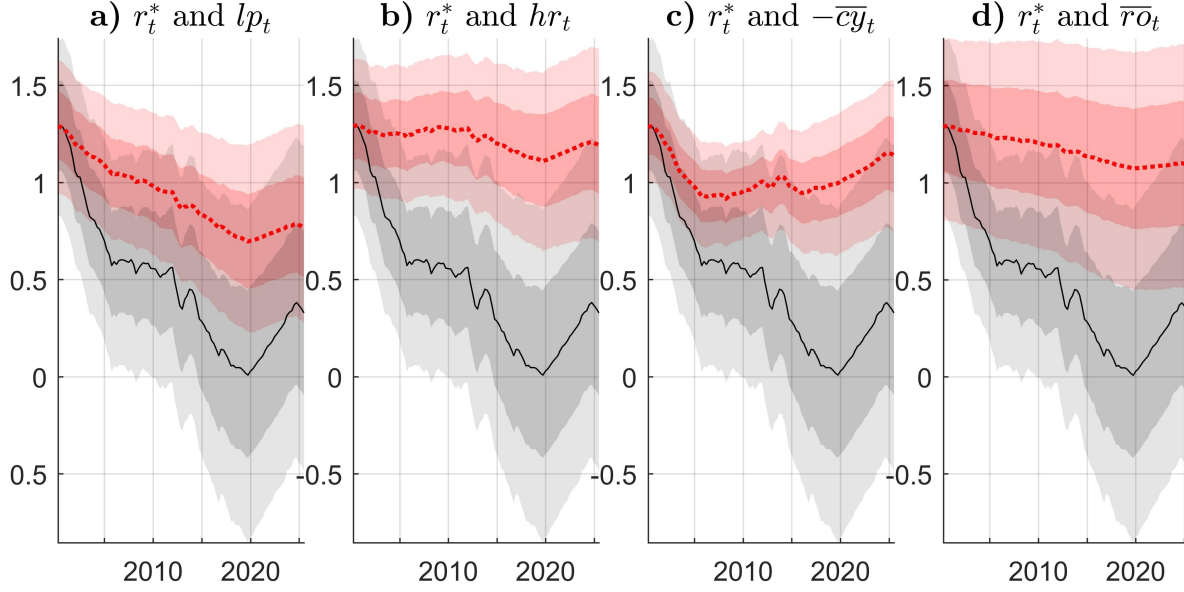


Figure 10: Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. This model uses the ICE BofA BBB 10+ years corporate bond index for the identification of the convenience yield. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

5 Conclusions

The natural rate of interest is inherently a model-dependent concept. This paper shows that both its estimation and its economic interpretation depend importantly on the econometric framework used to identify it. At the same time, our results consistently identify labor productivity growth as the primary driver of the long-run evolution of the euro-area natural rate, while labor input and financial factors play comparatively smaller roles.

From a methodological perspective, the paper demonstrates that incorporating a richer information set, stochastic volatility, and fat-tailed innovations within a parsimonious hybrid common-trend VAR framework can substantially improve inference on r^* . These features enable the model to better capture important characteristics of macroeconomic data, particularly during periods of elevated uncertainty, and yield considerably more precise estimates of the natural rate. More broadly, the findings suggest that a non-negligible

share of the uncertainty surrounding r^* is econometric rather than purely economic and can therefore be reduced through more flexible empirical specifications. From a monetary policy perspective, considering a range of alternative model specifications provides a more comprehensive assessment of model uncertainty, leading to more robust and reliable policy decisions.

Overall, the results suggest that further progress in understanding r^* requires advances on two fronts: improved identification of its underlying economic drivers and richer econometric frameworks capable of extracting low-frequency signals from increasingly complex macroeconomic data. Looking ahead, rapid advances in artificial intelligence, industrial automation, and computing technologies may reshape productivity growth and, consequently, the long-run trajectory of the natural rate. Determining whether these developments represent a persistent shift in r^* is likely to become one of the central challenges for monetary economics in the years ahead.

SUPPLEMENTARY MATERIAL

Online Appendix: The Online Appendix contains the estimation algorithm, details of the prior specification, additional empirical results, and a comprehensive prior sensitivity analysis (PDF).

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The econometrics of the euro-area natural rate of interest

ONLINE APPENDIX

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Bank of Greece

July 15, 2026

The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Greece. Any remaining errors are the sole responsibility of the author.

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F Bayesian estimation

To facilitate the presentation of the estimating algorithms we rewrite the HCT-VAR model:

$$\mathbf{y}_t = \boldsymbol{\tau}_t + \sum_{l=1}^p \mathbf{B}_l (\mathbf{y}_{t-l} - \boldsymbol{\tau}_{t-l}) + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \lambda_t \exp(h_t) \boldsymbol{\Sigma}) \quad (\text{F.1})$$

$$\boldsymbol{\tau}_t = \mathbf{S}_\gamma \boldsymbol{\gamma} + \mathbf{S}_\mu \boldsymbol{\mu}_t \quad (\text{F.2})$$

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\varepsilon}_t^\mu, \quad \boldsymbol{\varepsilon}_t^\mu \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \quad (\text{F.3})$$

$$h_t = h_{t-1} + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \kappa^2) \quad (\text{F.4})$$

$$\lambda_t | \nu \sim \mathcal{IG}(\nu/2, \nu/2) \quad (\text{F.5})$$

where, $\mathbf{y}_t = (y_{1,t}, \dots, y_{n,t})'$ is an $n \times 1$ vector of endogenous variables and $\mathbf{B}_1, \dots, \mathbf{B}_p$ are autoregressive coefficient matrices. The factor h_t is a common log-volatility component evolving as a random walk in (F.4), while the inverse-gamma latent variable λ_t implies that $\boldsymbol{\varepsilon}_t$ is marginally Student's t -distributed with ν degrees of freedom. The unconditional mean, $\mathbb{E}(\mathbf{y}_t) = \boldsymbol{\tau}_t$, is decomposed into a time-invariant component $\boldsymbol{\gamma} \in \mathbb{R}^{n_\gamma}$ and a time-varying component $\boldsymbol{\mu}_t \in \mathbb{R}^{n_\mu}$. Selection matrices $\mathbf{S}_\gamma \in \mathbb{R}^{n \times n_\gamma}$ and $\mathbf{S}_\mu \in \mathbb{R}^{n \times n_\mu}$ determine which variables have constant or time-varying means and how variables load on the common trends. The trend component $\boldsymbol{\mu}_t$ follows the driftless random walk in (F.3) with covariance matrix $\mathbf{Q}$. We further assume that the shocks in (F.1), (F.3), and (F.4) are mutually orthogonal.

F.1 Priors

Bayesian estimation requires specifying prior distributions for all parameters. We begin with the priors for the stationary VAR component defined in (F.1), (F.4), and (F.5) (see, e.g., Chan, 2020a):

$$\boldsymbol{\beta} | \boldsymbol{\Sigma} \sim \mathcal{N}(\boldsymbol{\beta}_0, \boldsymbol{\Sigma} \otimes \mathbf{V}_B) I(\boldsymbol{\beta}), \quad (\text{F.6})$$

$$\boldsymbol{\Sigma} \sim \mathcal{IW}(w_\varepsilon, (w_\varepsilon + n + 1) \underline{\boldsymbol{\Sigma}}), \quad (\text{F.7})$$

$$\kappa^2 \sim \mathcal{IG}(\nu_{h0}, S_{h0}), \quad (\text{F.8})$$

$$\nu \sim \mathcal{U}(a, b). \quad (\text{F.9})$$

The priors for $\boldsymbol{\beta} = \text{vec}(\mathbf{B}_1, \dots, \mathbf{B}_p)'$ and $\boldsymbol{\Sigma}$ follow a normal-inverse-Wishart structure ($\mathcal{IW}$) with w_ε degrees-of-freedom and mode $\underline{\boldsymbol{\Sigma}}$. The indicator function $I(\boldsymbol{\beta})$ enforces stationarity by restricting attention to stable VAR systems. The variance of the common volatility factor, κ^2 , follows an inverse Gamma distribution, while the degrees-of-freedom parameter ν is assigned a uniform distribution with upper and lower bounds a and b , respectively.

We next specify priors for the unconditional mean component, $\boldsymbol{\tau}_t$, which consists of $\boldsymbol{\gamma}$ and $\boldsymbol{\mu}_t$. For the time-invariant component, $\boldsymbol{\gamma}$, we assume:

$$\boldsymbol{\gamma} \sim \mathcal{N}(\boldsymbol{\gamma}_0, \mathbf{V}_\gamma). \quad (\text{F.10})$$

For the time-varying component, $\boldsymbol{\mu}_t$, we specify priors for the initial state and the innovation covariance matrix:

$$\boldsymbol{\mu}_0 \sim \mathcal{N}(\bar{\boldsymbol{\mu}}_0, \mathbf{V}_{\mu 0}), \quad (\text{F.11})$$

$$\mathbf{Q} \sim \mathcal{IW}(w_{\varepsilon_\mu}, (w_{\varepsilon_\mu} + n_\mu + 1)\underline{\mathbf{Q}}). \quad (\text{F.12})$$

The choice of w_{ε_μ} and the scale matrix $\underline{\mathbf{Q}}$ in (F.12) governs the smoothness of the estimated trends. To ensure that $\boldsymbol{\mu}_t$ captures low-frequency movements in r^* and its drivers, rather than transitory business cycle fluctuations, the literature typically adopts conservative priors with large w_{ε_μ} and small diagonal elements in $\underline{\mathbf{Q}}$. These choices imply highly persistent trends, often corresponding to variations on the order of 0.5–1 percentage points per century at least for the U.S. economy (see, e.g., Del Negro et al., 2017; Maffei-Faccioli, 2025).

F.2 Estimation algorithm for the LSM-t-sv model

To estimate the model in (F.1)–(F.5), we extend the MCMC sampler of Louzis (2026) to incorporate fat-tailed errors using the sampler of Chan (2020a) and to account for the absence of a measurement equation for inflation expectations. We then sequentially draw

from the following conditional posterior densities:

1. $p(\boldsymbol{\mu}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \boldsymbol{\vartheta});$
2. $p(\boldsymbol{\gamma}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta});$
3. $p(\mathbf{B}, \boldsymbol{\Sigma}|\mathbf{y}, \boldsymbol{\gamma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta});$
4. $p(\mathbf{h}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta});$
5. $p(\kappa^2|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta}_{-\kappa^2});$
6. $p(\boldsymbol{\lambda}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \boldsymbol{\gamma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\vartheta});$
7. $p(\nu|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\vartheta}_{-\nu});$
8. $p(\boldsymbol{\mu}_0|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \boldsymbol{\gamma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta}_{-\boldsymbol{\mu}_0}) ;$
9. $p(\mathbf{Q}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \boldsymbol{\gamma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta}_{-\mathbf{Q}}) ;$

where $\mathbf{y} = (\mathbf{y}'_1, \dots, \mathbf{y}'_T)'$ is a $nT \times 1$ vector of data, $\boldsymbol{\mu}$, $\mathbf{h}$, $\boldsymbol{\lambda}$ are defined similarly, $\boldsymbol{\vartheta} = (\kappa^2, \nu, \boldsymbol{\mu}_0, \mathbf{Q})$ and $\boldsymbol{\vartheta}_{-x}$ denotes vector $\boldsymbol{\vartheta}$ without x .¹

Step 1: To derive the posterior density of $\boldsymbol{\mu}$, we first rewrite the equation (F.1) in matrix notation that enable us to implement the precision matrix algorithm:

$$\tilde{\mathbf{y}}_{\boldsymbol{\gamma}} = \mathbf{H}_B \bar{\mathbf{S}}_{\boldsymbol{\mu}} \boldsymbol{\mu} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega} \otimes \boldsymbol{\Sigma}) \quad (\text{F.13})$$

where $\tilde{\mathbf{y}}_{\boldsymbol{\gamma}} = \mathbf{H}_B \bar{\mathbf{y}}_{\boldsymbol{\gamma}}$, $\bar{\mathbf{y}}_{\boldsymbol{\gamma}} = \mathbf{y} - \bar{\mathbf{S}}_{\boldsymbol{\gamma}} \boldsymbol{\gamma}$, $\bar{\mathbf{S}}_{\boldsymbol{\gamma}} = \mathbf{1}_T \otimes \mathbf{S}_{\boldsymbol{\gamma}}$, $\bar{\mathbf{S}}_{\boldsymbol{\mu}} = \mathbf{I}_T \otimes \mathbf{S}_{\boldsymbol{\mu}}$, $\boldsymbol{\Omega} = \text{diag}(\lambda_1 \exp(h_1), \dots, \lambda_T \exp(h_T))$ and $\mathbf{H}_B$ is the following square ($nT \times nT$) band matrix:

$$\mathbf{H}_B = \begin{bmatrix} \mathbf{I}_n & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ -\mathbf{B}_1 & \mathbf{I}_n & \mathbf{0} & \dots & \mathbf{0} \\ -\mathbf{B}_2 & -\mathbf{B}_1 & \mathbf{I}_n & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & -\mathbf{B}_p & \dots & -\mathbf{B}_1 & \mathbf{I}_n \end{bmatrix} \quad (\text{F.14})$$

¹ $\boldsymbol{\mu}$ is a $n_{\boldsymbol{\mu}}T \times 1$ vector, while $\mathbf{h}$ and $\boldsymbol{\lambda}$ are of size $T \times 1$.

By a simple change of variable we write $\tilde{\mathbf{y}}_\gamma | \mathbf{B}, \Sigma, \Omega, \boldsymbol{\mu}, \gamma \sim \mathcal{N}(\mathbf{H}_B \bar{\mathbf{S}}_\mu \boldsymbol{\mu}, \Omega \otimes \Sigma)$ implying the following log likelihood:

$$\log p(\tilde{\mathbf{y}}_\gamma | \mathbf{B}, \Sigma, \mathbf{h}, \boldsymbol{\mu}, \gamma) \propto -\frac{1}{2}(\tilde{\mathbf{y}}_\gamma - \mathbf{H}_B \bar{\mathbf{S}}_\mu \boldsymbol{\mu})'(\Omega^{-1} \otimes \Sigma^{-1})(\tilde{\mathbf{y}}_\gamma - \mathbf{H}_B \bar{\mathbf{S}}_\mu \boldsymbol{\mu}) \quad (\text{F.15})$$

To derive the conditional posterior distribution of $\boldsymbol{\mu}$ we need its prior distribution. Thus, we rewrite the state equation in (F.3) as:

$$\mathbf{H}\boldsymbol{\mu} = \mathbf{a}_\mu + \boldsymbol{\varepsilon}^\mu, \quad \boldsymbol{\varepsilon}^\mu \sim \mathcal{N}(\mathbf{0}, \mathbf{G}) \quad (\text{F.16})$$

where $\mathbf{a}_\mu = (\boldsymbol{\mu}'_0, \mathbf{0}, \dots, \mathbf{0})'$, $\mathbf{G} = \mathbf{I}_T \otimes \mathbf{Q}$ and $\mathbf{H}$ is a $n_\mu T \times n_\mu T$ matrix defined as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{n_\mu} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ -\mathbf{I}_{n_\mu} & \mathbf{I}_{n_\mu} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_{n_\mu} & \mathbf{I}_{n_\mu} & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & -\mathbf{I}_{n_\mu} & \mathbf{I}_{n_\mu} \end{bmatrix} \quad (\text{F.17})$$

Both $\mathbf{H}_B$ and $\mathbf{H}$ are sparse matrices, i.e. the vast majority of entries are zero, where the non-zero elements are arranged below the main unitary diagonal. This structure leads to significant computational gains and since it holds that $|\mathbf{H}_B| = |\mathbf{H}| = 1 \ \forall \mathbf{B}_i$, the matrices are invertible meaning that we can rephrase (F.16) as Chan and Jeliazkov (2009); Chan (2013):^{2,3}

$$\boldsymbol{\mu} = \tilde{\boldsymbol{\mu}} + \tilde{\boldsymbol{\varepsilon}}^\mu, \quad \tilde{\boldsymbol{\varepsilon}}^\mu \sim \mathcal{N}(\mathbf{0}, \tilde{\mathbf{G}}) \quad (\text{F.18})$$

where $\tilde{\boldsymbol{\mu}} = \mathbf{H}^{-1}\mathbf{a}_\mu = \mathbf{1}_T \otimes \boldsymbol{\mu}_0$, $\tilde{\boldsymbol{\varepsilon}}^\mu = \mathbf{H}^{-1}\boldsymbol{\varepsilon}^\mu$ and $\tilde{\mathbf{G}} = (\mathbf{H}'\mathbf{G}^{-1}\mathbf{H})^{-1}$, implying a Gaussian

² |.| denotes the determinant. See also Chan (2017a) for a textbook treatment of fast band matrix algorithm methods.

³First, it is trivial to show that $\tilde{\Sigma} = \text{Cov}(\tilde{\boldsymbol{\varepsilon}}) = \text{Cov}(\mathbf{H}_B^{-1}\boldsymbol{\varepsilon}) = \mathbf{H}_B^{-1}\text{Cov}(\boldsymbol{\varepsilon})\mathbf{H}_B^{-1'} = \mathbf{H}_B^{-1}(\mathbf{I}_T \otimes \Sigma)\mathbf{H}_B^{-1'}$ using standard covariance properties. Second, for computational efficiency reasons it is often more convenient to write $\tilde{\Sigma} = (\tilde{\Sigma}^{-1})^{-1} = (\mathbf{H}_B'(\mathbf{I}_T \otimes \Sigma^{-1})\mathbf{H}_B)^{-1}$. This last result follows from the fact that for any three invertible square matrices holds that $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$ and also that $(\mathbf{I}_T \otimes \Sigma)^{-1} = \mathbf{I}_T \otimes \Sigma^{-1}$.

prior distribution $\boldsymbol{\mu}|\boldsymbol{\mu}_0, \mathbf{G} \sim \mathcal{N}(\tilde{\boldsymbol{\mu}}, \tilde{\mathbf{G}})$ with log density:

$$\log p(\boldsymbol{\mu}|\boldsymbol{\mu}_0, \mathbf{G}) \propto -\frac{1}{2}(\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}})' \tilde{\mathbf{G}}^{-1}(\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}}) \quad (\text{F.19})$$

We derive the conditional posterior distribution for $\boldsymbol{\mu}$ by combining the likelihood (F.15) with the prior (F.19). We ignore any term that does not include $\boldsymbol{\mu}$ and we apply the *completing-the-squares* technique to get:

$$\begin{aligned} \log p(\boldsymbol{\mu}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\mu}_0) &\propto -\frac{1}{2}(\tilde{\mathbf{y}}_\gamma - \mathbf{H}_B \bar{\mathbf{S}}_\mu \boldsymbol{\mu})'(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1})(\tilde{\mathbf{y}}_\gamma - \mathbf{H}_B \bar{\mathbf{S}}_\mu \boldsymbol{\mu}) \\ &\quad -\frac{1}{2}(\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}})' \tilde{\mathbf{G}}^{-1}(\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}}) \\ &\propto -\frac{1}{2} \left\{ -2\boldsymbol{\mu} \left[\bar{\mathbf{S}}_\mu' \mathbf{H}_B' (\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1}) \tilde{\mathbf{y}}_\gamma + \tilde{\mathbf{G}}^{-1} \tilde{\boldsymbol{\mu}} \right] \dots \right. \\ &\quad \left. + \boldsymbol{\mu}' \left[\bar{\mathbf{S}}_\mu' \mathbf{H}_B' (\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1}) \mathbf{H}_B \bar{\mathbf{S}}_\mu + \tilde{\mathbf{G}}^{-1} \right] \boldsymbol{\mu} \right\} \\ &\propto -\frac{1}{2} (-2\boldsymbol{\mu}' \mathbf{K}_\mu^{-1} \hat{\boldsymbol{\mu}} + \boldsymbol{\mu}' \mathbf{K}_\mu^{-1} \boldsymbol{\mu} + \hat{\boldsymbol{\mu}}' \mathbf{K}_\mu^{-1} \hat{\boldsymbol{\mu}}) \\ &\propto -\frac{1}{2}(\boldsymbol{\mu} - \hat{\boldsymbol{\mu}})' \mathbf{K}_\mu^{-1}(\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}) \end{aligned} \quad (\text{F.20})$$

which is a multivariate Gaussian kernel with mean $\hat{\boldsymbol{\mu}}$ and covariance matrix $\mathbf{K}_\mu$, implying the following Gaussian conditional posterior distribution for $\boldsymbol{\mu}$:

$$\boldsymbol{\mu}|\mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\lambda}, \boldsymbol{\gamma}, \boldsymbol{\mu}_0, \mathbf{Q} \sim \mathcal{N}(\hat{\boldsymbol{\mu}}, \mathbf{K}_\mu) \quad (\text{F.21})$$

where $\mathbf{K}_\mu = [(\mathbf{H}_B \bar{\mathbf{S}}_\mu)'(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1})(\mathbf{H}_B \bar{\mathbf{S}}_\mu) + \tilde{\mathbf{G}}^{-1}]^{-1}$ and $\hat{\boldsymbol{\mu}} = \mathbf{K}_\mu^{-1}[(\mathbf{H}_B \bar{\mathbf{S}}_\mu)'(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1})\tilde{\mathbf{y}}_\gamma + \tilde{\mathbf{G}}^{-1}\tilde{\boldsymbol{\mu}}]$.

Step 2: We proceed as in step 1 and we rewrite (F.1) as a standard linear regression using matrix notation:

$$\tilde{\mathbf{y}}_\mu = \mathbf{H}_B \bar{\mathbf{S}}_\gamma \boldsymbol{\gamma} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega} \otimes \boldsymbol{\Sigma}) \quad (\text{F.22})$$

where $\tilde{\mathbf{y}}_\mu = \mathbf{H}_B \bar{\mathbf{y}}_\mu$ and $\bar{\mathbf{y}}_\mu = \mathbf{y} - \bar{\mathbf{S}}_\mu \boldsymbol{\mu}$ with conditional distribution $\tilde{\mathbf{y}}_\mu | \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda} \sim \mathcal{N}(\mathbf{H}_B \bar{\mathbf{S}}_\gamma \boldsymbol{\gamma}, \boldsymbol{\Omega} \otimes \boldsymbol{\Sigma})$ and log density:

$$\log p(\tilde{\mathbf{y}}_\mu | \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \propto -\frac{1}{2}(\tilde{\mathbf{y}}_\mu - \mathbf{H}_B \bar{\mathbf{S}}_\gamma \boldsymbol{\gamma})'(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1})(\tilde{\mathbf{y}}_\mu - \mathbf{H}_B \bar{\mathbf{S}}_\gamma \boldsymbol{\gamma}) \quad (\text{F.23})$$

Using the Gaussian prior for $\boldsymbol{\gamma}$ in (F.10) and applying standard regression results, we can show that the posterior distribution of $\boldsymbol{\gamma}$ is:

$$\boldsymbol{\gamma} | \mathbf{y}, \mathbf{B}, \boldsymbol{\Sigma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\lambda} \sim \mathcal{N}(\hat{\boldsymbol{\gamma}}, \mathbf{K}_\gamma) \quad (\text{F.24})$$

where $\mathbf{K}_\gamma = [(\mathbf{H}_B \bar{\mathbf{S}}_\gamma)'(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1})(\mathbf{H}_B \bar{\mathbf{S}}_\gamma) + \mathbf{V}_\gamma^{-1}]^{-1}$ and $\hat{\boldsymbol{\gamma}} = \mathbf{K}_\gamma^{-1}[(\mathbf{H}_B \bar{\mathbf{S}}_\gamma)'(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Sigma}^{-1})\tilde{\mathbf{y}}_\mu + \mathbf{V}_\gamma^{-1}\boldsymbol{\gamma}_0]$.⁴

Step 3: Once $\boldsymbol{\mu}$ and $\boldsymbol{\gamma}$ have been drawn from their posterior distributions, we compute the demeaned vector of endogenous variables, i.e. $\mathbf{y}_t^* = \mathbf{y}_t - \boldsymbol{\tau}_t$, and jointly sample $(\mathbf{B}, \boldsymbol{\Sigma})$ using standard methods (see, e.g., Chan, 2020a). To this end, we rewrite (F.1) as:

$$\mathbf{Y}^* = \mathbf{X}^* \mathbf{B} + \mathbf{E} \quad \text{vec}(\mathbf{E}) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma} \otimes \boldsymbol{\Omega}) \quad (\text{F.25})$$

where $\mathbf{B} = (\mathbf{B}_1, \dots, \mathbf{B}_p)'$ is the $np \times n$ coefficient matrix, $\mathbf{Y}^*$, $\mathbf{X}^*$ and $\mathbf{E}$ are $T \times n$, $T \times np$ and $T \times n$ matrices which are constructed by stacking over t the vectors $\mathbf{y}_t^*$, $\mathbf{x}_t^* = (\mathbf{y}_{t-1}^*, \dots, \mathbf{y}_{t-p}^*)$ and $\boldsymbol{\varepsilon}_t^*$.⁵ Using the normal-inverse Wishart natural conjugate prior distribution in (F.6) and (F.7), the corresponding posterior distribution becomes:

$$\boldsymbol{\Sigma} | \mathbf{Y}^*, \boldsymbol{\Omega} \sim \mathcal{IW}(\hat{\mathbf{S}}, \nu_0 + T) \quad \text{vec}(\mathbf{B}) | \mathbf{Y}^*, \boldsymbol{\Sigma}, \boldsymbol{\Omega} \sim \mathcal{N}(\text{vec}(\hat{\mathbf{B}}), \boldsymbol{\Sigma} \otimes \mathbf{K}_B) \quad (\text{F.26})$$

where $\mathbf{K}_B = \mathbf{V}_B^{-1} + \mathbf{X}^{*'} \boldsymbol{\Omega}^{-1} \mathbf{X}^*$, $\hat{\mathbf{B}} = \mathbf{K}_B^{-1}(\mathbf{V}_B^{-1} \mathbf{B}_0 + \mathbf{X}^{*'} \boldsymbol{\Omega}^{-1} \mathbf{Y}^*)$ and $\hat{\mathbf{S}} = (w_\varepsilon + n + 1) \underline{\mathbf{S}} + \mathbf{B}_0' \mathbf{V}_B^{-1} \mathbf{B}_0 + \mathbf{Y}^{*'} \boldsymbol{\Omega}^{-1} \mathbf{Y}^* - \hat{\mathbf{B}}' \mathbf{K}_B \hat{\mathbf{B}}$.

⁴In practice, the quantities $\mathbf{H}_B \bar{\mathbf{S}}_\mu$, $\mathbf{H}_B \bar{\mathbf{S}}_\gamma$ and $\mathbf{H}_B \bar{\mathbf{y}}_\gamma$ are computed efficiently as shown in Louzis (2026).

⁵The $\text{vec}(\cdot)$ operator converts a matrix into a column vector by stacking the columns.

Step 4: We use the acceptance-rejection Metropolis-Hastings step of Chan (2020a) to draw h_t conditional on other parameters.⁶.

Step 5: We draw κ^2 from the following inverse-Gamma conditional posterior:

$$\kappa^2 | \mathbf{y}, \mathbf{B}, \mathbf{\Sigma}, \mathbf{h}, \boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\vartheta}_{-\kappa^2} \sim \mathcal{IG}(\nu_{h0} + T/2, (S_{h0} + \boldsymbol{\varepsilon}^{h'} \boldsymbol{\varepsilon}^h)/2) \quad (\text{F.27})$$

where $\boldsymbol{\varepsilon}^h = (\varepsilon_1^h, \dots, \varepsilon_T^h)'$.

Steps 6 and 7: We draw λ_t conditional on other parameters from:

$$\lambda_t | \mathbf{y}, \mathbf{B}, \mathbf{\Sigma}, \boldsymbol{\gamma}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\vartheta} \sim \mathcal{IG}((\nu + n)/2, (\boldsymbol{\varepsilon}_t' \mathbf{\Sigma} \boldsymbol{\varepsilon}_t / \exp(h_t) + \nu)/2) \quad (\text{F.28})$$

Degrees-of-freedom, ν , are sampled via an independence-chain Metropolis-Hastings step (see Chan, 2020a, and references therein).

Step 8: To sample the starting values of the trend component, we first observe that $\boldsymbol{\mu}_0$ appears only in the first state equation i.e.:

$$\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0 + \boldsymbol{\varepsilon}_1^\mu \quad \boldsymbol{\varepsilon}_1^\mu \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \quad (\text{F.29})$$

Assuming the Gaussian prior distribution in (F.10) and applying standard regression results we derive the full conditional posterior density:

$$\boldsymbol{\mu}_0 | \mathbf{y}, \boldsymbol{\mu}_1, \mathbf{Q}, \sim \mathcal{N}(\hat{\boldsymbol{\mu}}_0, \mathbf{K}_{\mu 0}^{-1}) \quad (\text{F.30})$$

where $\mathbf{K}_{\mu 0} = \mathbf{V}_{\mu 0}^{-1} + \mathbf{Q}^{-1}$ and $\hat{\boldsymbol{\mu}}_0 = \mathbf{K}_{\mu 0}^{-1}(\mathbf{V}_{\mu 0}^{-1} \bar{\boldsymbol{\mu}}_0 + \mathbf{Q}^{-1} \boldsymbol{\mu}_1)$.

Step 9: The conditional posterior distribution for covariance matrix $\mathbf{Q}$ is:

$$\mathbf{Q} \sim \mathcal{IW}(w_{\varepsilon_\mu} + T, (w_{\varepsilon_\mu} + n + 1)\underline{\mathbf{Q}} + \boldsymbol{\varepsilon}^{\mu'} \boldsymbol{\varepsilon}^\mu) \quad (\text{F.31})$$

⁶We refer the interested reader to Chan (2017b) for details

where $\boldsymbol{\varepsilon}^\mu = (\boldsymbol{\varepsilon}_1^\mu, \dots, \boldsymbol{\varepsilon}_T^\mu)'$.

The proposed sampler can be easily modified to allow for constant volatility and/or Gaussian error terms. In practice, we set $\exp(h_t) = 1$ and/or $\lambda_t = 1 \forall t$ to estimate -t, -csv or models with standard Gaussian errors and constant volatility.

G Choice of priors

Prior specification follows established practice in the Bayesian VAR literature (Chan, 2020a; Del Negro et al., 2017). We set $\boldsymbol{\beta} = \mathbf{0}$ and specify $\mathbf{V}_B$ to mimic a Minnesota prior with overall tightness equal to 0.2. For the covariance matrix $\boldsymbol{\Sigma}$, we employ a diffuse inverse-Wishart prior with $w_\varepsilon = n + 3$ degrees of freedom, corresponding to the minimum value that ensures the existence of the prior mean. The scale matrix is assumed diagonal and calibrated using residual information from the data, namely

$$\underline{\boldsymbol{\Sigma}} = \text{diag}(\mathbf{s}_\varepsilon),$$

where $\mathbf{s}_\varepsilon$ denotes the vector of mean squared residuals obtained from a VAR(4) estimated by ordinary least squares over the full sample. For the stochastic volatility block, we set $\nu_{h0} = 5$, $S_{h0} = 0.01$, while for the degrees of freedom, ν we specify $a = 2$, and $b = 50$.

Turning to the specification of priors for $\boldsymbol{\mu}_t$, we set the prior means for the initial state, $\boldsymbol{\mu}_0$, according to the second column of Table G.1 with prior covariance matrix $\mathbf{V}_{\mu 0} = 0.1 \times \mathbf{I}_{n_\mu}$. The diagonal elements of $\underline{\mathbf{Q}}$ are reported in the third column of Table G.1. We impose highly persistent trend dynamics by assigning small innovation variances to the elements of $\underline{\mathbf{Q}}$. Because the natural rate of interest is decomposed into four distinct components, i.e. $\overline{lp}_t$, $\overline{hr}_t$, $\overline{cy}_t$, and $\overline{ro}_t$, we calibrate the corresponding variances to be four times smaller than the benchmark value of 1/400 adopted by Del Negro et al. (2017) in their baseline specification without such a decomposition. For $\overline{ro}_t$, we impose an even tighter prior variance (1/3200) to mitigate potential identification problems arising from the absence of a directly observable counterpart. The prior variances assigned to the remaining trend components are chosen according to the same principle of ensuring smooth low-frequency dynamics. Finally, we set $w_{\epsilon_\mu} = 100$ which translates in a highly informative

prior as in Del Negro et al. (2017).

Table G.1: Prior specification for trends

Trends	$\bar{\boldsymbol{\mu}}_0$	diag($\underline{\mathbf{Q}}$)
$\overline{lp}_t$	2	1/1600
$\overline{hr}_t$	0.25	1/1600
$\overline{\pi}_t$	2	1/200
$\overline{\pi}_t^d$	0	1/400
$\overline{ro}_t$	0	1/3200
$\overline{tp}_t$	0.75	1/400
$\overline{co}_t$	0	1/3200
$\overline{cy}_t$	1	1/1600
$\overline{ur}_t$	6.75	1/400

Finally, for the steady-state $\boldsymbol{\gamma}$, prior means are calibrated using pre-sample averages computed over 40 observations whenever sufficient historical data are available. The prior covariance matrix is set to $\mathbf{V}_{\boldsymbol{\gamma}} = \mathbf{I}_{n_{\boldsymbol{\gamma}}}$ which is informative enough as it is required in steady-state models Villani (2009).

H Additional results

This section presents additional empirical results for the estimated trend components, $\boldsymbol{\mu}_t$, and unconditional means, $\boldsymbol{\tau}_t$, across all model specifications. We also report the estimated contributions of the drivers of r^* for all models, with the exception of the LSM-t-sv specification, whose results are presented in the main text.

H.1 Estimates of μ_t and τ_t across alternative Models

H.1.1 BM (baseline) model

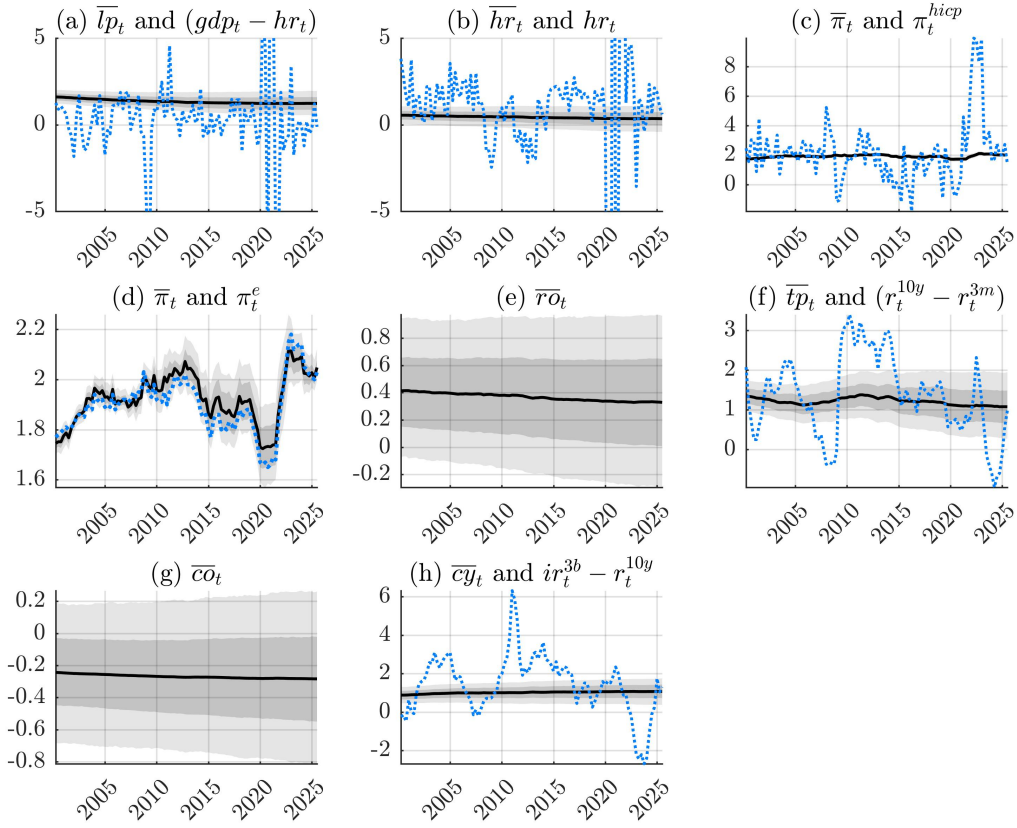


Figure H.1: Estimates of μ_t based on the BM specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

H.1.2 BM-t model

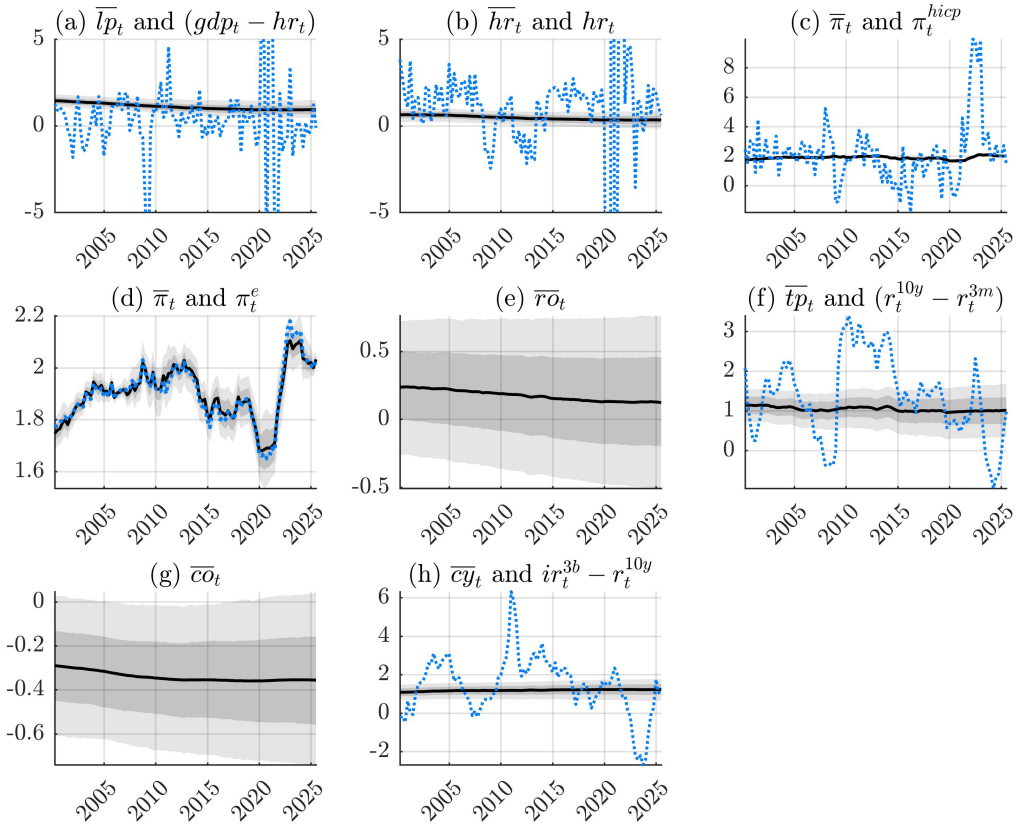


Figure H.2: Estimates of $\boldsymbol{\mu}_t$ based on the BM-t specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

H.1.3 BM-sv model

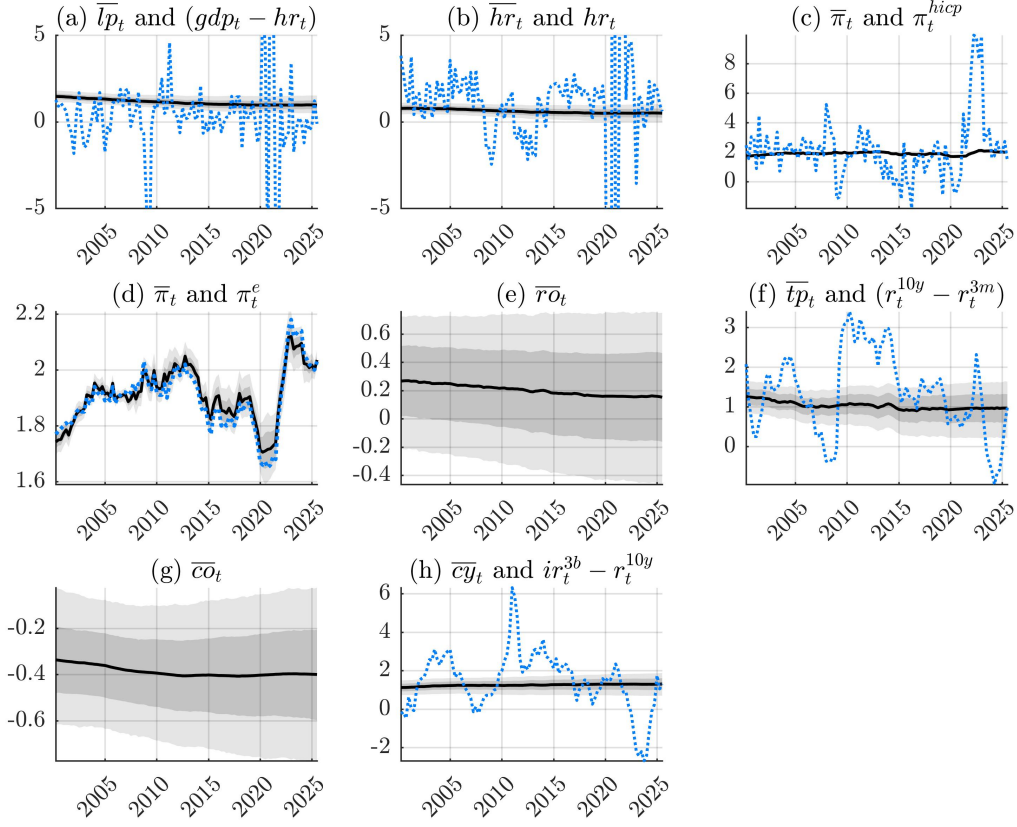


Figure H.3: Estimates of μ_t based on the BM-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

H.1.4 BM-sv-t model

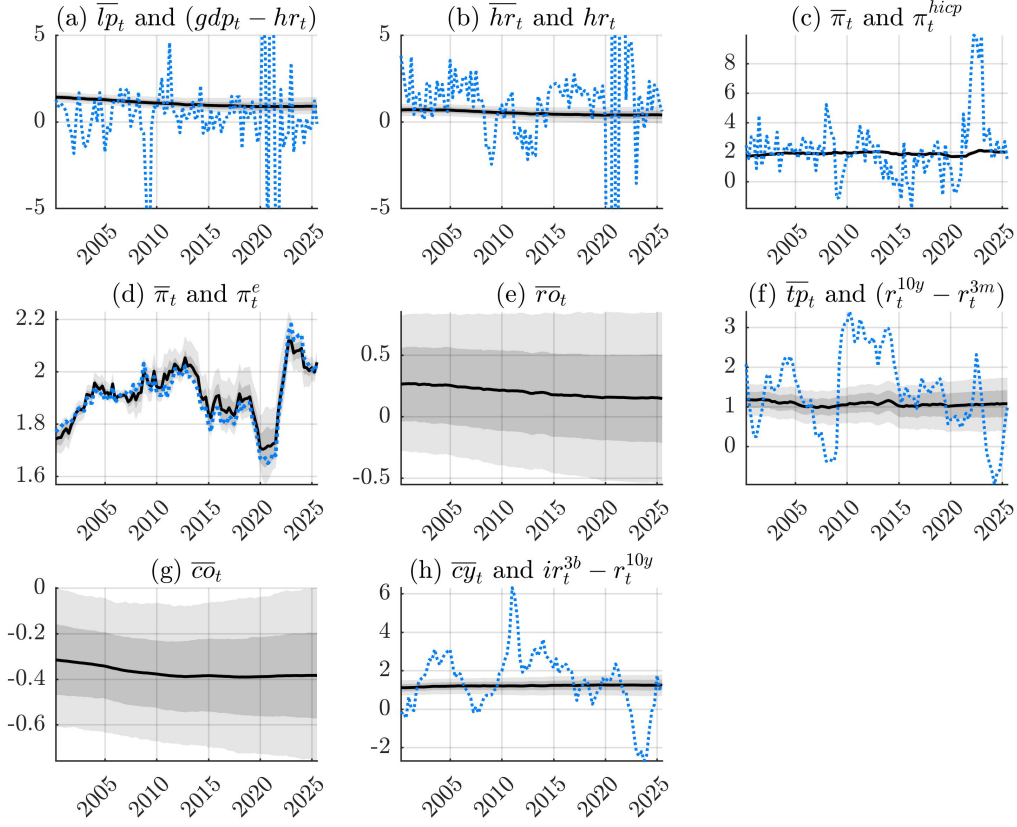


Figure H.4: Estimates of $\boldsymbol{\mu}_t$ based on the BM-t-sv specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

H.1.5 LSM model

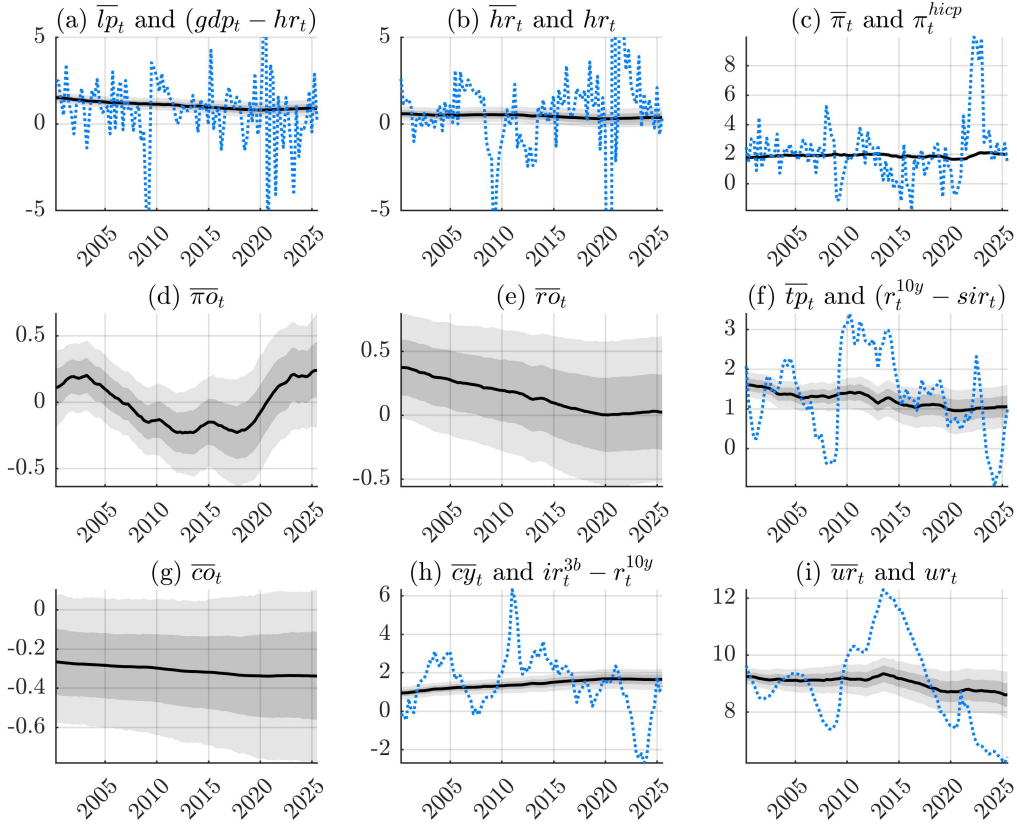


Figure H.5: Estimates of μ_t based on the LSM specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

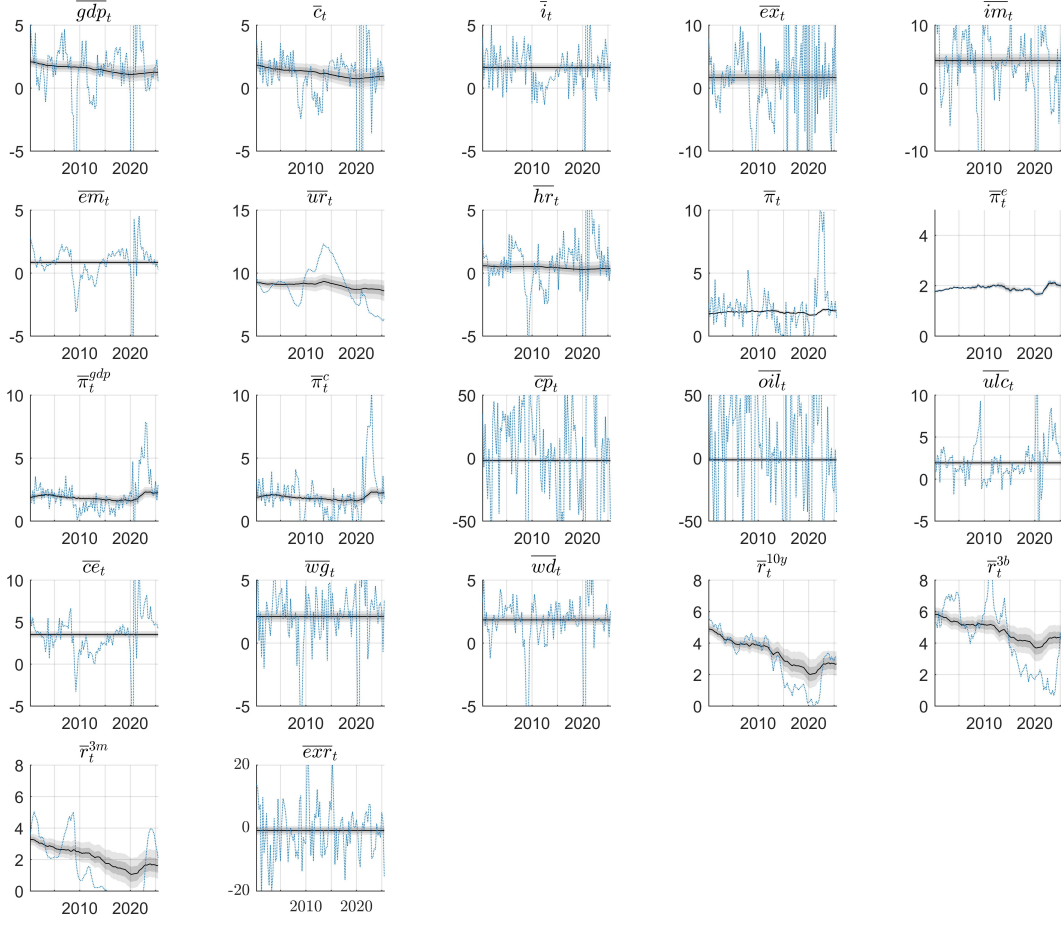


Figure H.6: Estimates of τ_t based on the LSM specification. Black lines denote the posterior mean of the elements of τ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables.

H.1.6 LSM-t model

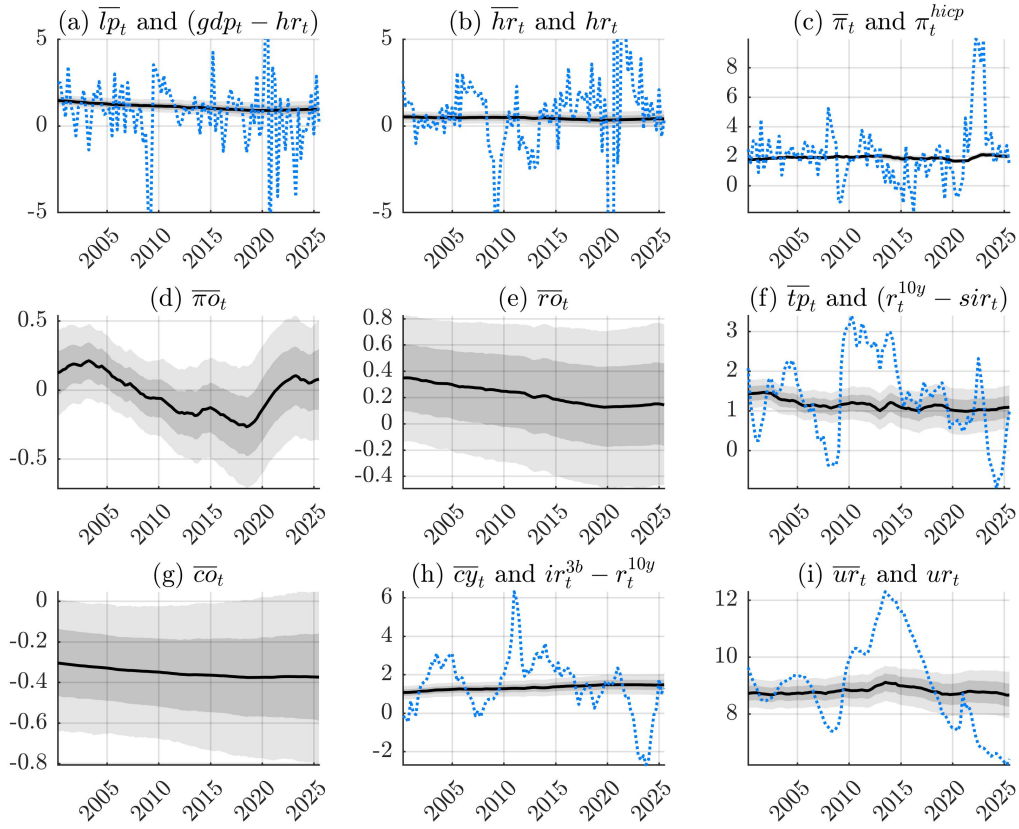


Figure H.7: Estimates of μ_t based on the LSM-t specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

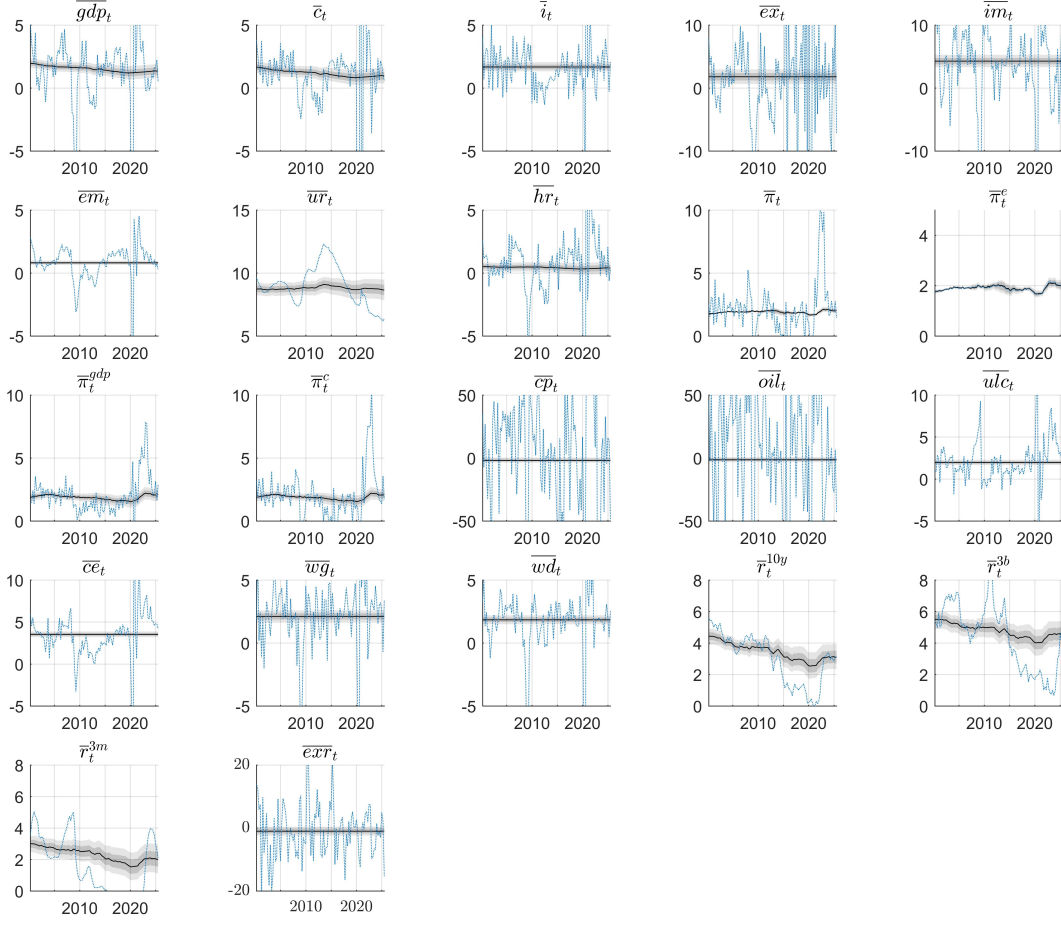


Figure H.8: Estimates of τ_t based on the LSM-t specification. Black lines denote the posterior mean of the elements of τ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables.

H.1.7 LSM-sv model

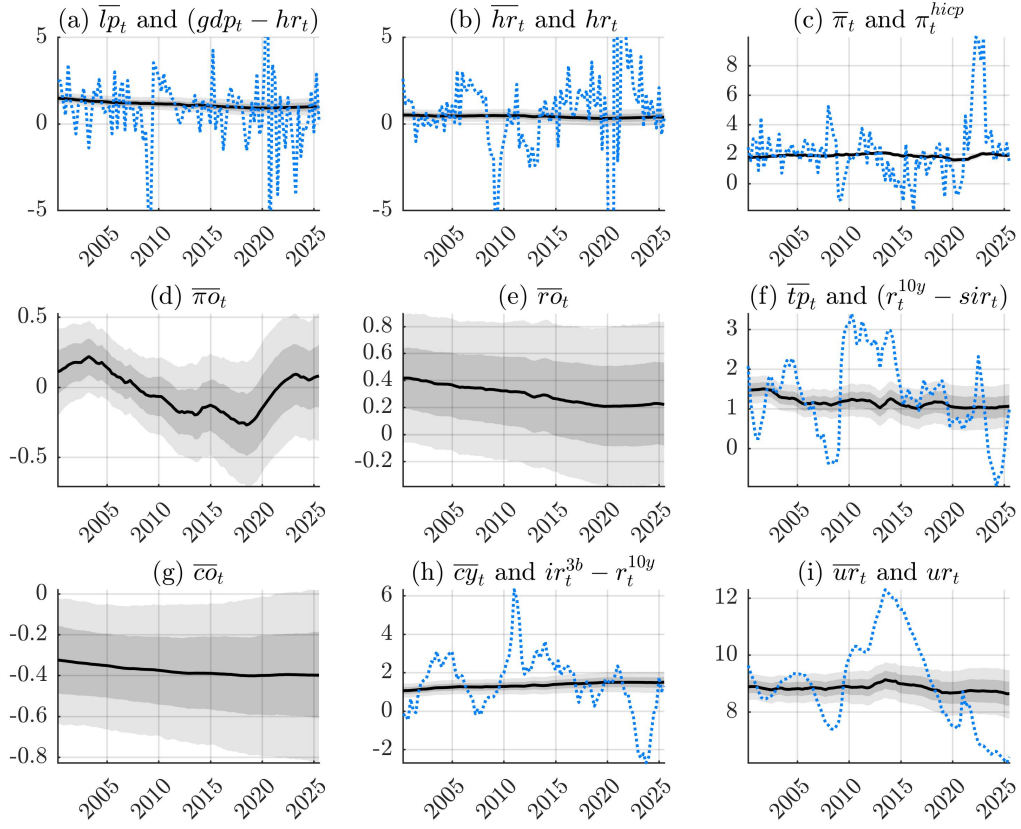


Figure H.9: Estimates of μ_t based on the LSM-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

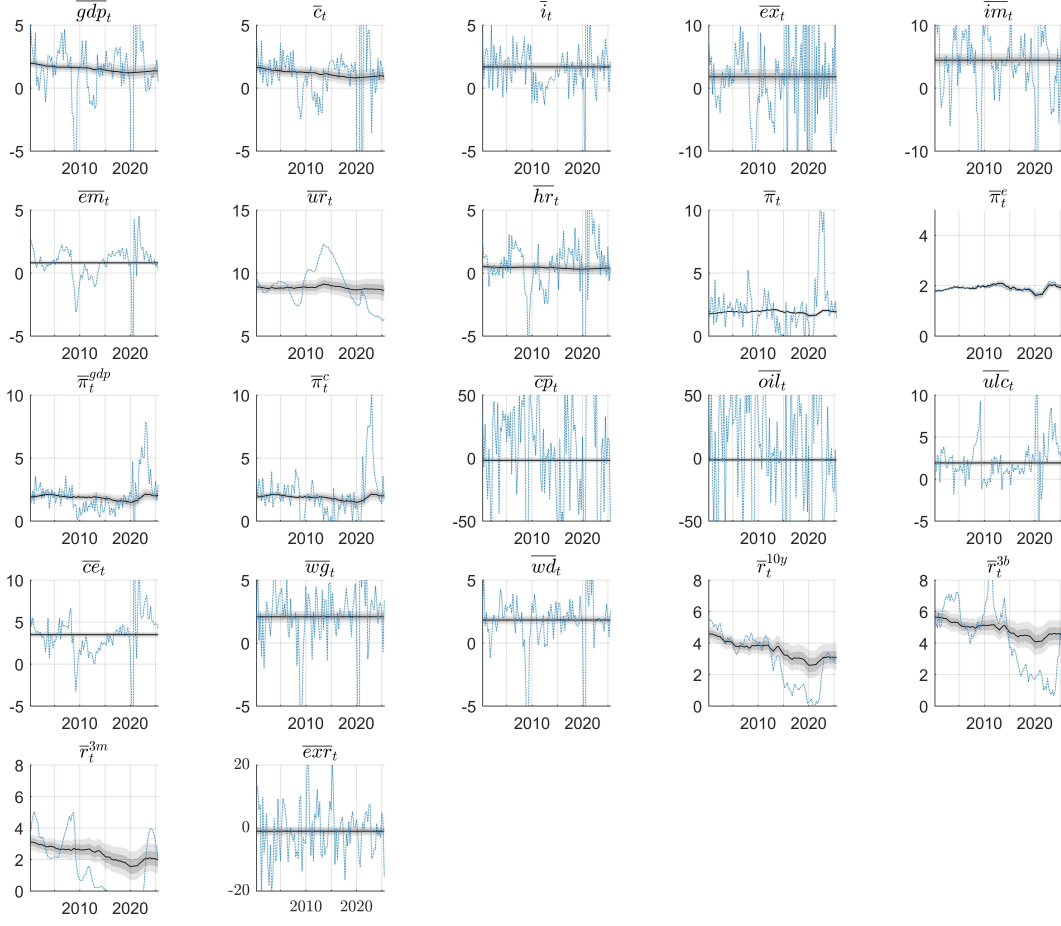


Figure H.10: Estimates of τ_t based on the LSM-sv specification. Black lines denote the posterior mean of the elements of τ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables.

H.1.8 LSM-sv-t model

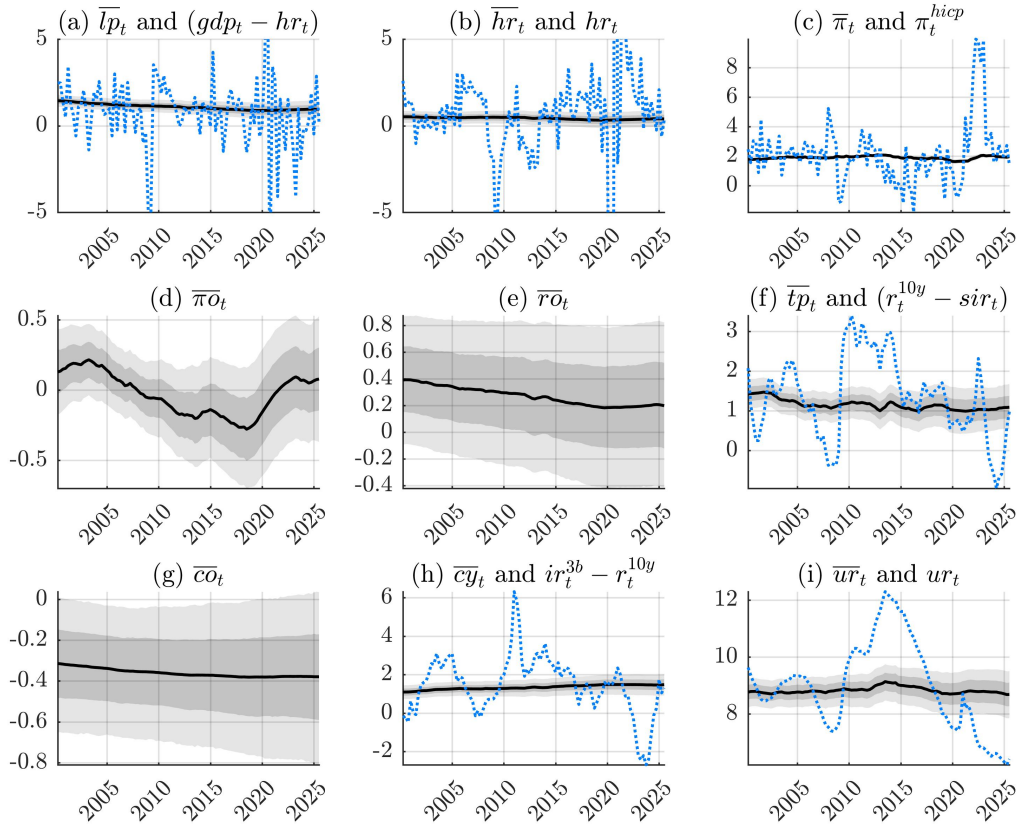


Figure H.11: Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

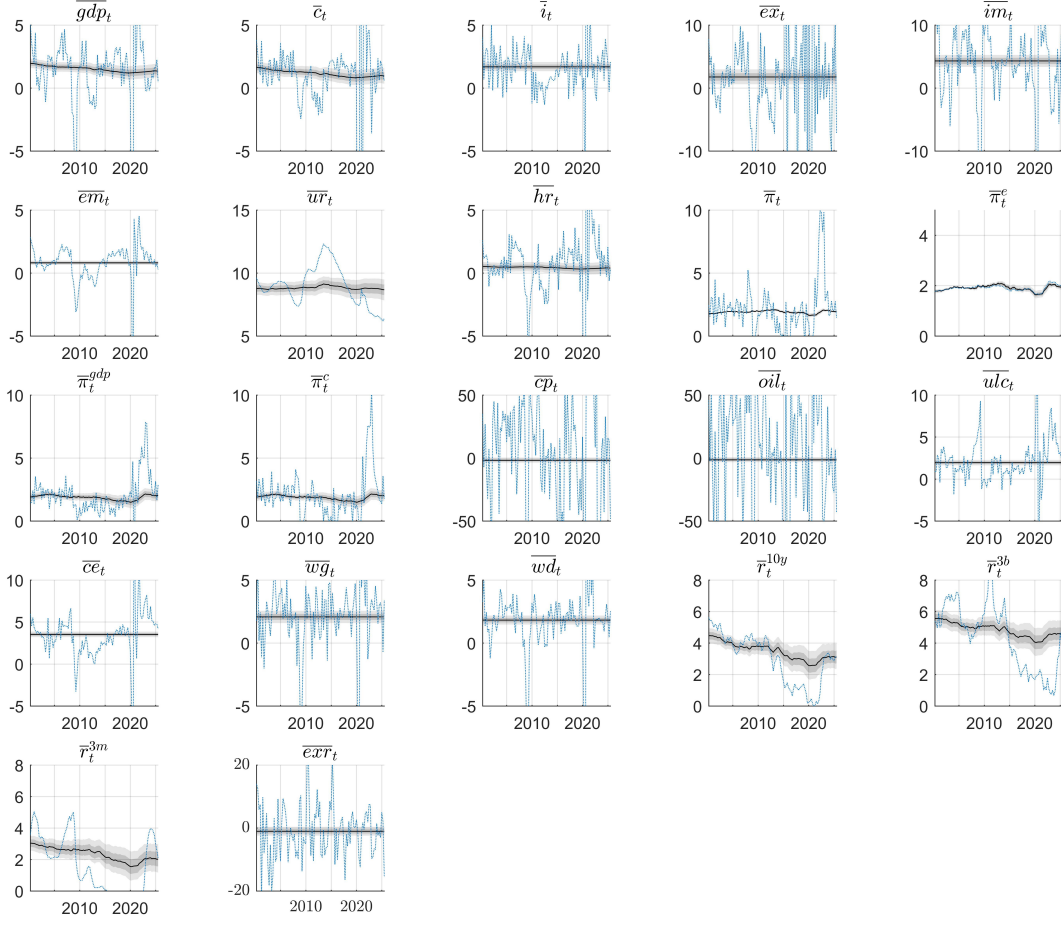


Figure H.12: Estimates of τ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of τ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables.

H.2 Estimates of the drivers of r^* across alternative models

H.2.1 BM (baseline) model

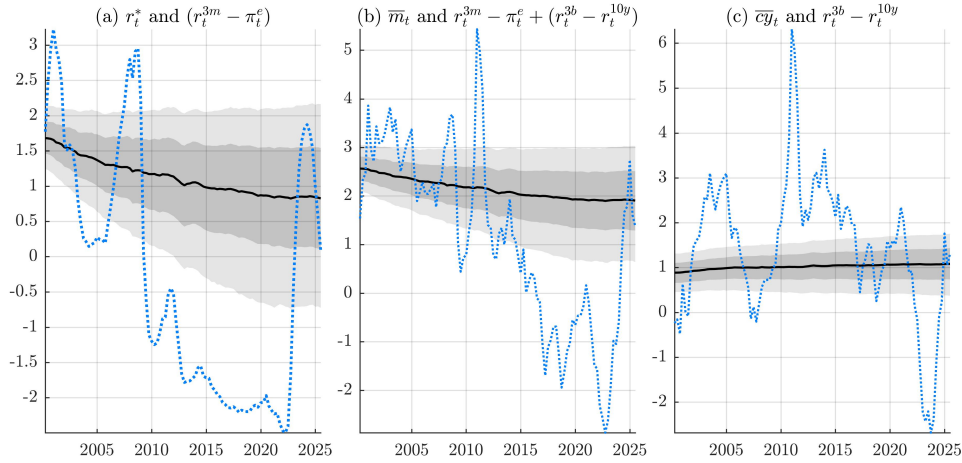


Figure H.13: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

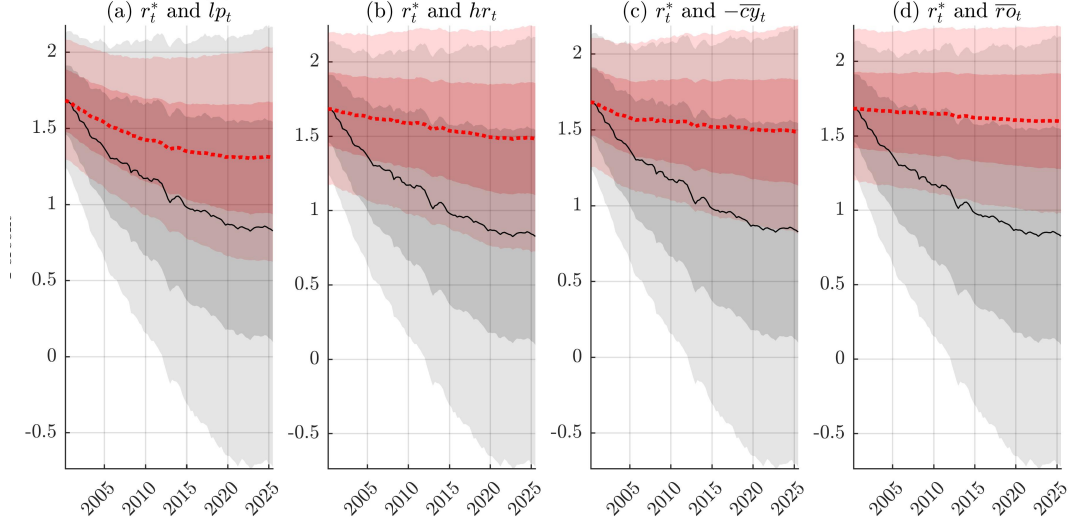


Figure H.14: Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

H.2.2 BM-t model

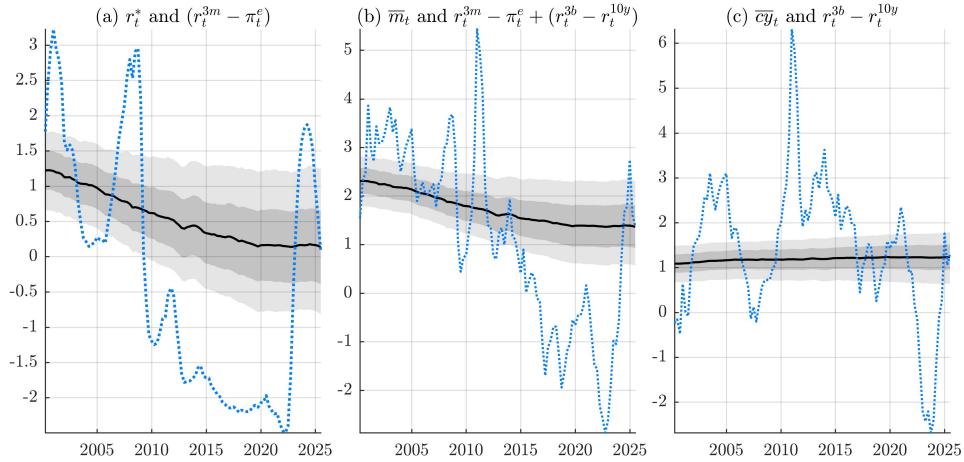


Figure H.15: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM-t specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

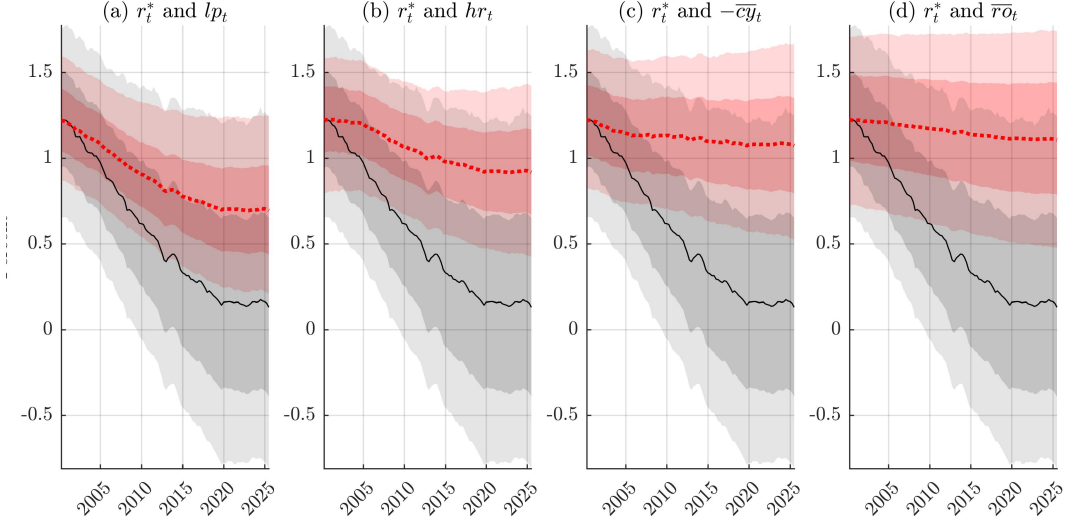


Figure H.16: Decomposition of the natural interest rate, r_t^* , based on the BM-t specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

H.2.3 BM-sv model

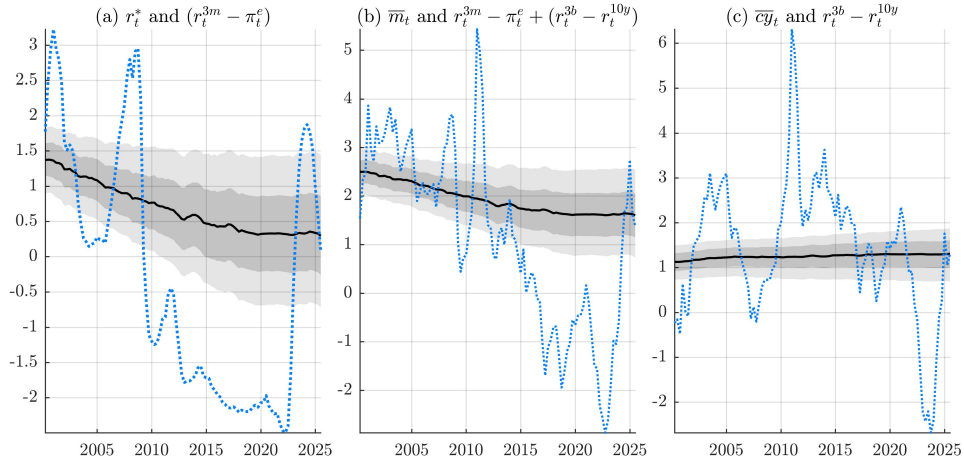


Figure H.17: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

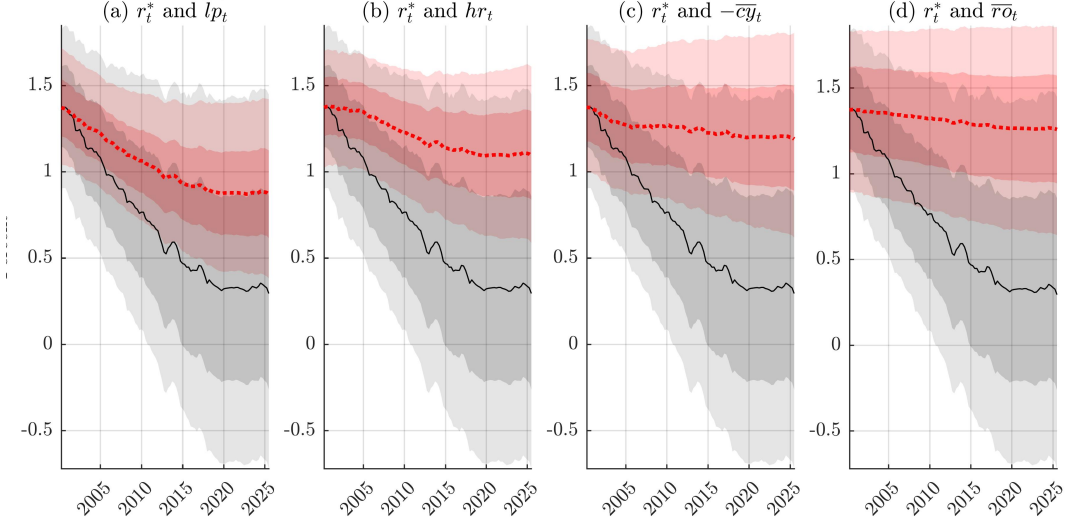


Figure H.18: Decomposition of the natural interest rate, r_t^* , based on the BM-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

H.2.4 BM-t-sv model

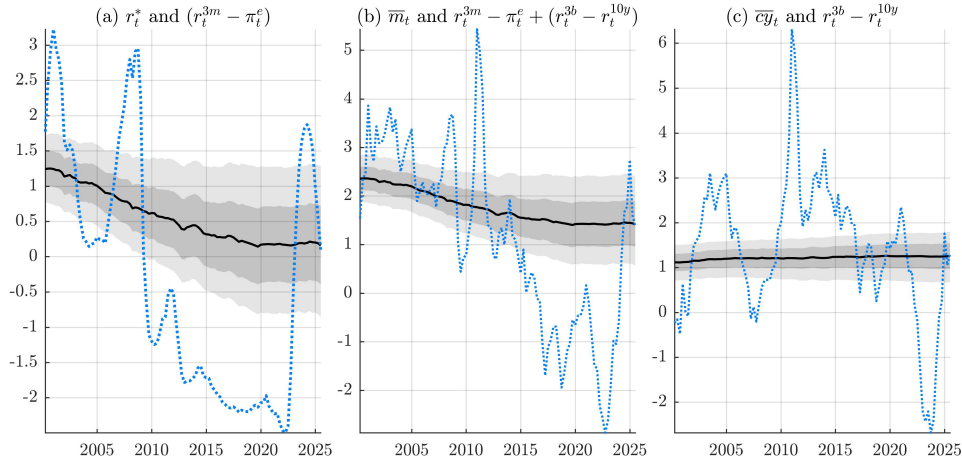


Figure H.19: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

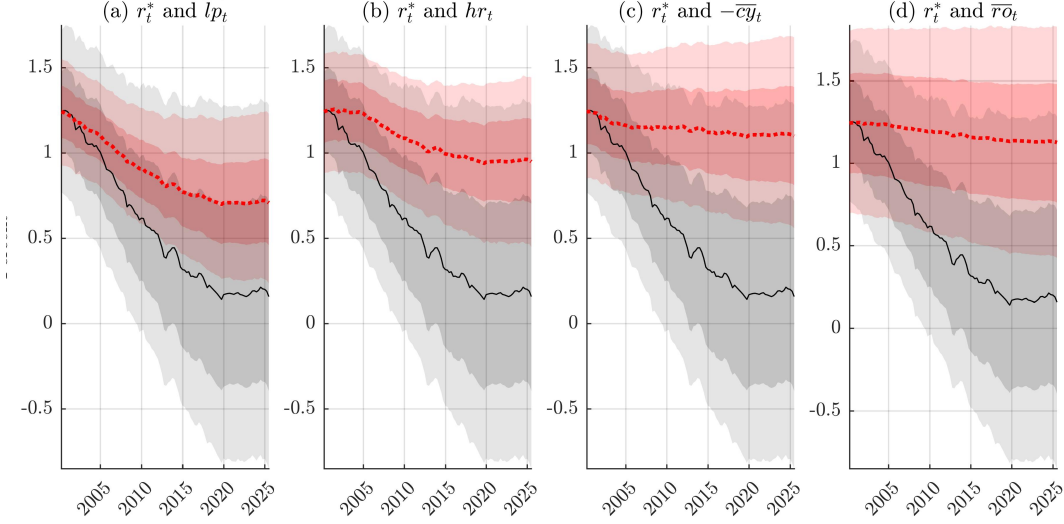


Figure H.20: Decomposition of the natural interest rate, r_t^* , based on the BM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\overline{l p}_t$, $\overline{h r}_t$, $-\overline{c y}_t$, and $\overline{r o}_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\overline{l p}_t$, $\overline{h r}_t$, $-\overline{c y}_t$, and $\overline{r o}_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

H.2.5 LSM model

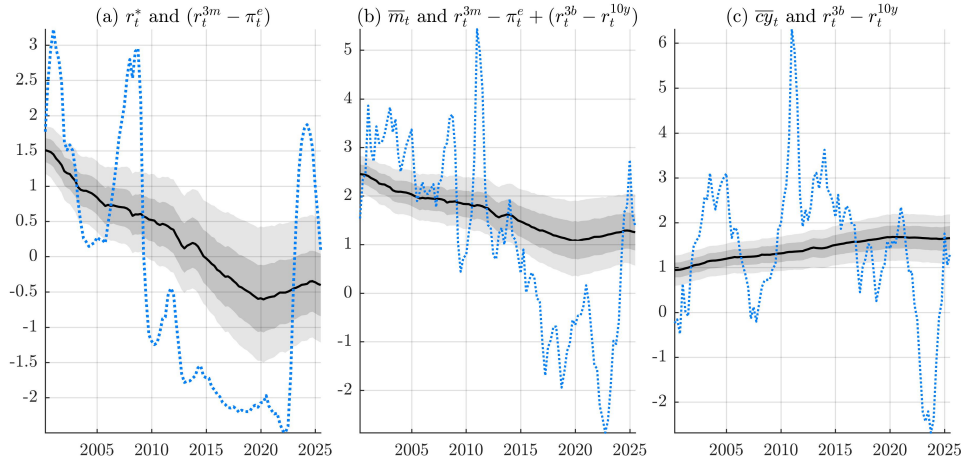


Figure H.21: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\overline{m}_t$, and the convenience yield, $\overline{c y}_t$, based on the LSM specification. Black lines denote the posterior means of r_t^* , $\overline{m}_t$, and $\overline{c y}_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

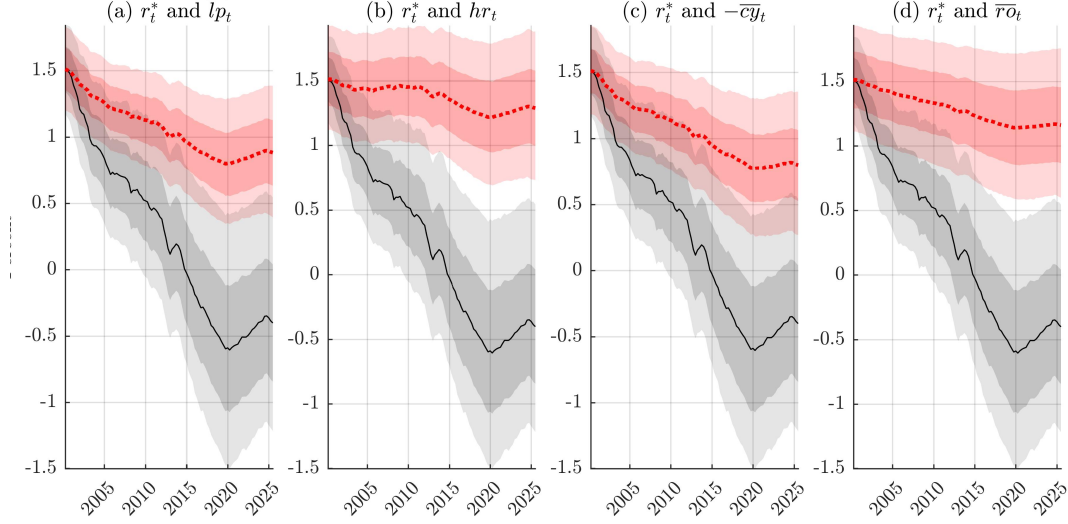


Figure H.22: Decomposition of the natural interest rate, r_t^* , based on the LSM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

H.2.6 LSM-t model

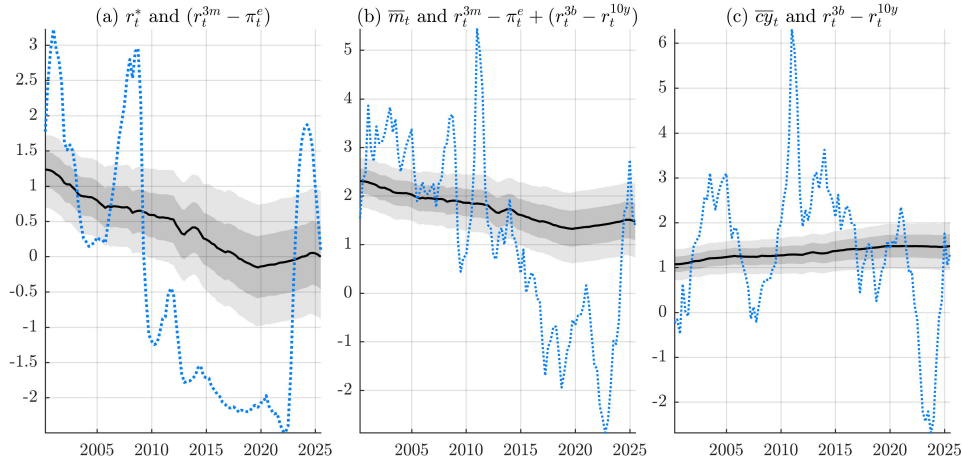


Figure H.23: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

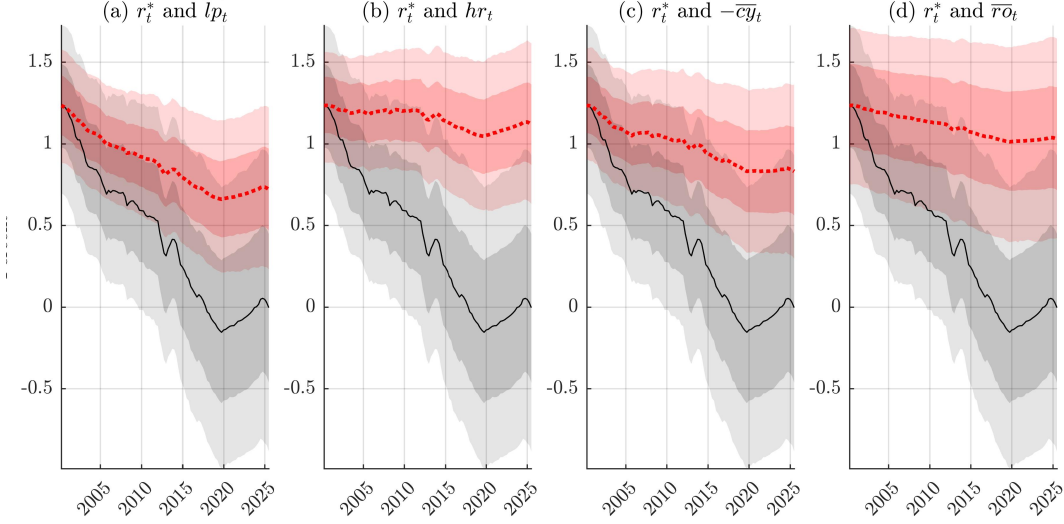


Figure H.24: Decomposition of the natural interest rate, r_t^* , based on the LSM-t specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

H.2.7 LSM-sv model

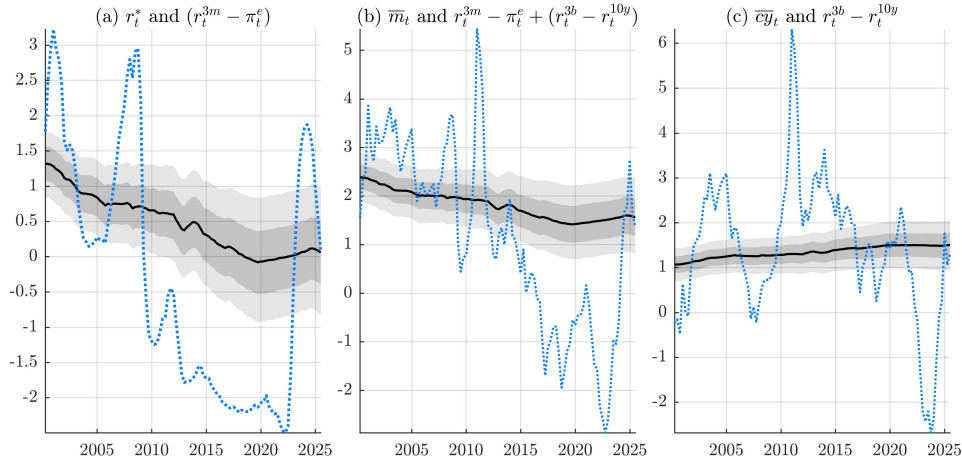


Figure H.25: Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

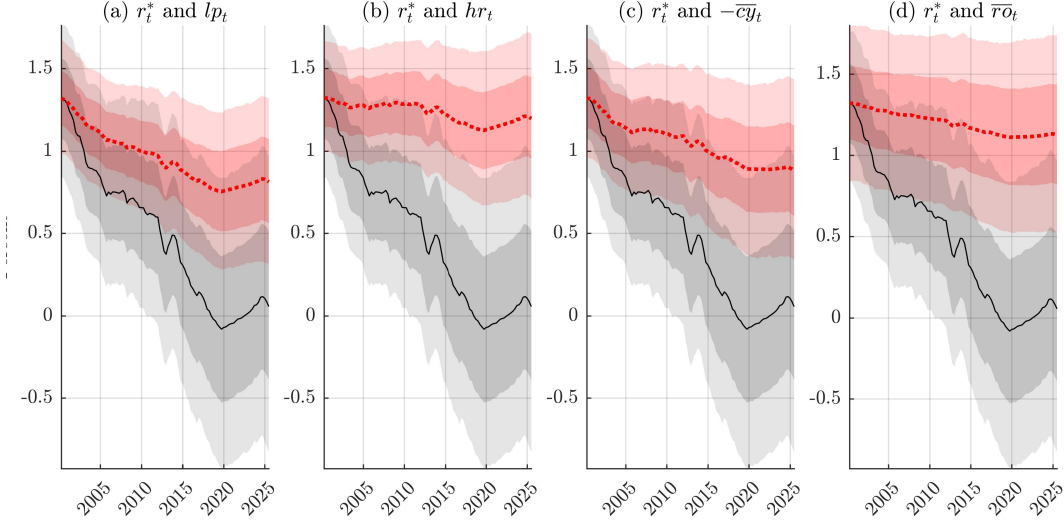


Figure H.26: Decomposition of the natural interest rate, r_t^* , based on the LSM-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

I Sensitivity of r^* estimates to the prior choice of $\mathbf{Q}$

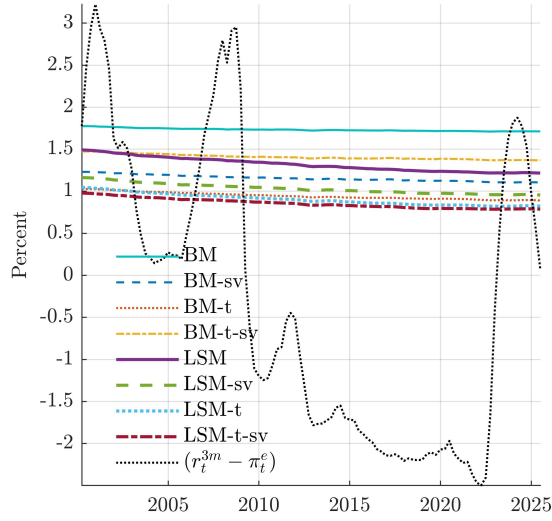
This section presents a comprehensive sensitivity analysis of the prior specification for the innovation covariance matrix, $\mathbf{Q}$, which governs the evolution of the trend component, $\boldsymbol{\mu}_t$.

To assess the robustness of the estimated models, we consider alternative values for both the scale matrix, $\mathbf{Q}$, and the degrees-of-freedom parameter, w_{ε_μ} , of the inverse-Wishart prior, and re-estimate all model specifications. Specifically, we define an alternative scale matrix as $\mathbf{Q}^* = k\mathbf{Q}$, where $k \in \{0.1, 0.5, 2, 10\}$ and $\mathbf{Q}$ is specified as in Table G.1. This allows us to examine priors with scale matrices that are 10 and 2 times smaller, as well as 2 and 10 times larger, than the baseline specification used in the main text.

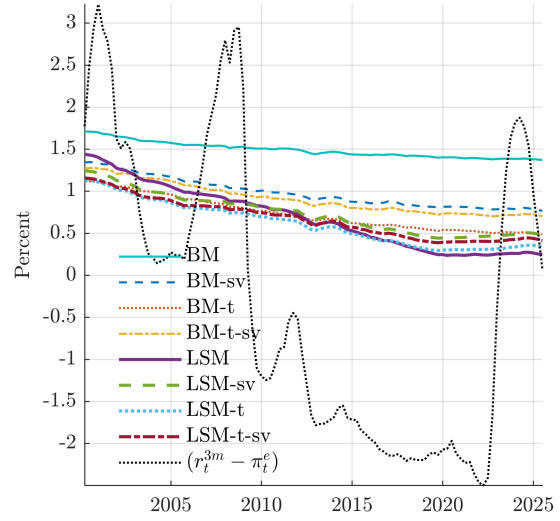
In addition, we vary the degrees-of-freedom parameter by considering $w_{\varepsilon_\mu} \in \{9, 14, 50, 200\}$ for the small-scale models and $w_{\varepsilon_\mu} \in \{24, 29, 50, 200\}$ for the large-scale models. The values 9 and 24 correspond to the minimum admissible degrees-of-freedom, namely $w_{\varepsilon_\mu} = n_\mu + 2$, required for a well-defined inverse-Wishart prior. The values 14 and 29 imply weakly informative priors, while $w_{\varepsilon_\mu} = 50$ yields a moderately informative prior that is less

informative than the baseline specification adopted in the main text ($w_{\varepsilon_\mu} = 100$). Finally, $w_{\varepsilon_\mu} = 200$ corresponds to a highly informative prior.

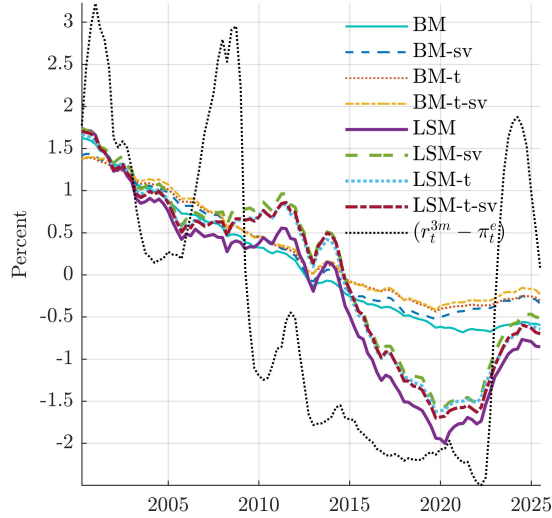
I.1 Estimates of r^* using alternative scale parameters, $\underline{Q}$



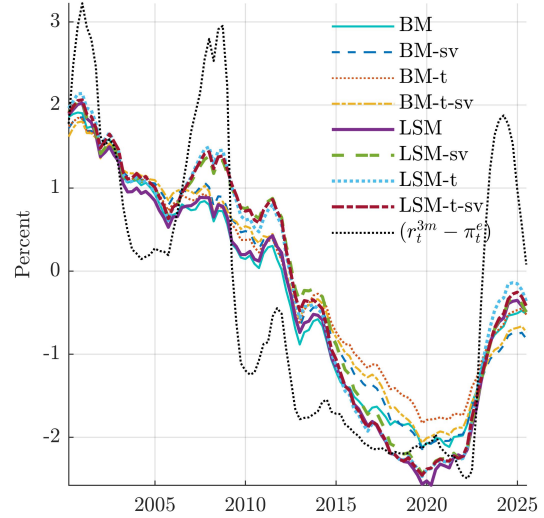
(a) $\underline{Q}^* = 0.1 \times \underline{Q}$



(b) $\underline{Q}^* = 0.5 \times \underline{Q}$



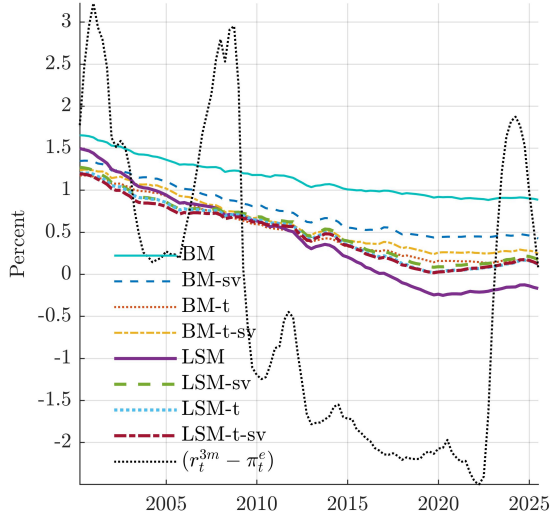
(c) $\underline{Q}^* = 2 \times \underline{Q}$



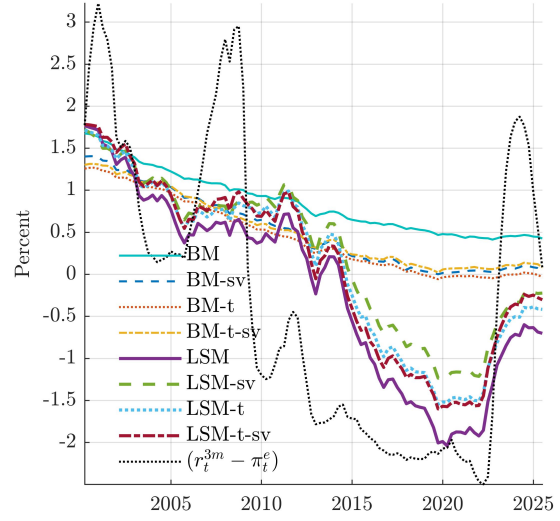
(d) $\underline{Q}^* = 10 \times \underline{Q}$

Figure I.1: Posterior means of r_t^* for alternative values of scale parameter $\underline{Q}$

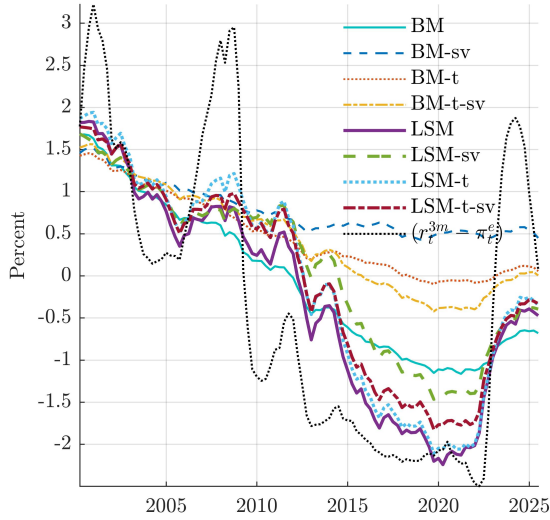
I.2 Estimates of r^* using alternative degrees-of-freedom, w_{ε_μ}



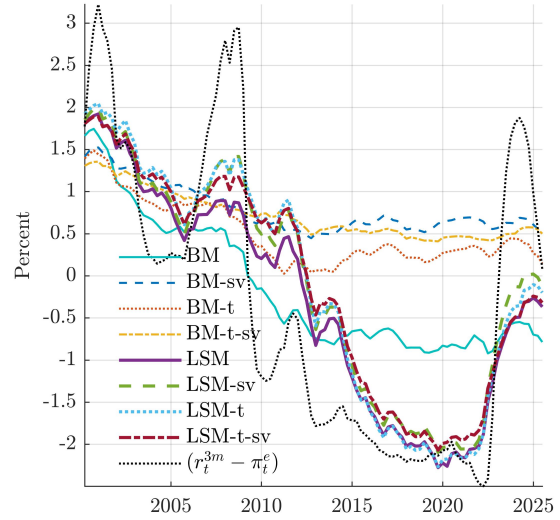
(a) $w_{\varepsilon_\mu} = 200$



(b) $w_{\varepsilon_\mu} = 50$



(c) $w_{\varepsilon_\mu} = 29$ and 14 for LSM- and BM- models, respectively



(d) $w_{\varepsilon_\mu} = 24$ and 9 for LSM- and BM- models, respectively

Figure I.2: Posterior means of r_t^* for alternative values of degrees-of-freedom, w_{ε_μ}

I.3 Comparing the estimates of r^* using standard (see Table G.1) and alternative scale parameters $\underline{Q}$

I.3.1 BM (baseline) model

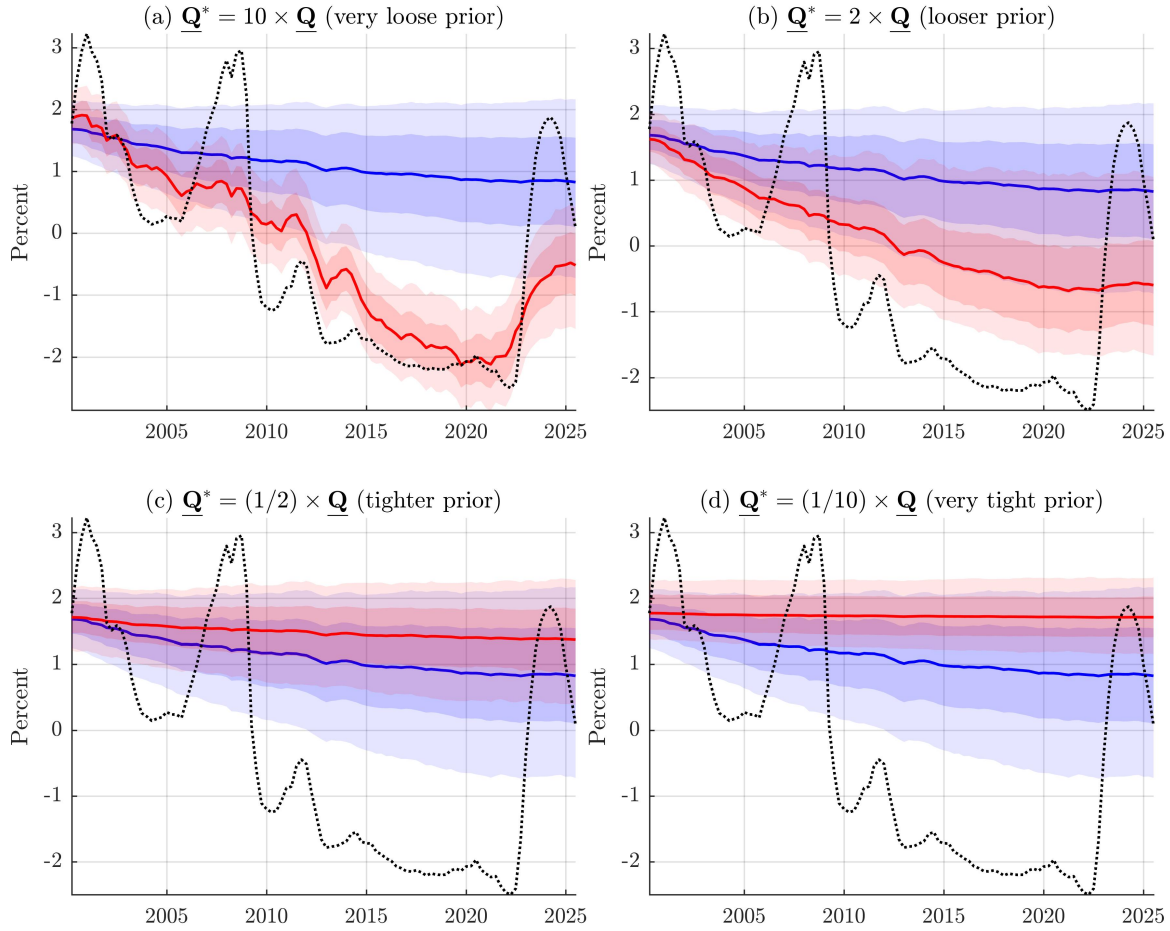


Figure I.3: Sensitivity analysis on prior choice for innovations covariance matrix $\underline{Q}$ using the BM (baseline) model. Blue line denotes the posterior mean of the natural interest rate, r_t^* , based on the standard $\underline{Q}$ specification of Table G.1. The blue (light blue) shaded areas represent the 68% (95%) posterior credible intervals. Red line denotes the posterior mean of the natural interest rate, r_t^* , based on alternative $\underline{Q}$ prior specifications using the BM model. The red (light red) shaded areas represent the 68% (95%) posterior credible intervals. The black dotted line shows the short-term real rate $r_t^{3m} - \pi_t^e$.

I.3.2 LSM-t-sv model

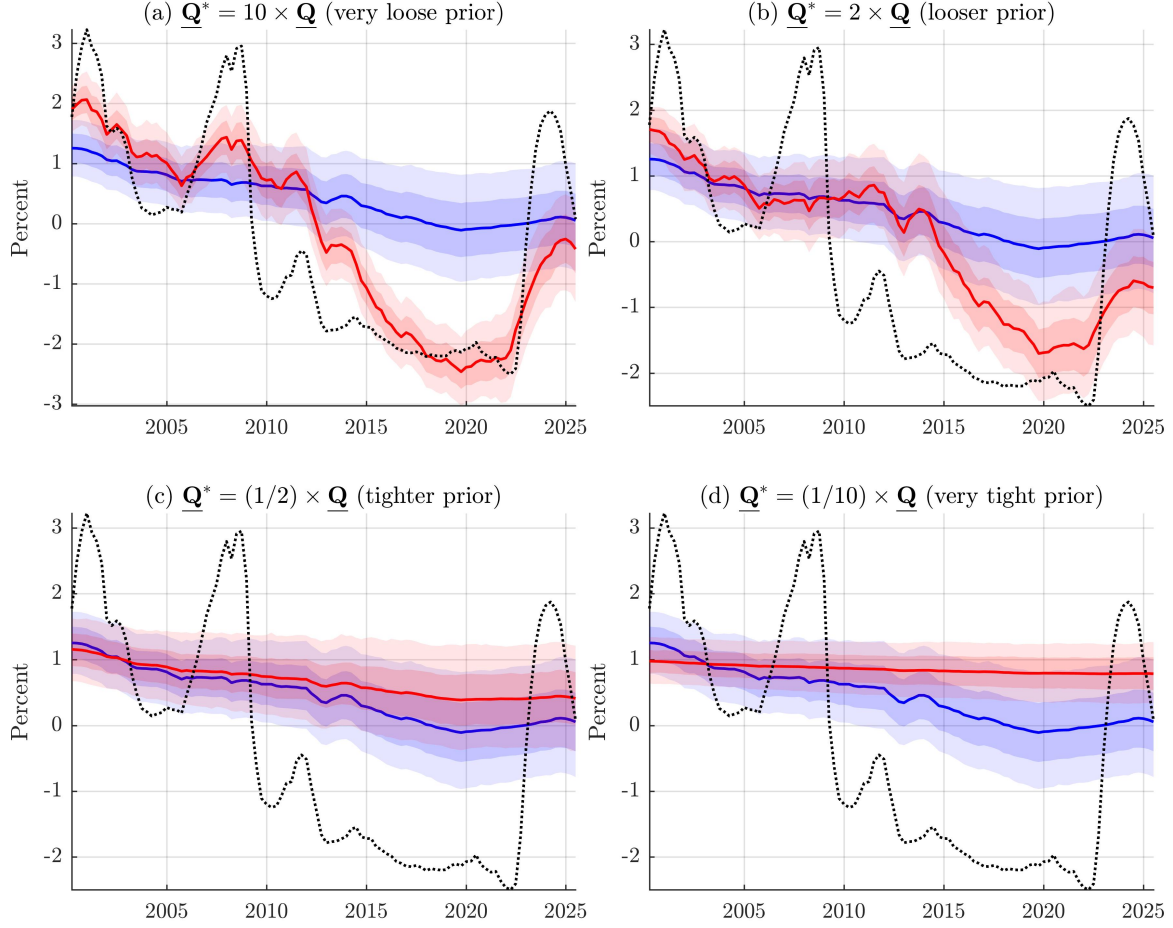


Figure I.4: Sensitivity analysis on prior choice for innovations covariance matrix $\underline{\mathbf{Q}}$ using the LSM-t-sv model. Blue line denotes the posterior mean of the natural interest rate, r_t^* , based on the standard $\underline{\mathbf{Q}}$ specification of Table G.1. The blue (light blue) shaded areas represent the 68% (95%) posterior credible intervals. Red line denotes the posterior mean of the natural interest rate, r_t^* , based on alternative $\underline{\mathbf{Q}}$ prior specifications. The red (light red) shaded areas represent the 68% (95%) posterior credible intervals. The black dotted line shows the short-term real rate $r_t^{3m} - \pi_t^e$.

I.4 Comparing estimates of r^* using standard ($w_{\varepsilon_\mu} = 100$) and alternative degrees-of-freedom, w_{ε_μ}

I.4.1 BM (baseline) model

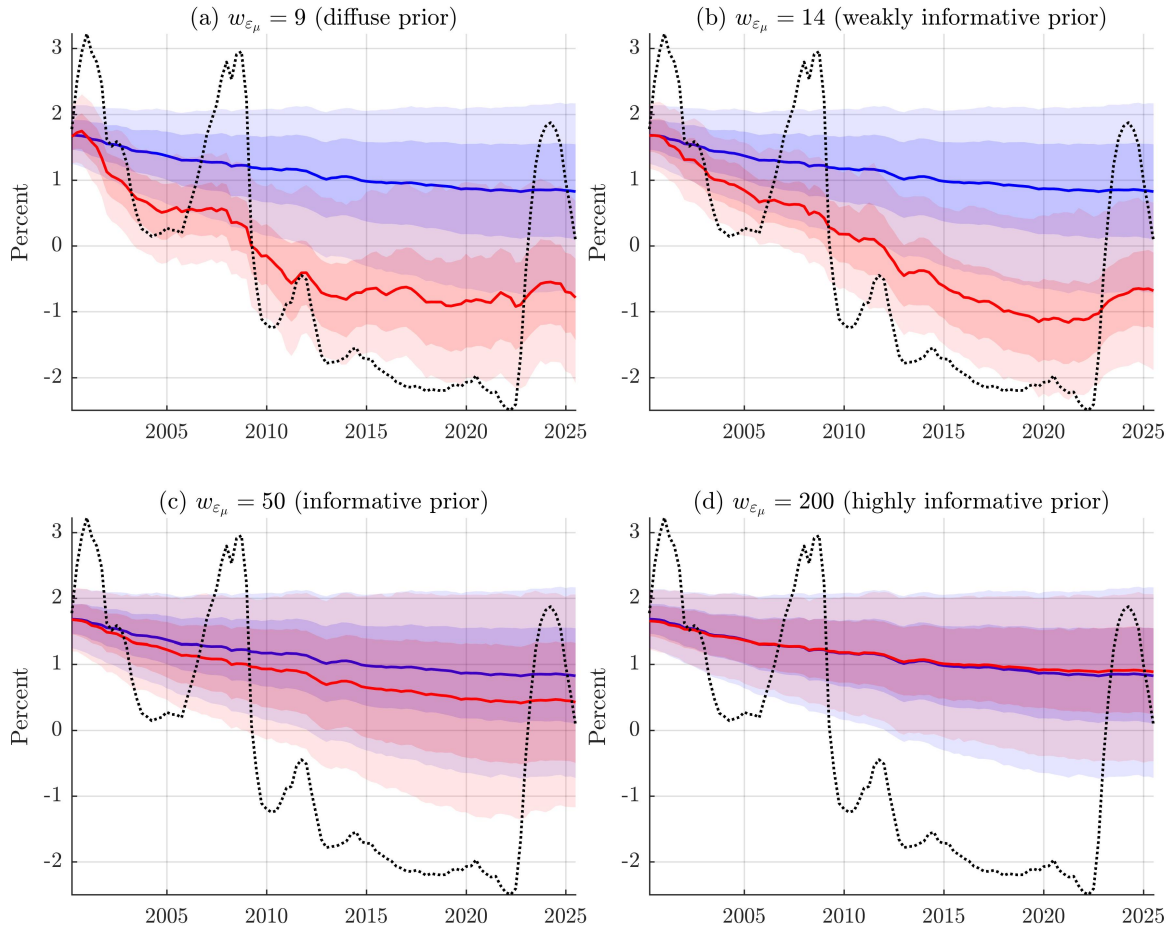


Figure I.5: Sensitivity analysis on prior choice for innovations covariance matrix $\mathbf{Q}$ using the BM (baseline) model. Blue line denotes the posterior mean of the natural interest rate, r_t^* , based on the standard $\mathbf{Q}$ specification of Table G.1. The blue (light blue) shaded areas represent the 68% (95%) posterior credible intervals. Red line denotes the posterior mean of the natural interest rate, r_t^* , based on alternative $\mathbf{Q}$ prior specifications using the BM model. The red (light red) shaded areas represent the 68% (95%) posterior credible intervals. The black dotted line shows the short-term real rate $r_t^{3m} - \pi_t^e$.

I.4.2 LSM-t-sv model

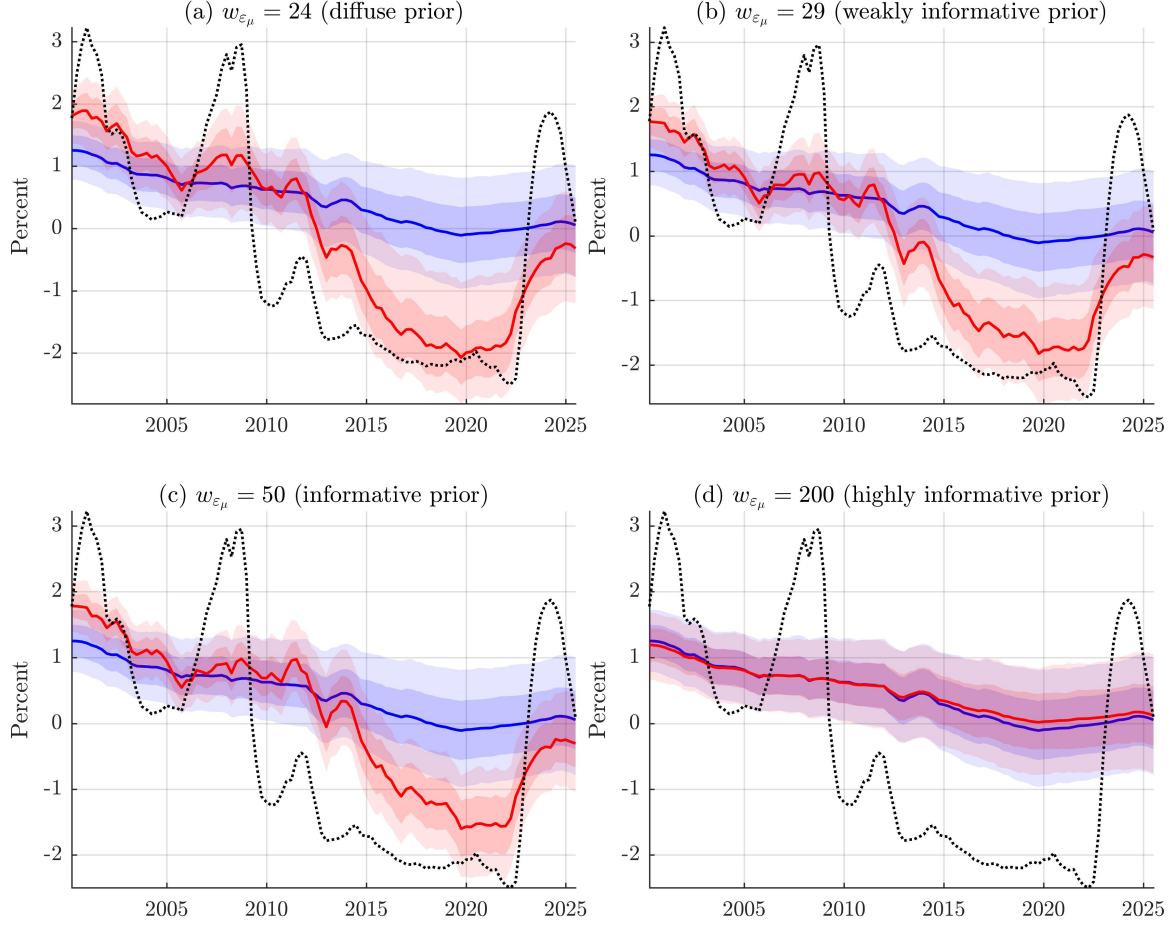
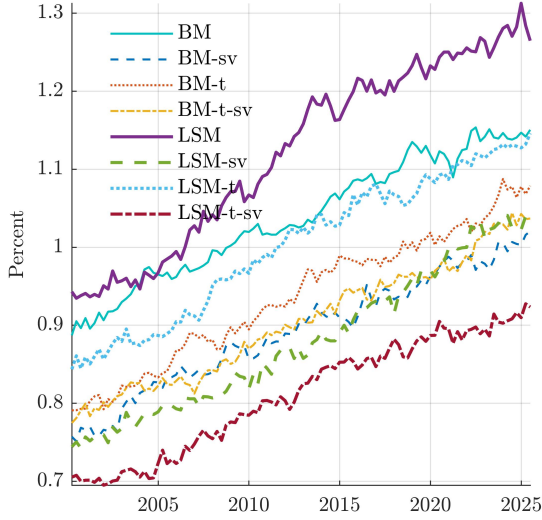


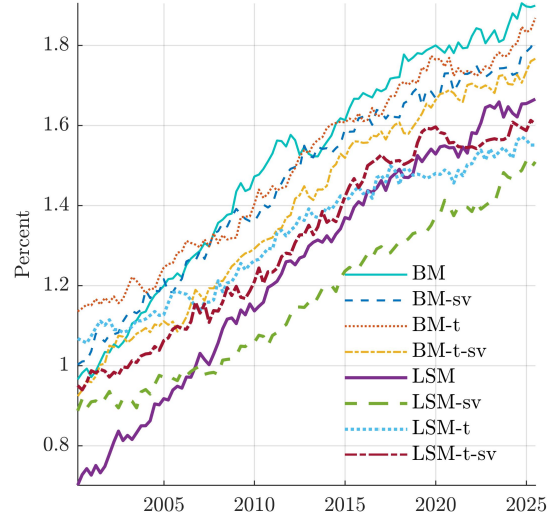
Figure I.6: Sensitivity analysis on prior choice for innovations covariance matrix $\mathbf{Q}$ using the LSM-t-sv model. Blue line denotes the posterior mean of the natural interest rate, r_t^* , based on the standard $\mathbf{Q}$ specification of Table G.1. The blue (light blue) shaded areas represent the 68% (95%) posterior credible intervals. Red line denotes the posterior mean of the natural interest rate, r_t^* , based on alternative $\mathbf{Q}$ prior specifications. The red (light red) shaded areas represent the 68% (95%) posterior credible intervals. The black dotted line shows the short-term real rate $r_t^{3m} - \pi_t^e$.

J Sensitivity of the width of credible intervals for r^* to the prior choice of $\underline{Q}$

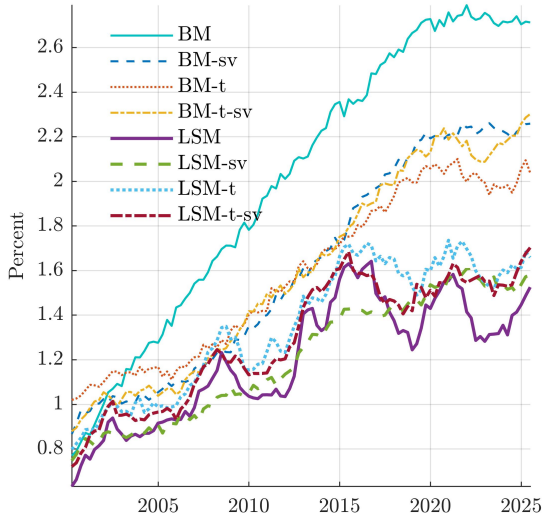
J.1 Width of credible intervals for r^* for alternative values of scale parameter, $\underline{Q}$



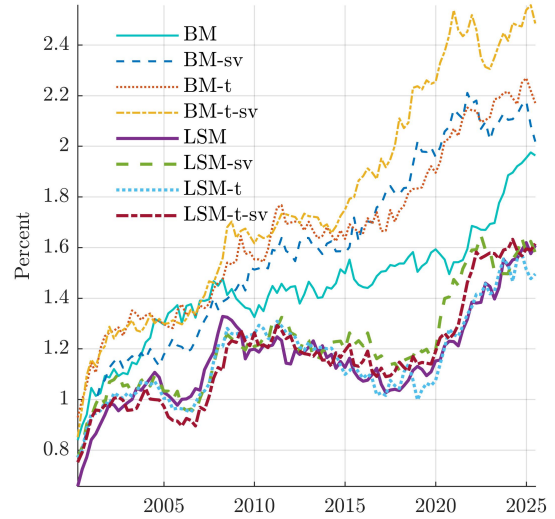
(a) $\underline{Q}^* = 0.1 \times \underline{Q}$



(b) $\underline{Q}^* = 0.5 \times \underline{Q}$



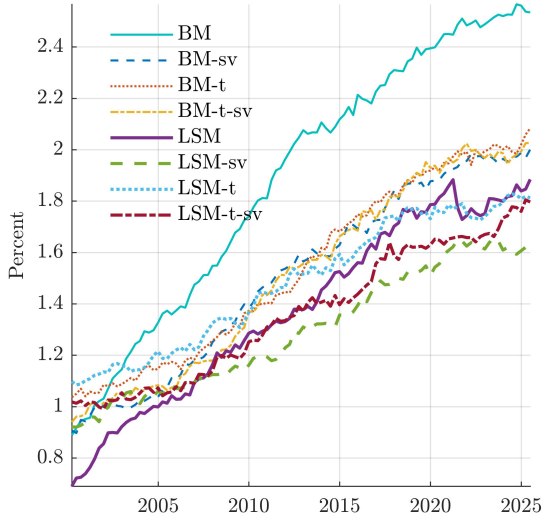
(c) $\underline{Q}^* = 2 \times \underline{Q}$



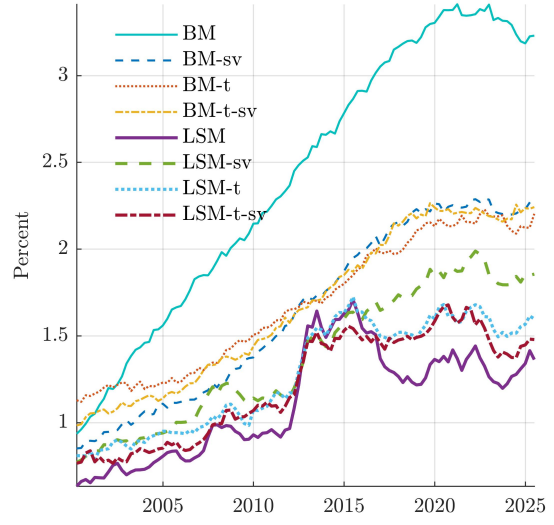
(d) $\underline{Q}^* = 10 \times \underline{Q}$

Figure J.1: Width of posterior 95% credible intervals for r_t^* for alternative values of scale parameter, $\underline{Q}$

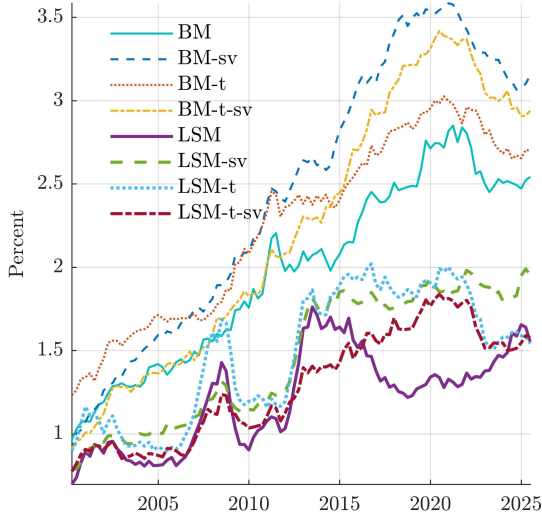
J.2 Width of credible intervals for r^* for alternative values of degrees-of-freedom w_{ε_μ}



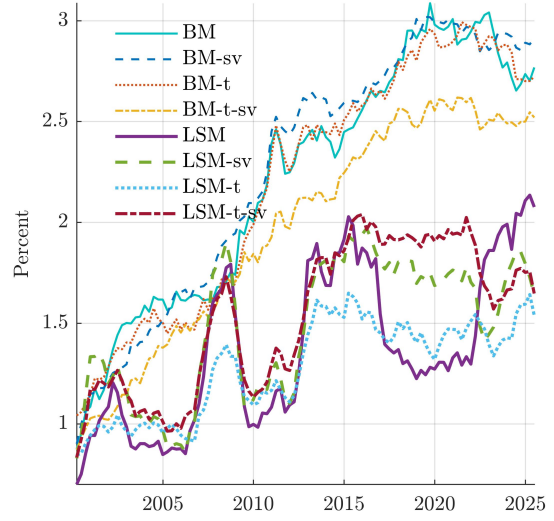
(a) $w_{\varepsilon_\mu} = 200$



(b) $w_{\varepsilon_\mu} = 50$



(c) $w_{\varepsilon_\mu} = 29$ and 14 for LSM- and BM- models, respectively



(d) $w_{\varepsilon_\mu} = 24$ and 9 for LSM- and BM- models, respectively

Figure J.2: Width of posterior 95% credible intervals for r_t^* for alternative values of degrees-of-freedom, w_{ε_μ}

K Sensitivity of estimates of the r^* drivers to the prior choice of $\mathbf{Q}$

K.1 Estimates of r^* , stochastic discount factor and convenience yield using alternative scale parameters, $\underline{\mathbf{Q}}$

K.1.1 BM (baseline) model

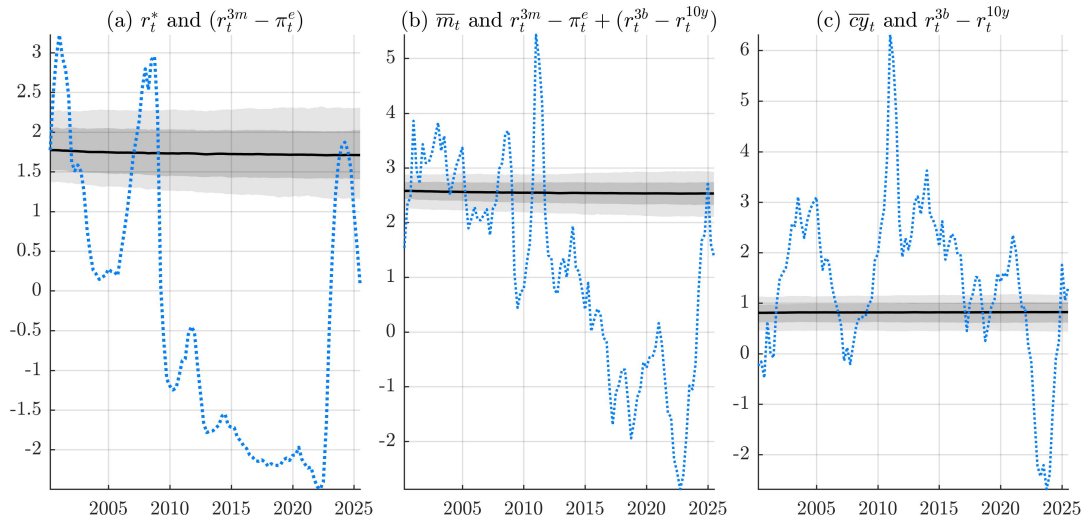


Figure K.1: **Results for: $\underline{\mathbf{Q}}^* = 0.1 \times \underline{\mathbf{Q}}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

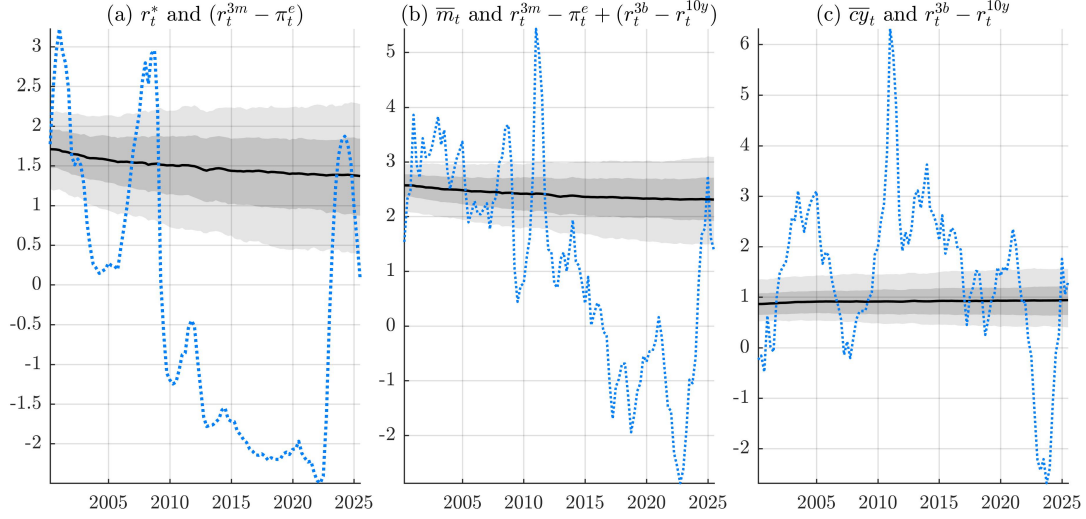


Figure K.2: **Results for: $\underline{Q}^* = 0.5 \times \underline{Q}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{cy}_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{cy}_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

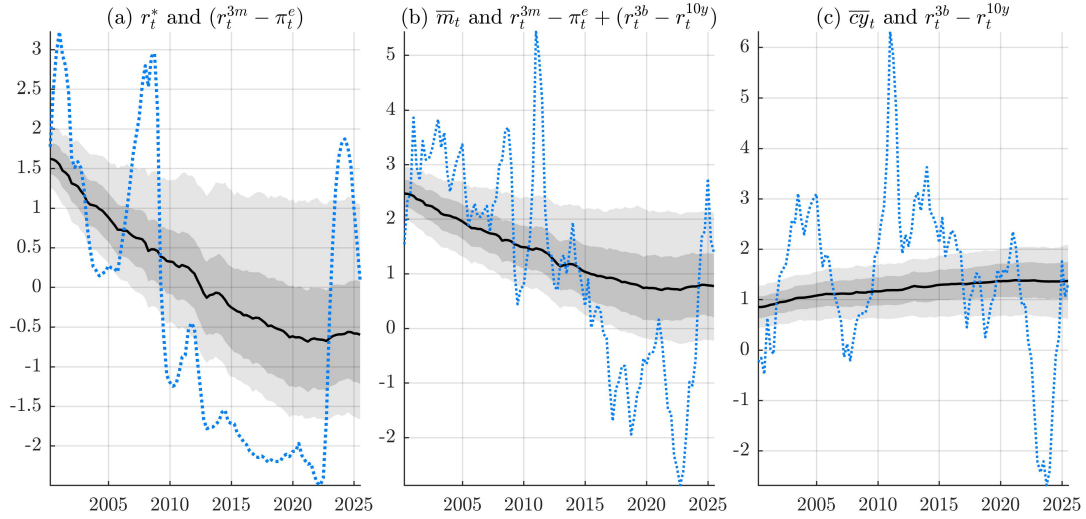


Figure K.3: **Results for: $\underline{Q}^* = 2 \times \underline{Q}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{cy}_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{cy}_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

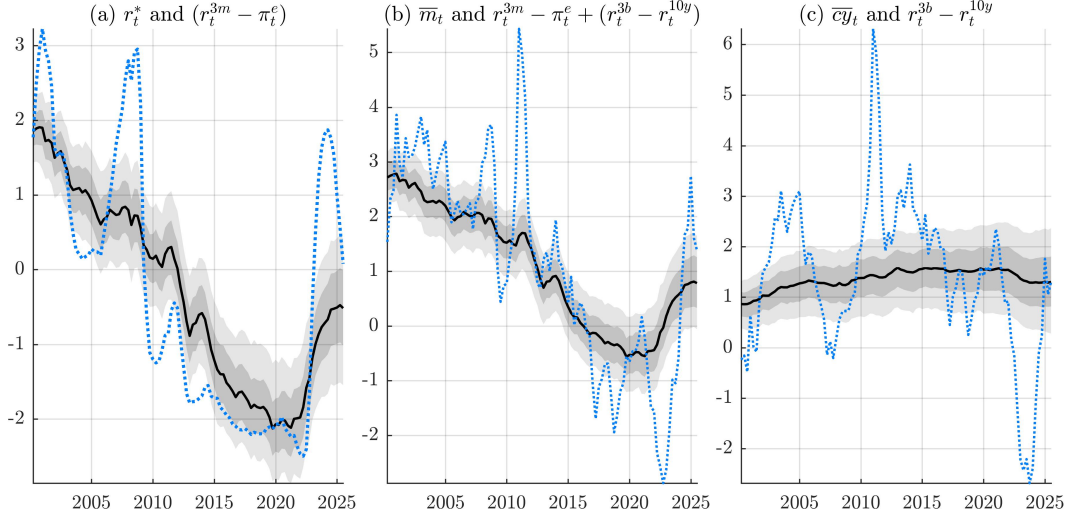


Figure K.4: **Results for: $\underline{\mathbf{Q}}^* = 10 \times \underline{\mathbf{Q}}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{cy}_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{cy}_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

K.1.2 LSM-t-sv model

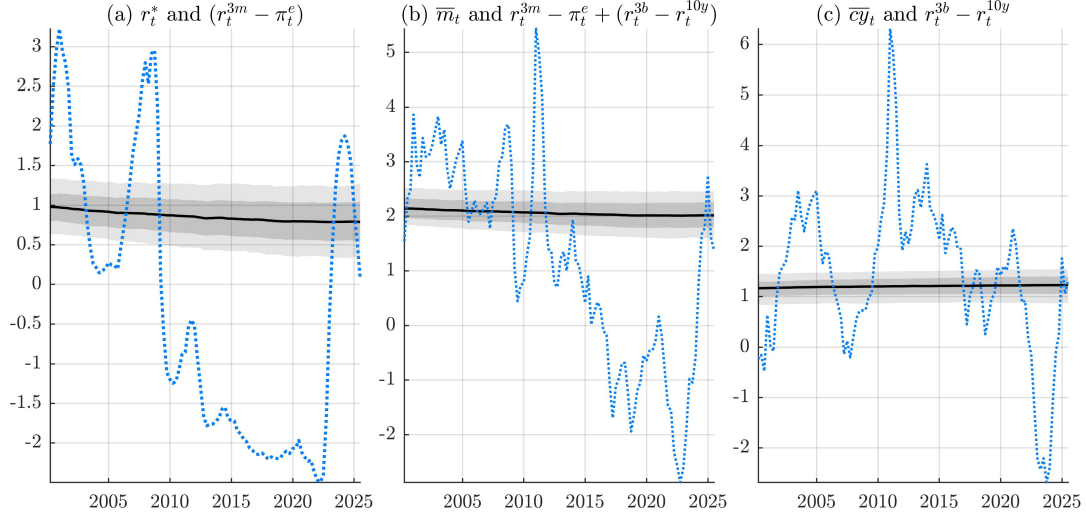


Figure K.5: **Results for: $\underline{\mathbf{Q}}^* = 0.1 \times \underline{\mathbf{Q}}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{cy}_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{cy}_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

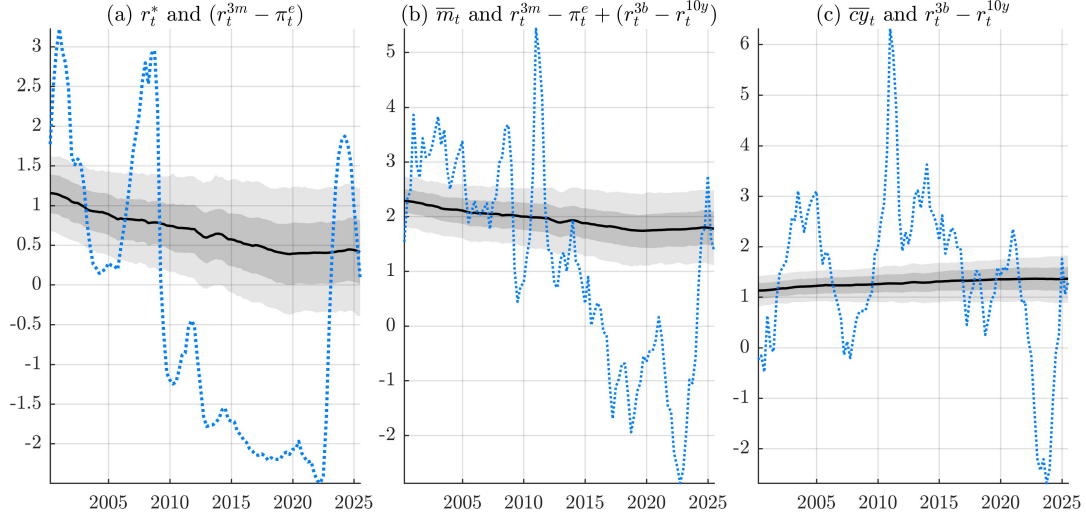


Figure K.6: **Results for: $\underline{\mathbf{Q}}^* = 0.5 \times \underline{\mathbf{Q}}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

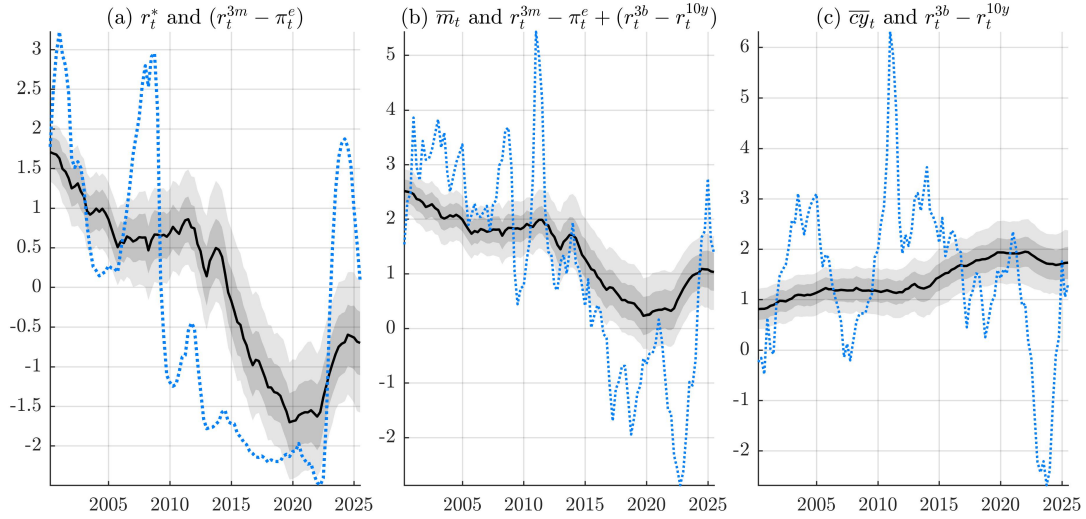


Figure K.7: **Results for: $\underline{\mathbf{Q}}^* = 2 \times \underline{\mathbf{Q}}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

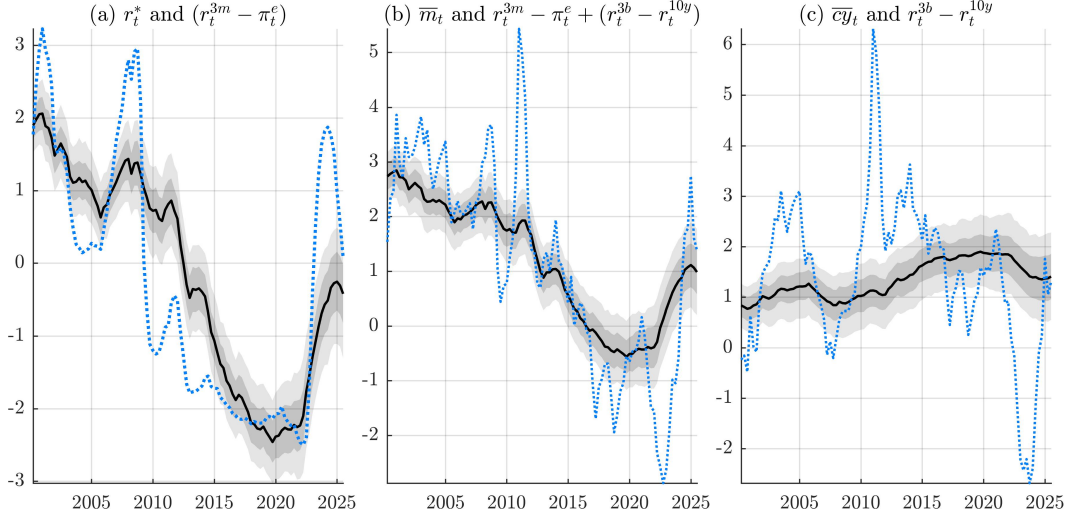


Figure K.8: **Results for: $\underline{\mathbf{Q}}^* = 10 \times \underline{\mathbf{Q}}$.** Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

K.2 Decomposition of r^* into trends of labor productivity, hours worked, convenience yield and other factors using alternative scale parameters, $\underline{\mathbf{Q}}$

K.2.1 BM (baseline) model

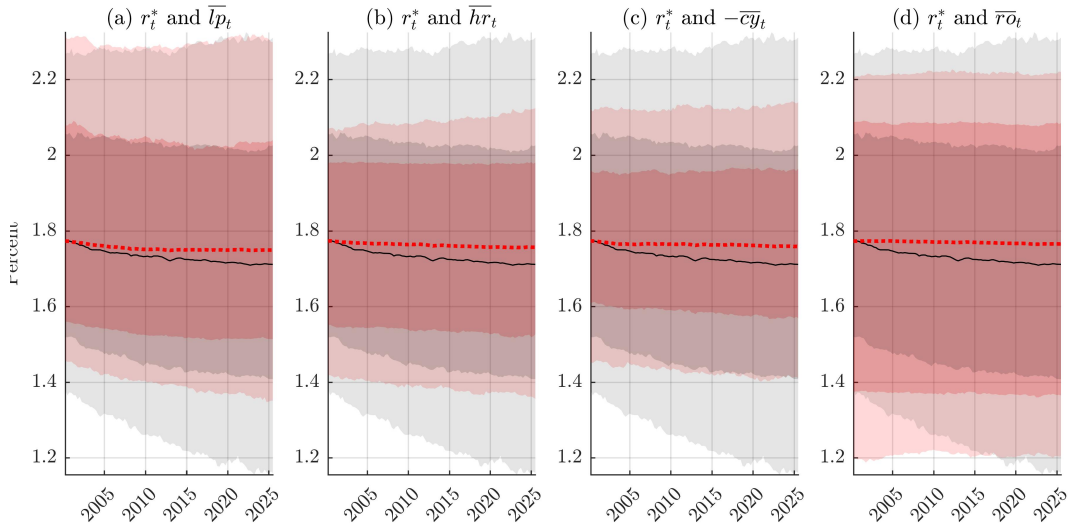


Figure K.9: **Results for: $\underline{\mathbf{Q}}^* = 0.1 \times \underline{\mathbf{Q}}$.** Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

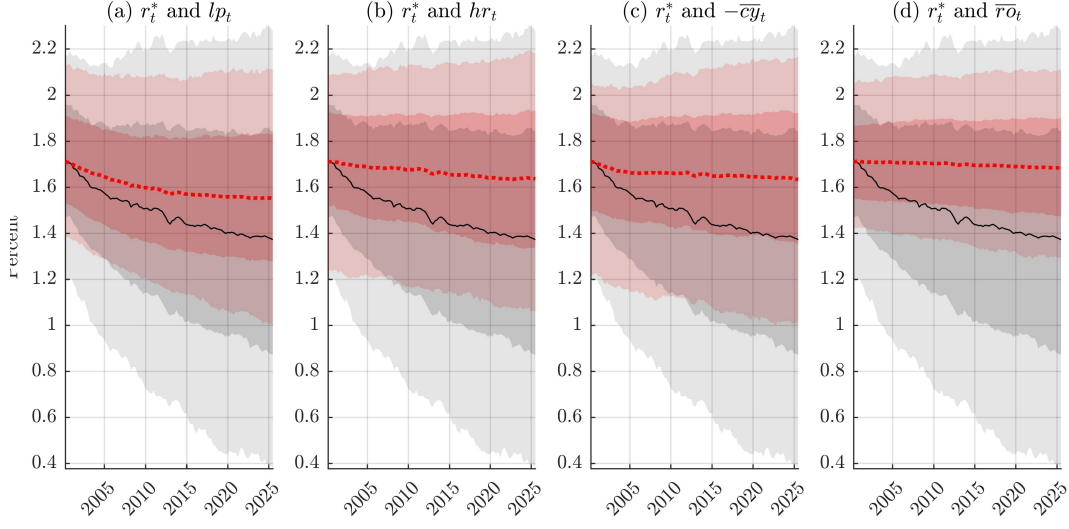


Figure K.10: **Results for: $\underline{Q}^* = 0.5 \times \underline{Q}$.** Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

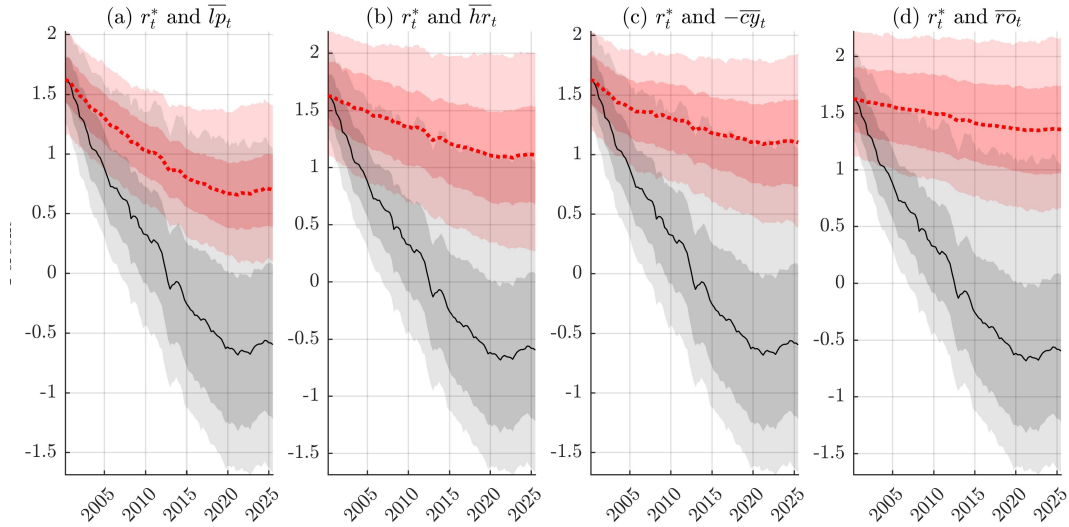


Figure K.11: **Results for: $\underline{Q}^* = 2 \times \underline{Q}$.** Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

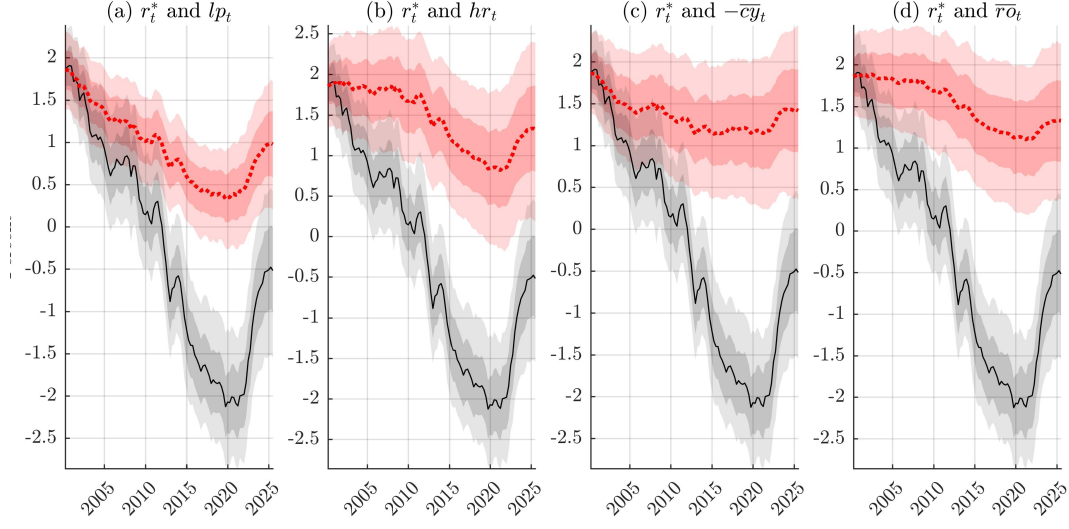


Figure K.12: **Results for: $\underline{Q}^* = 10 \times \underline{Q}$.** Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

K.2.2 LSM-t-sv model

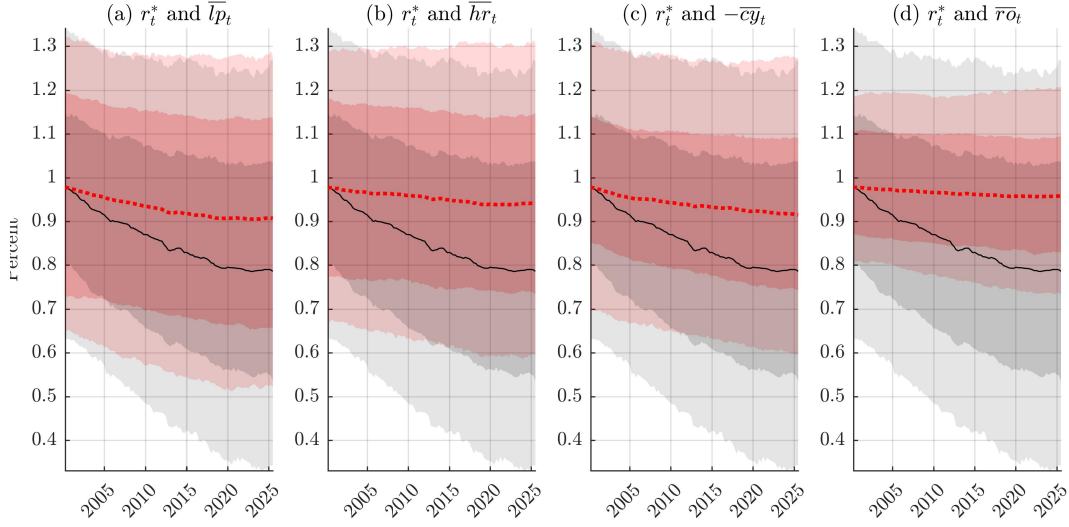


Figure K.13: **Results for: $\underline{\mathbf{Q}}^* = 0.1 \times \underline{\mathbf{Q}}$.** Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

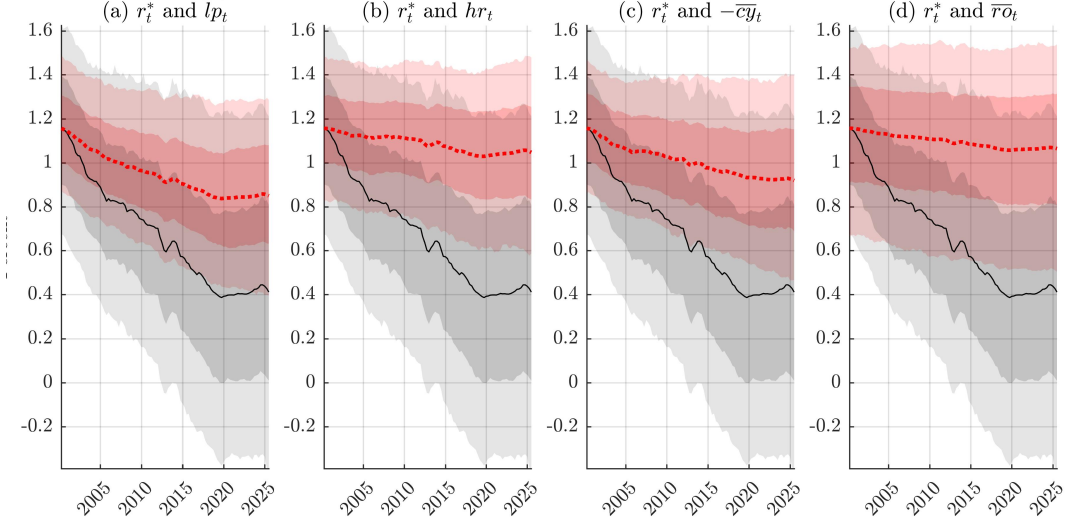


Figure K.14: **Results for: $\underline{Q}^* = 0.5 \times \underline{Q}$.** Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

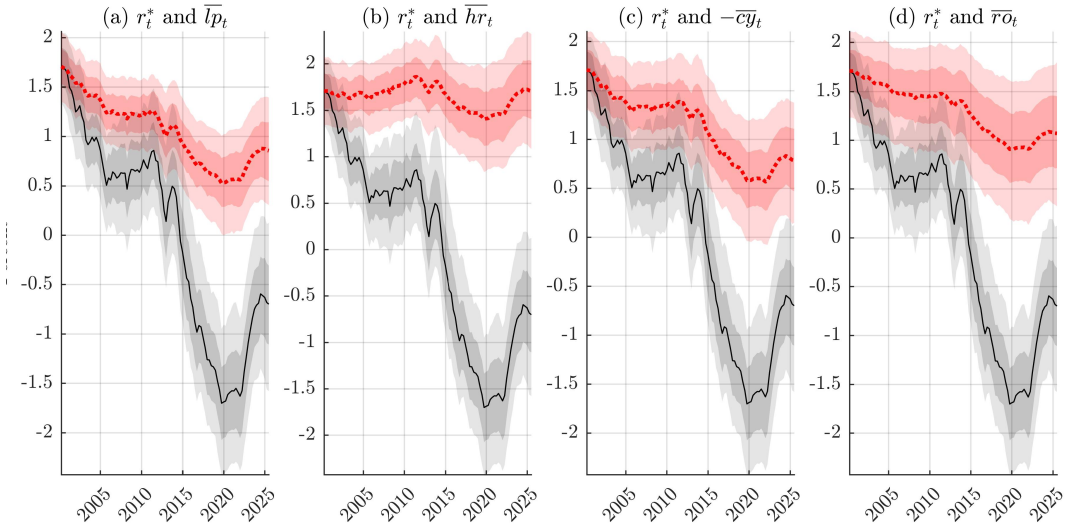


Figure K.15: **Results for: $\underline{Q}^* = 2 \times \underline{Q}$.** Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

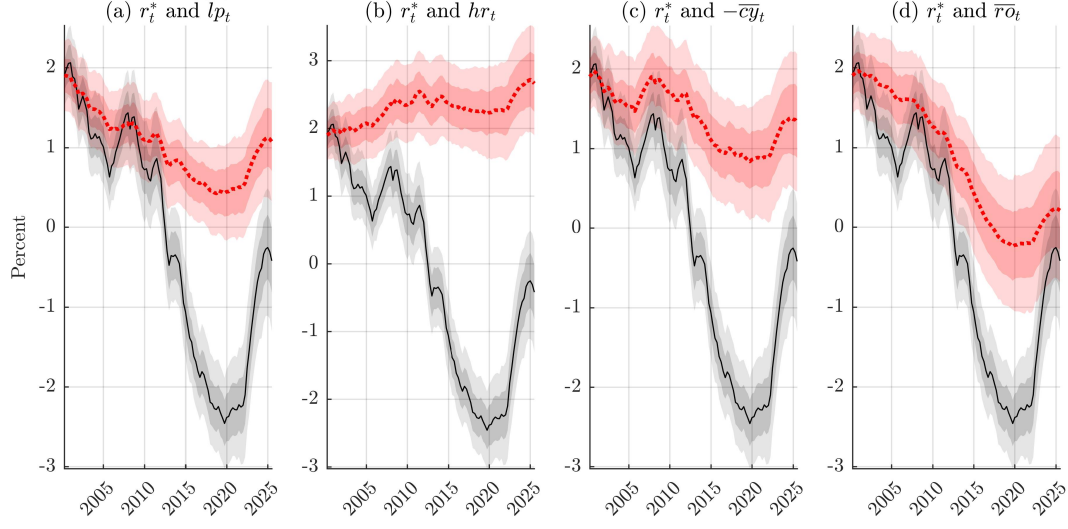


Figure K.16: **Results for: $\underline{\mathbf{Q}}^* = 10 \times \underline{\mathbf{Q}}$.** Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\overline{lp}_t$, $\overline{hr}_t$, $-\overline{cy}_t$, and $\overline{ro}_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

K.3 Estimates of μ_t using alternative scale parameters, $\underline{Q}$

K.3.1 BM (baseline) model

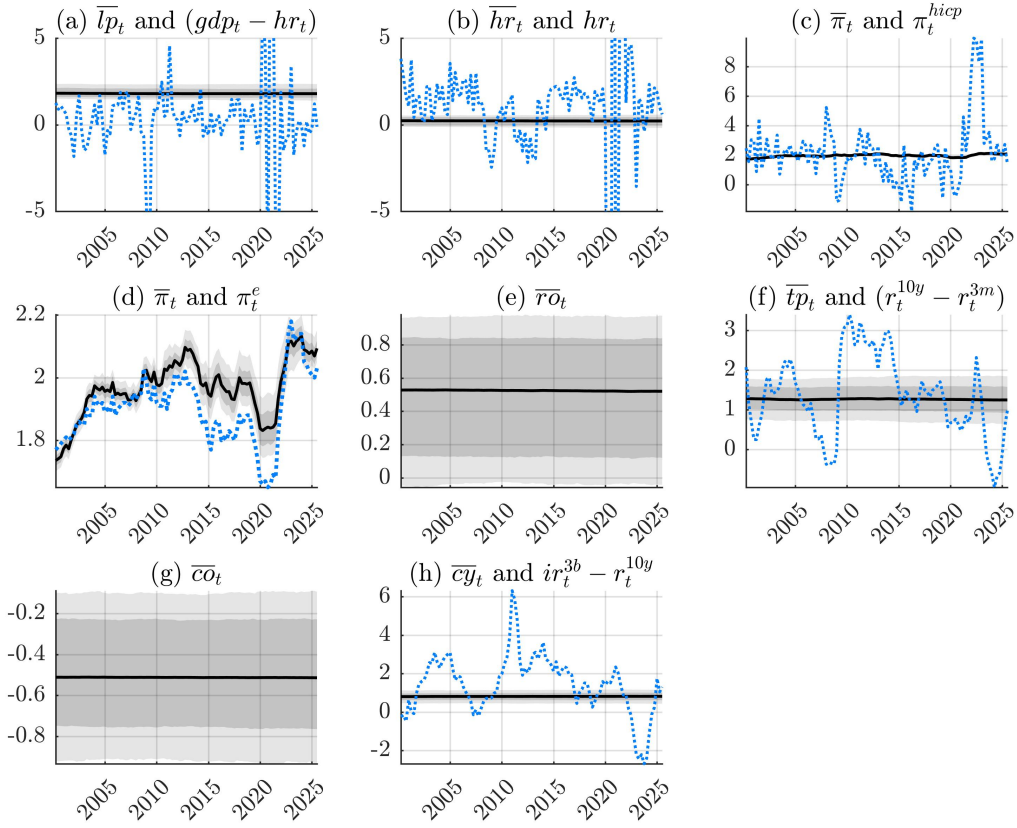


Figure K.17: **Results for: $\mathbf{Q}^* = 0.1 \times \mathbf{Q}$.** Estimates of μ_t based on the BM specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

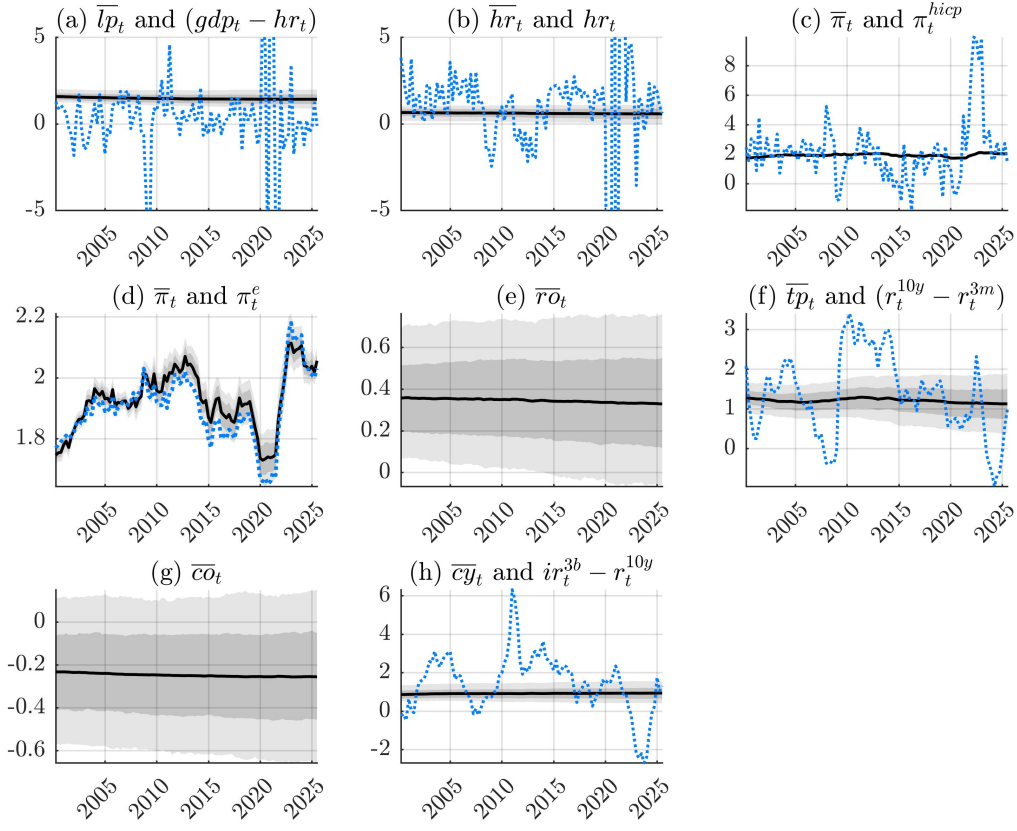


Figure K.18: **Results for: $\mathbf{Q}^* = 0.5 \times \mathbf{Q}$.** Estimates of $\boldsymbol{\mu}_t$ based on the BM specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

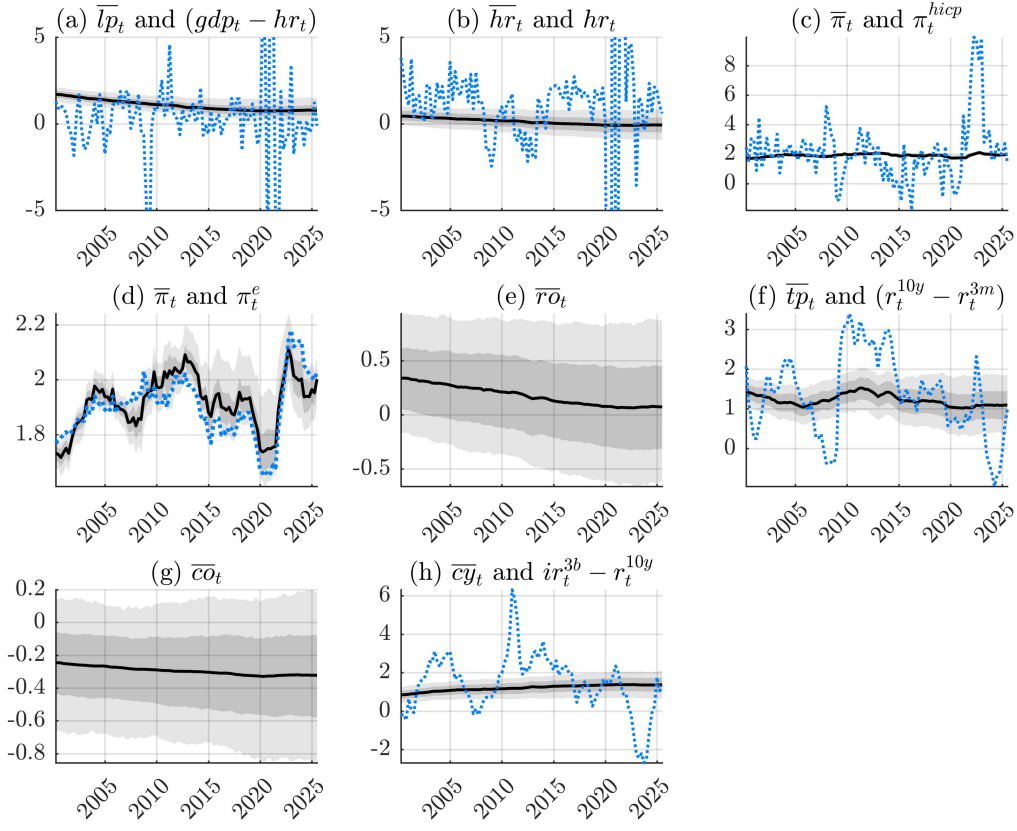


Figure K.19: **Results for: $\underline{Q}^* = 2 \times \underline{Q}$.** Estimates of $\boldsymbol{\mu}_t$ based on the BM specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

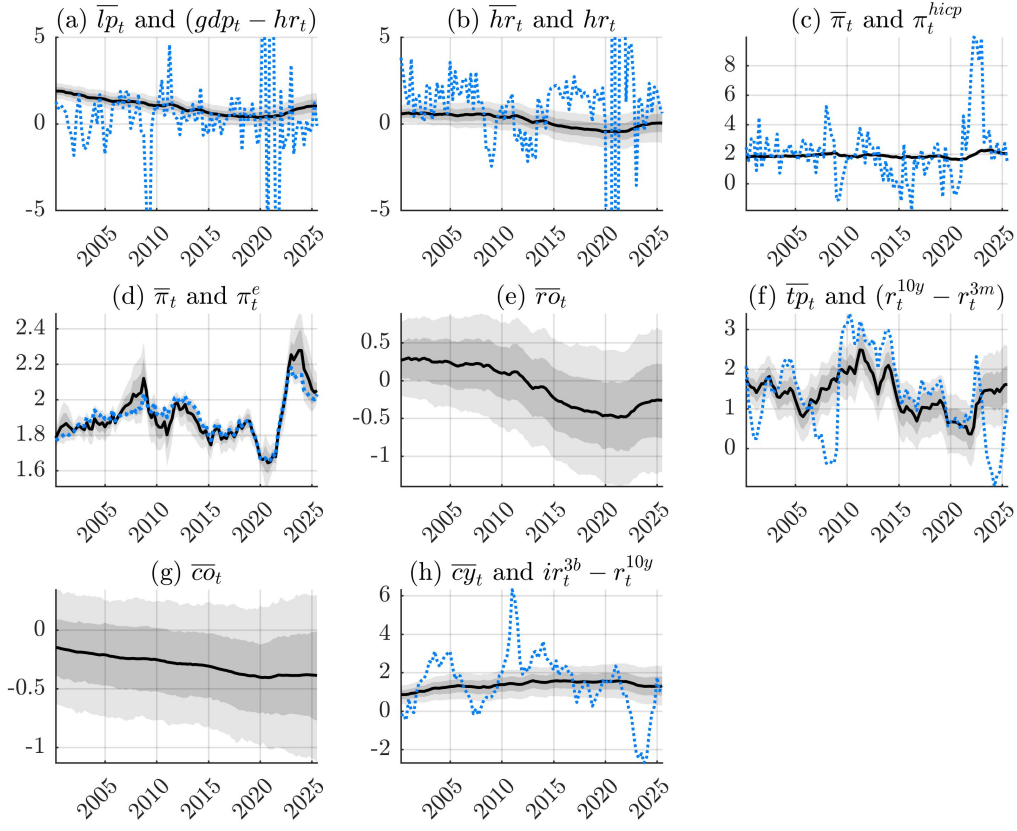


Figure K.20: **Results for: $Q^* = 10 \times Q$.** Estimates of μ_t based on the BM specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

K.3.2 LSM-t-sv model

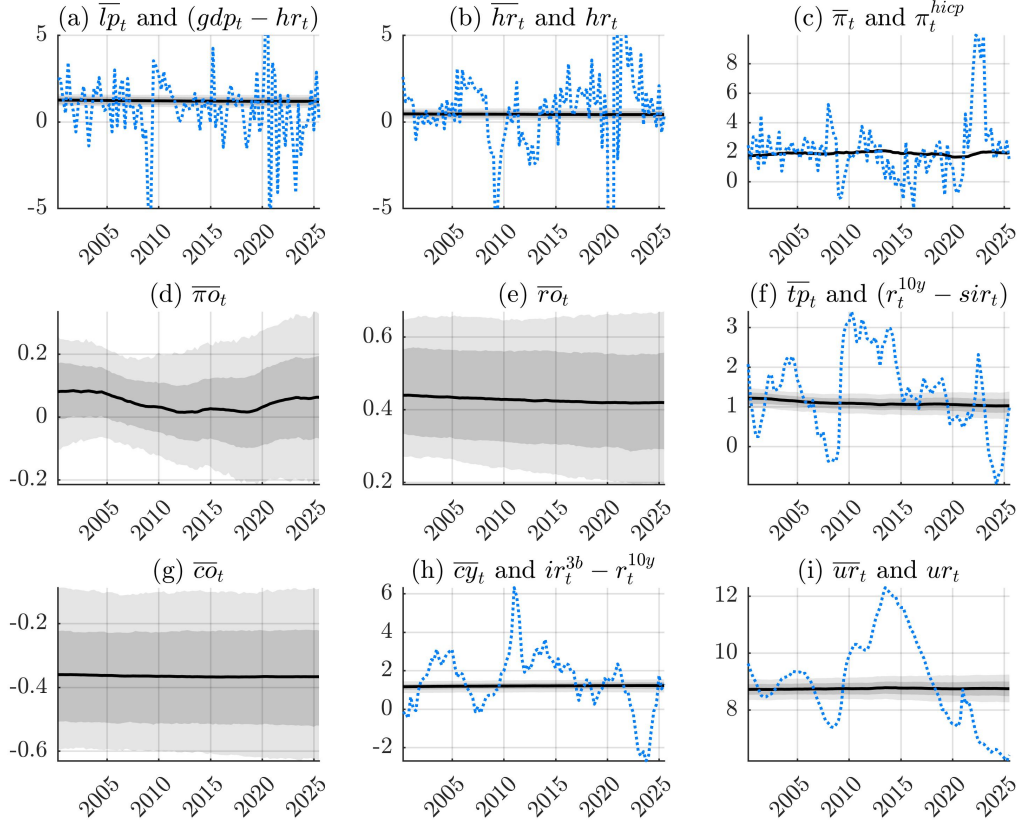


Figure K.21: **Results for: $\underline{Q}^* = 0.1 \times \underline{Q}$.** Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

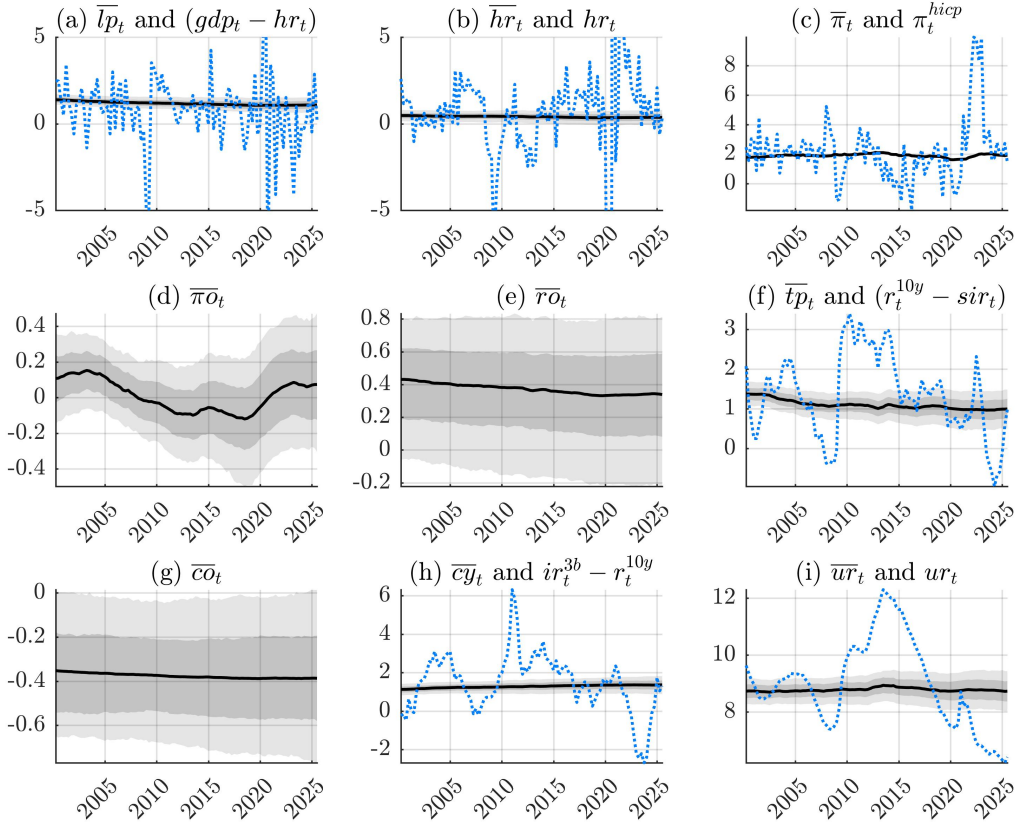


Figure K.22: **Results for: $\mathbf{Q}^* = 0.5 \times \mathbf{Q}$.** Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

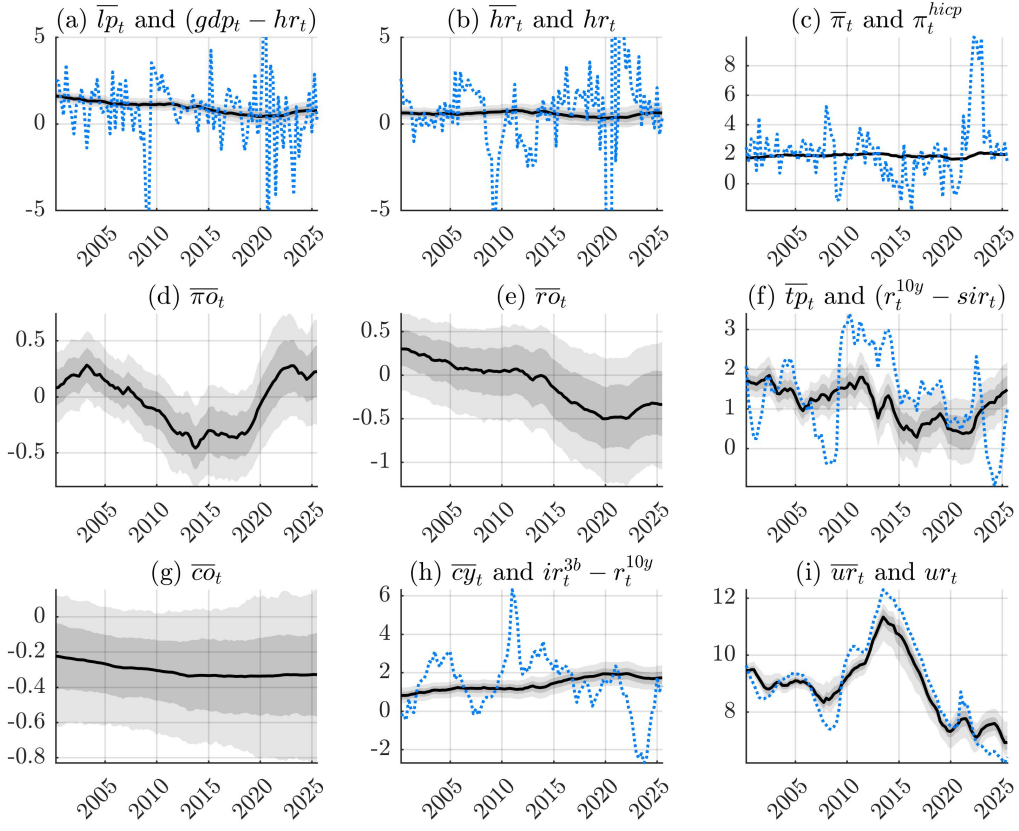


Figure K.23: **Results for: $\underline{Q}^* = 2 \times \underline{Q}$.** Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

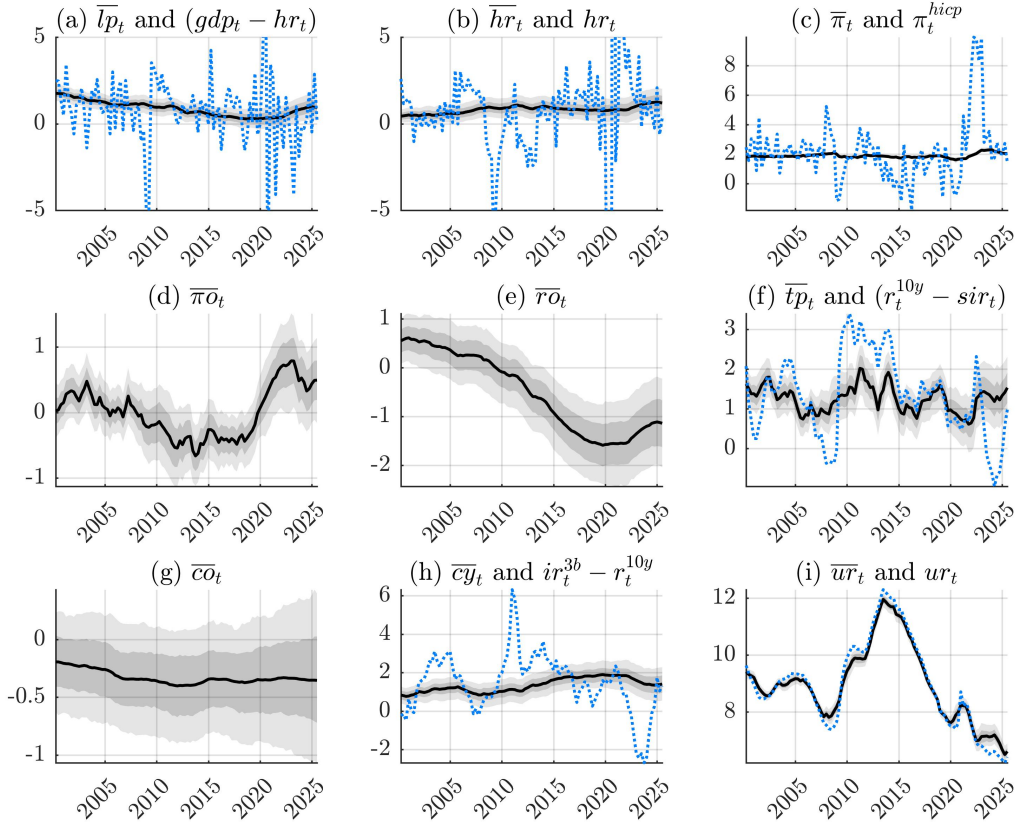


Figure K.24: **Results for: $\underline{\mathbf{Q}}^* = 10 \times \underline{\mathbf{Q}}$.** Estimates of $\boldsymbol{\mu}_t$ based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

K.4 Estimates of r^* , stochastic discount factor and convenience yield using alternative degrees-of-freedom, w_{ε_μ}

K.4.1 BM (baseline) model

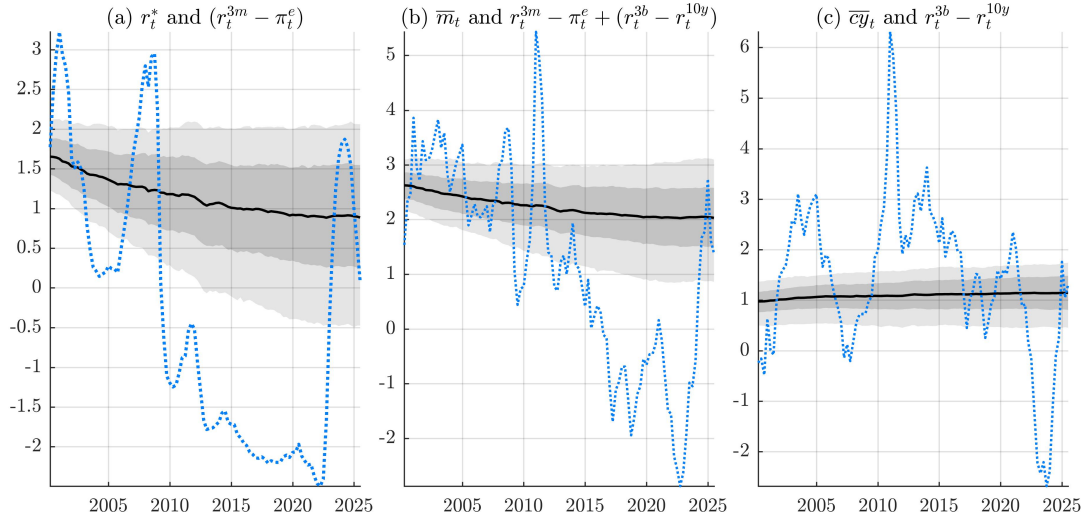


Figure K.25: **Results for:** $w_{\varepsilon_\mu} = 200$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{cy}_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{cy}_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

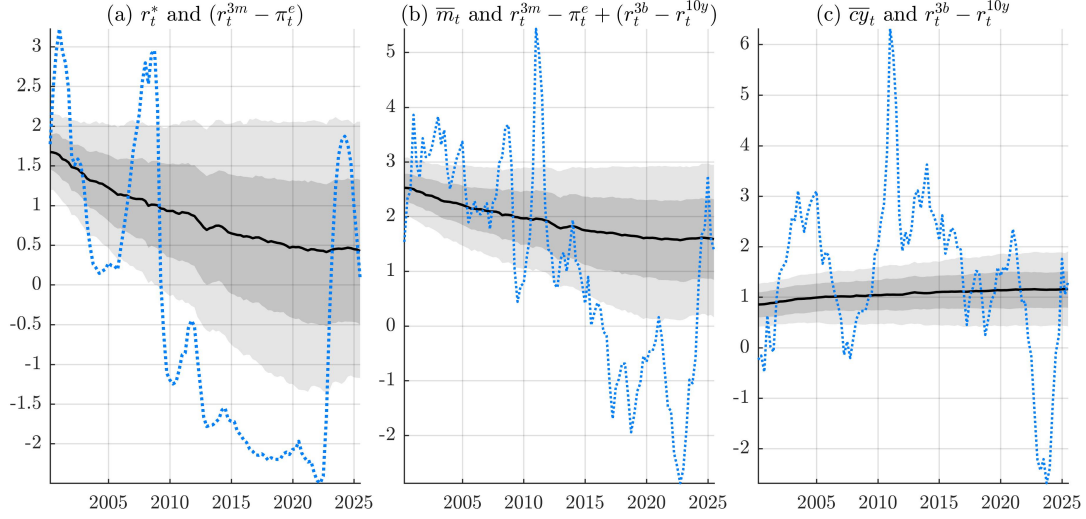


Figure K.26: **Results for:** $w_{\varepsilon_\mu} = 50$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

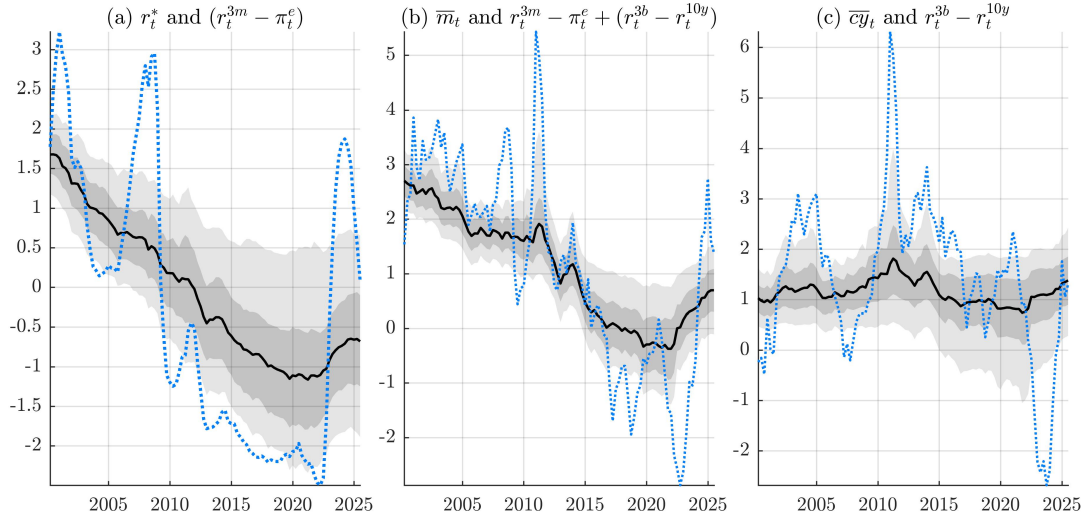


Figure K.27: **Results for:** $w_{\varepsilon_\mu} = 14$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

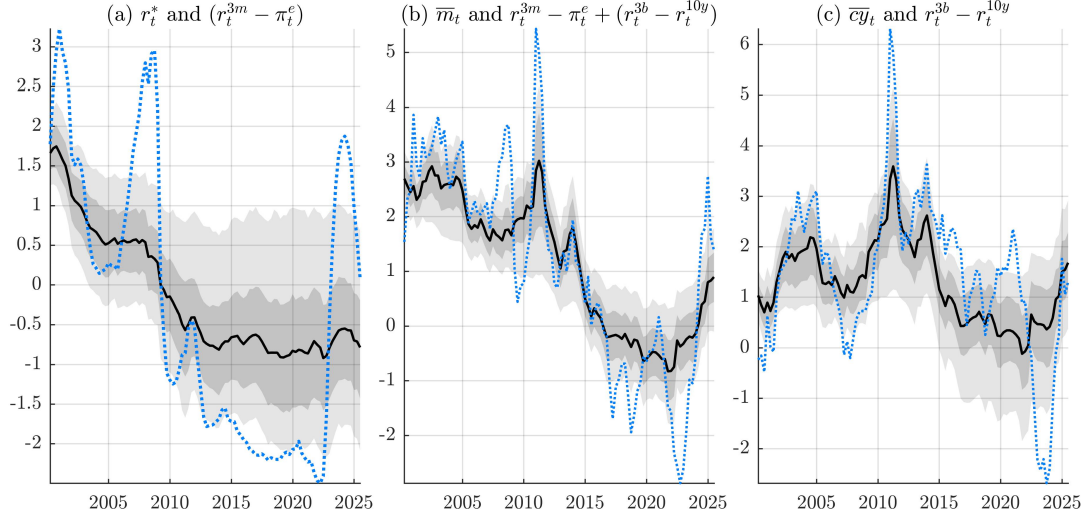


Figure K.28: **Results for:** $w_{\varepsilon_\mu} = 9$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the BM specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

K.4.2 LSM-t-sv model

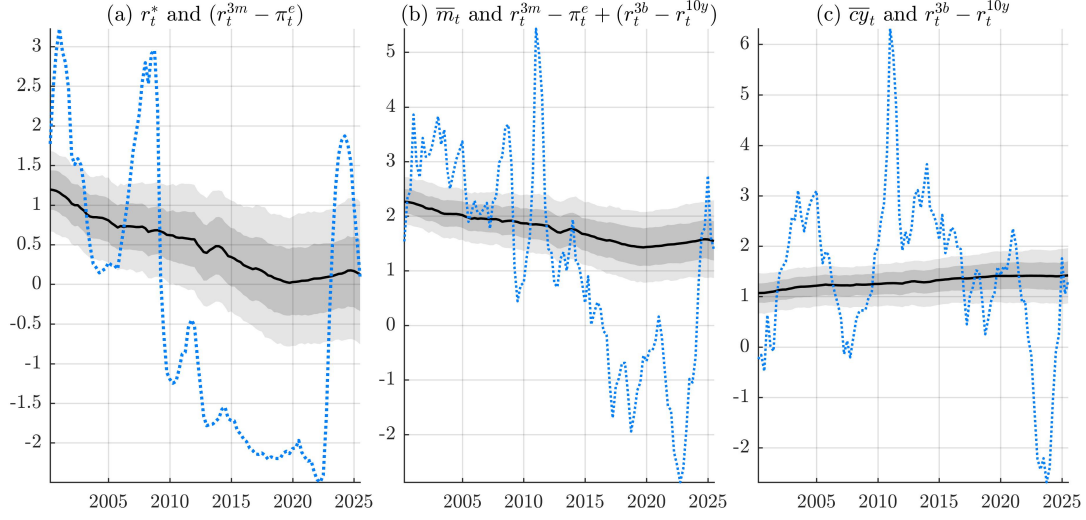


Figure K.29: **Results for:** $w_{\varepsilon_\mu} = 200$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

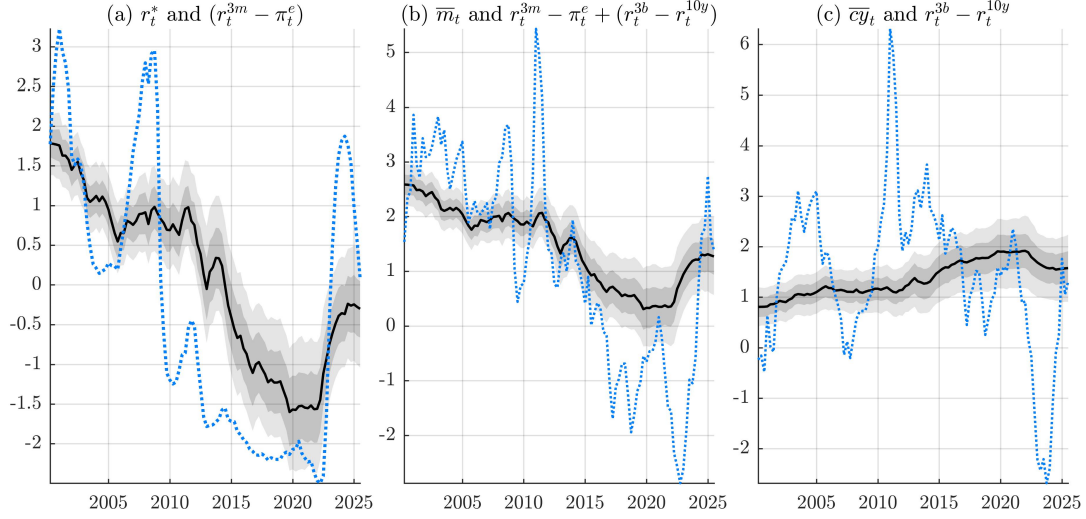


Figure K.30: **Results for:** $w_{\varepsilon\mu} = 50$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

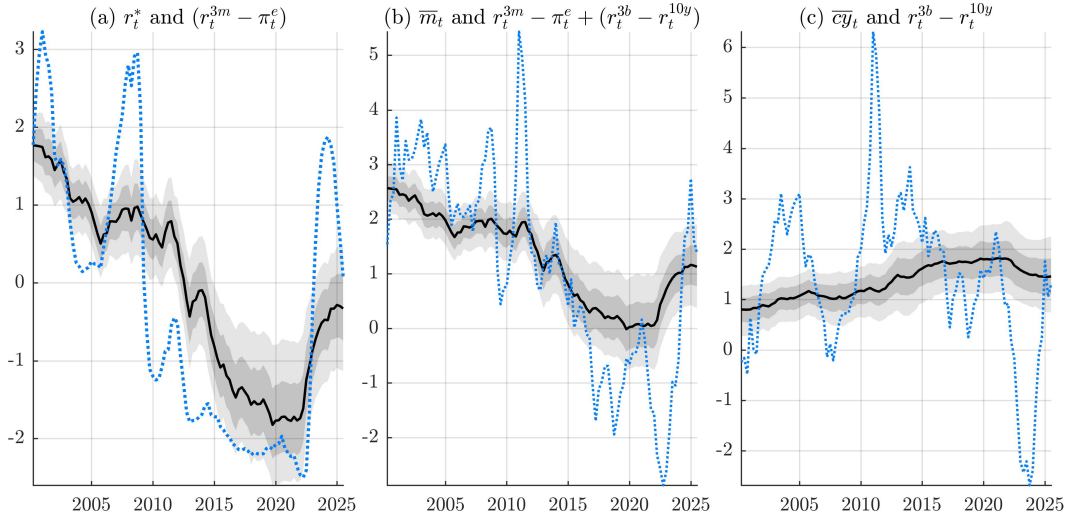


Figure K.31: **Results for:** $w_{\varepsilon\mu} = 29$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

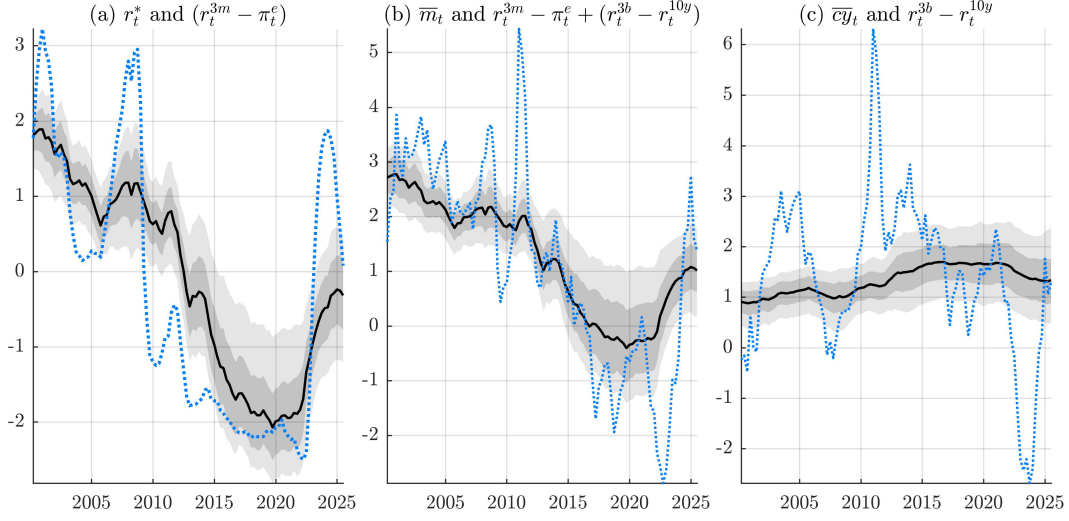


Figure K.32: **Results for:** $w_{\varepsilon\mu} = 24$. Estimates of the natural interest rate, r_t^* , the stochastic discount factor, $\bar{m}_t$, and the convenience yield, $\bar{c}y_t$, based on the LSM-t-sv specification. Black lines denote the posterior means of r_t^* , $\bar{m}_t$, and $\bar{c}y_t$ in panels (a), (b), and (c), respectively. Light-blue dotted lines show the corresponding observables calculated as $(r_t^{3m} - \pi_t^e)$, $[r_t^{3m} - \pi_t^e + (r_t^{3b} - r_t^{10y})]$, and $(r_t^{3b} - r_t^{10y})$ in panels (a), (b), and (c), respectively. The dark (light) shaded areas denote the 68% (95%) posterior credible intervals.

K.5 Decomposition of r^* into trends of labor productivity, hours worked, convenience yield and other factors using alternative degrees-of-freedom, w_{ε_μ}

K.5.1 BM (baseline) model

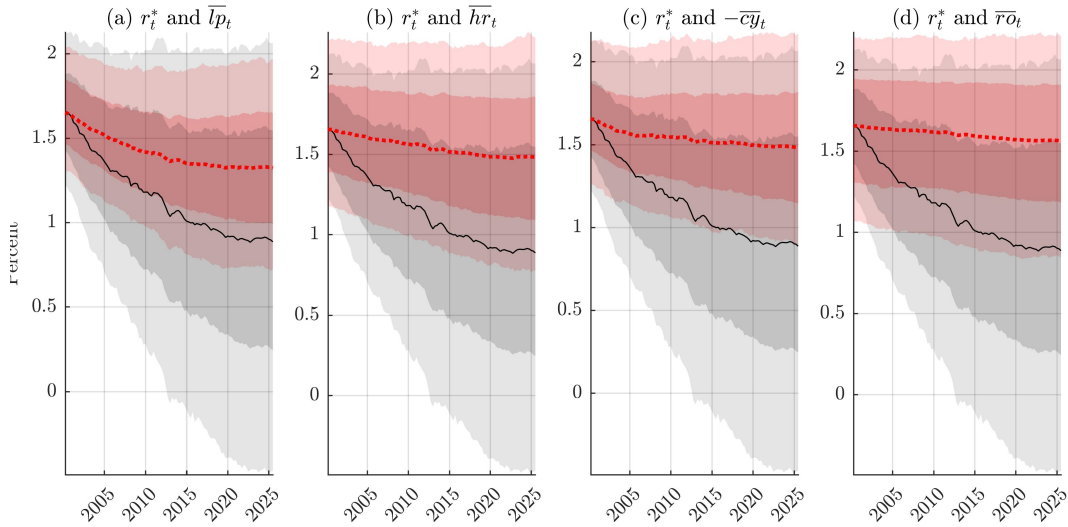


Figure K.33: **Results for:** $w_{\varepsilon_\mu} = 200$. Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

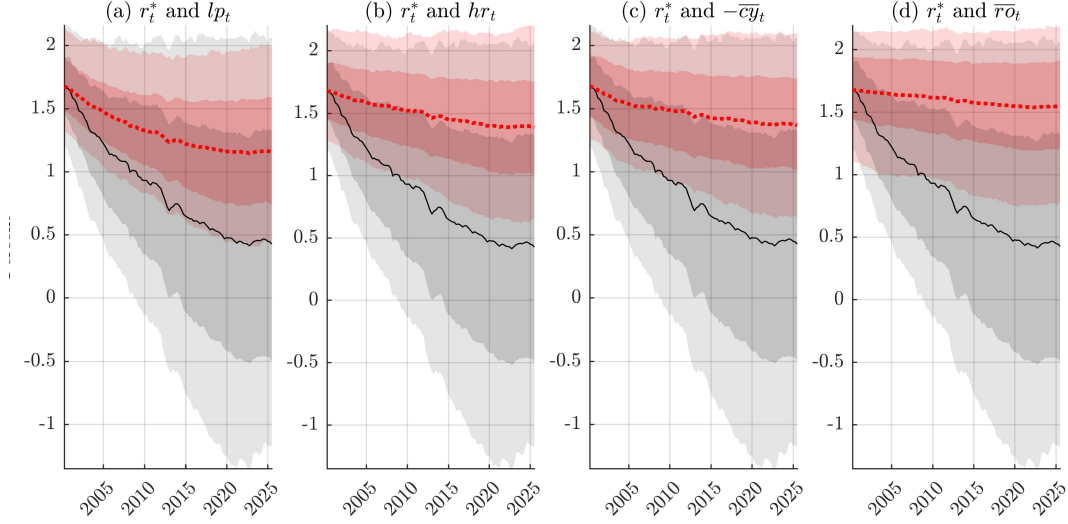


Figure K.34: **Results for:** $w_{\varepsilon_\mu} = 50$. Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

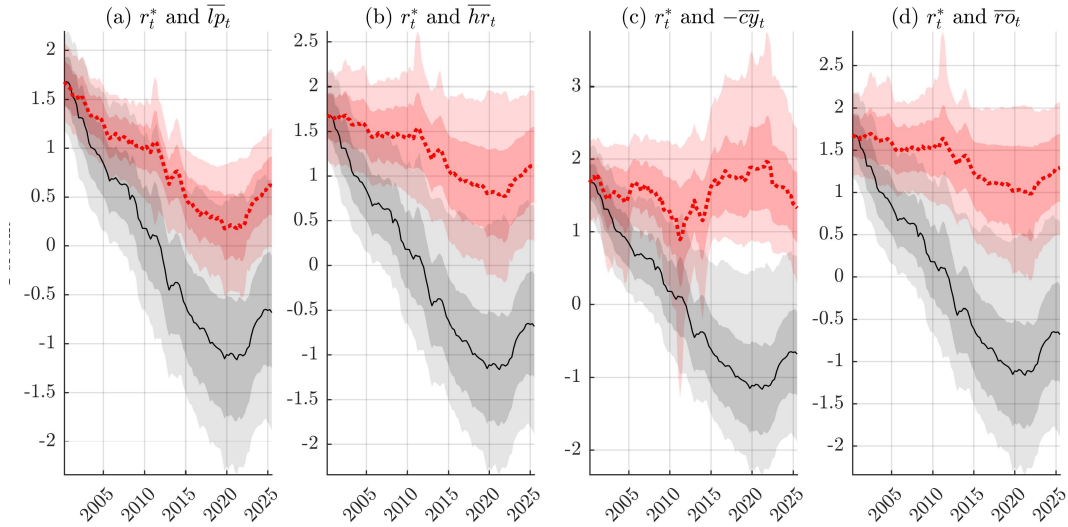


Figure K.35: **Results for:** $w_{\varepsilon_\mu} = 14$. Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

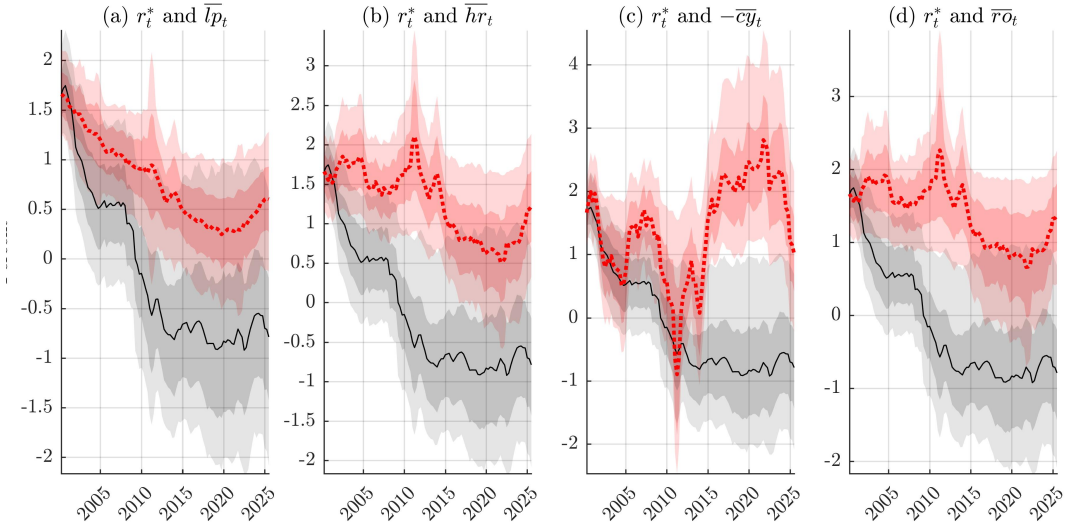


Figure K.36: **Results for:** $w_{\varepsilon_\mu} = 9$. Decomposition of the natural interest rate, r_t^* , based on the BM specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

K.5.2 LSM-t-sv model

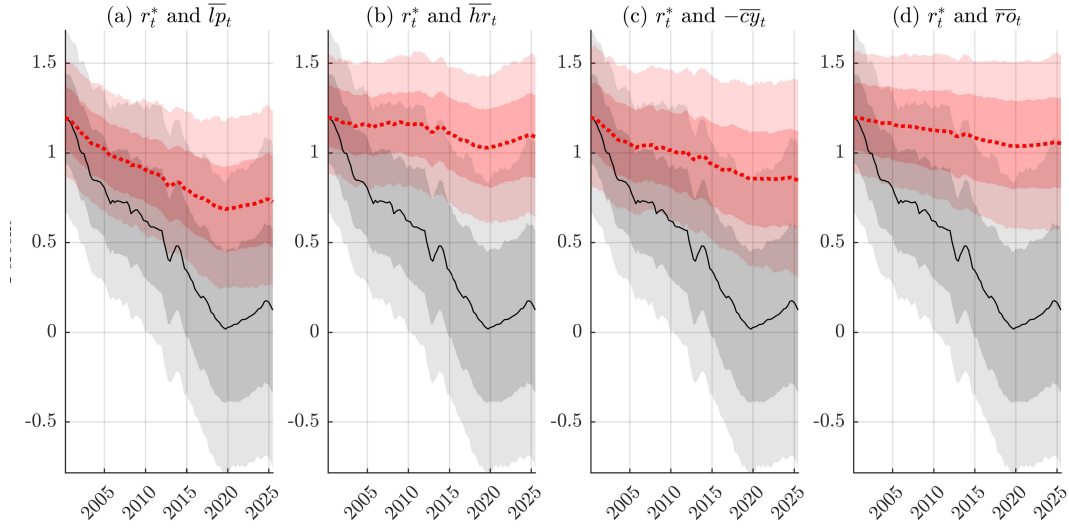


Figure K.37: **Results for:** $w_{\varepsilon_\mu} = 200$. Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

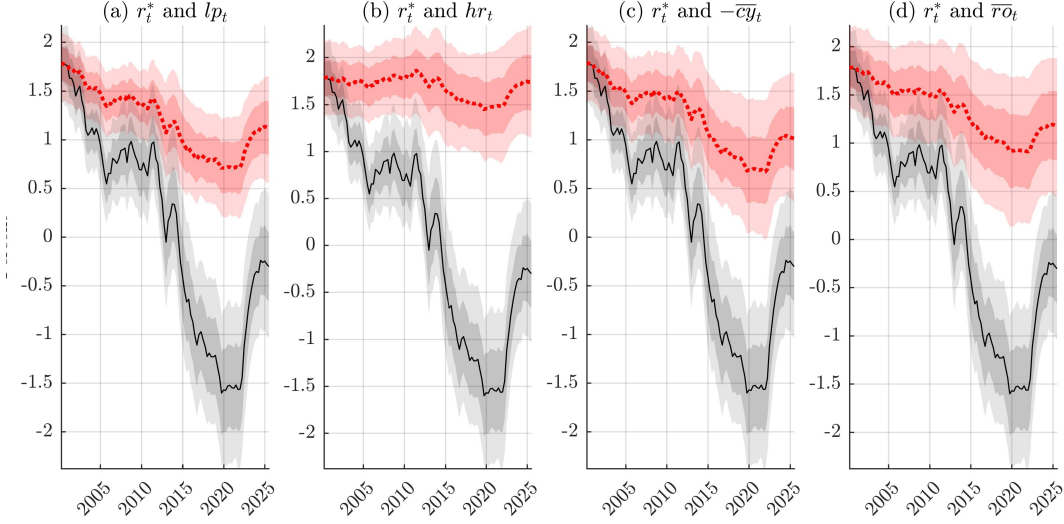


Figure K.38: **Results for:** $w_{\varepsilon_\mu} = 50$. Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

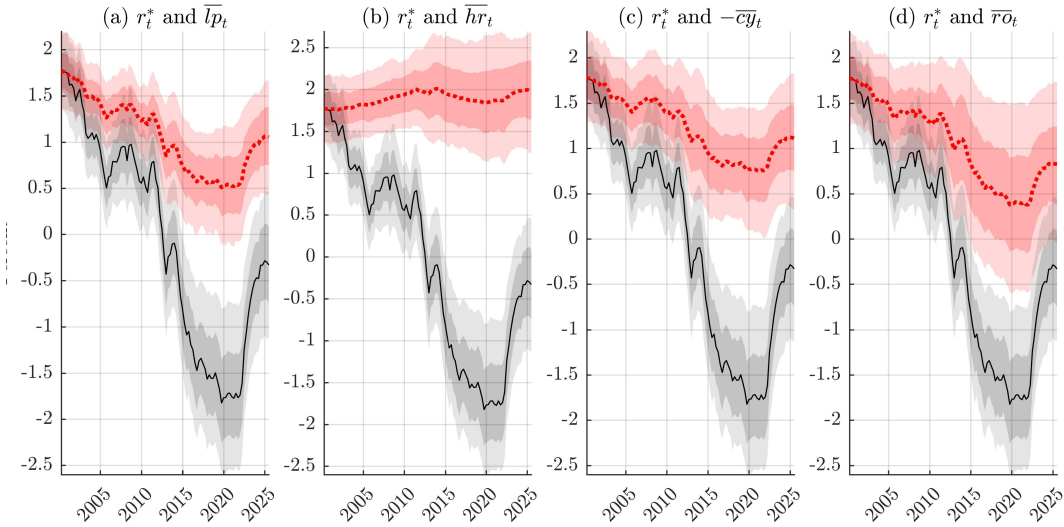


Figure K.39: **Results for:** $w_{\varepsilon_\mu} = 29$. Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

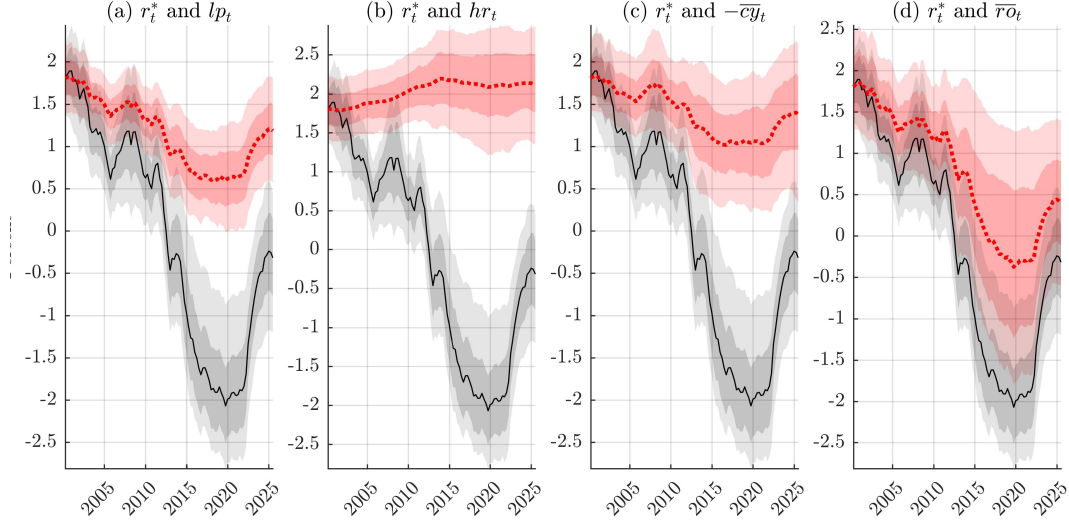


Figure K.40: **Results for:** $w_{\varepsilon_\mu} = 24$. Decomposition of the natural interest rate, r_t^* , based on the LSM-t-sv specification. The black line denotes the posterior mean of r_t^* , while the red dotted lines denote the posterior means of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ in panels (a), (b), (c), and (d), respectively. The dark (light) shaded areas represent the 68% (95%) posterior credible intervals. The levels of $\bar{l}p_t$, $\bar{h}r_t$, $-\bar{c}y_t$, and $\bar{r}o_t$ are normalized so that their posterior mean in 2000:Q1 equals $r_{2000:Q1}^*$.

K.6 Estimates of μ_t using alternative alternative degrees-of-freedom,

$$w_{\varepsilon_\mu}$$

K.6.1 BM (baseline) model

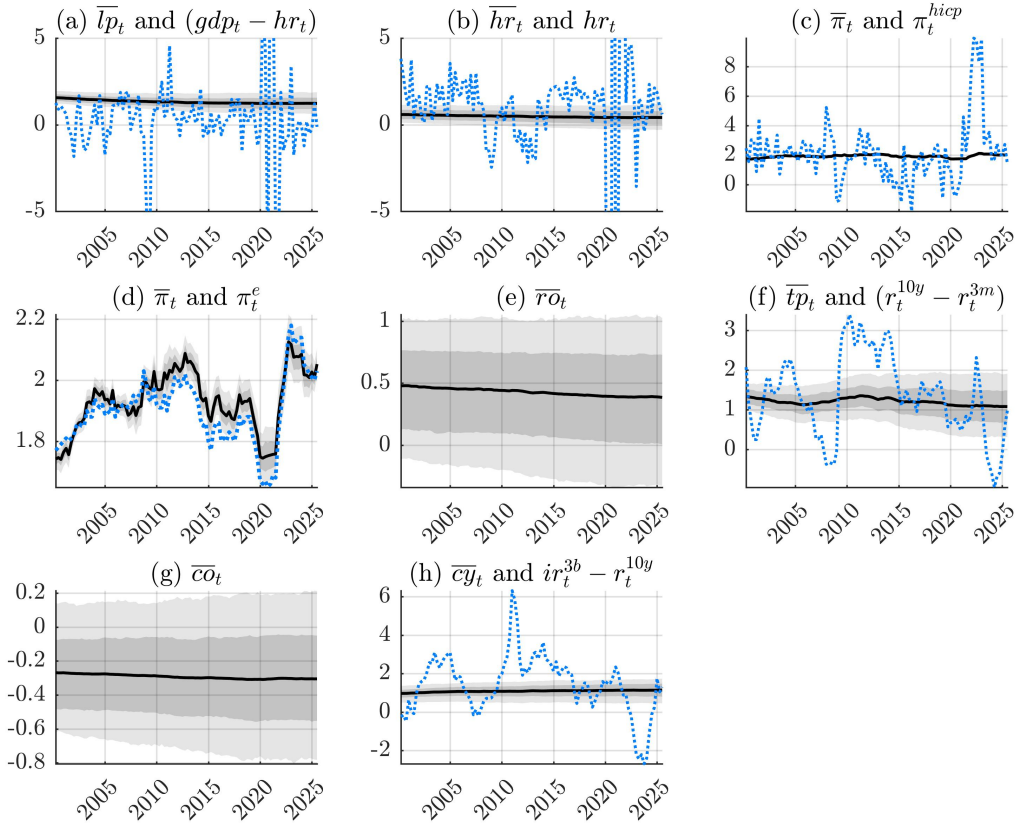


Figure K.41: **Results for:** $w_{\varepsilon_\mu} = 200$ based on the BM specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

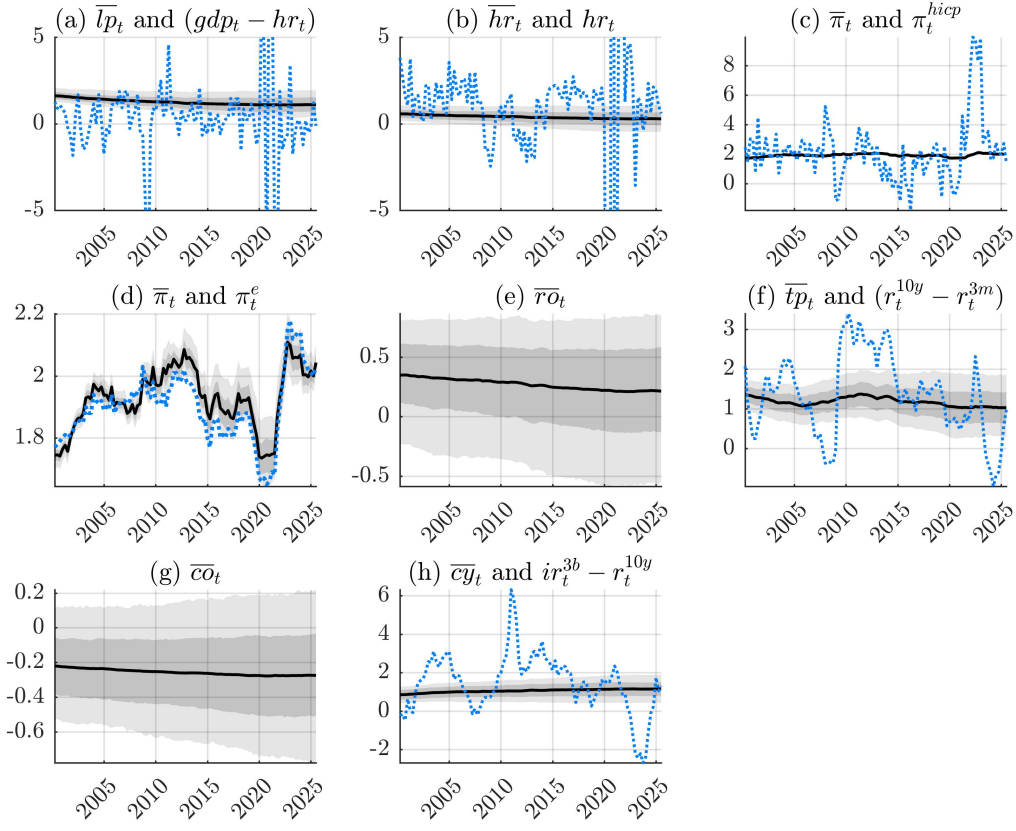


Figure K.42: **Results for:** $w_{\varepsilon\mu} = 50$ based on the BM specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

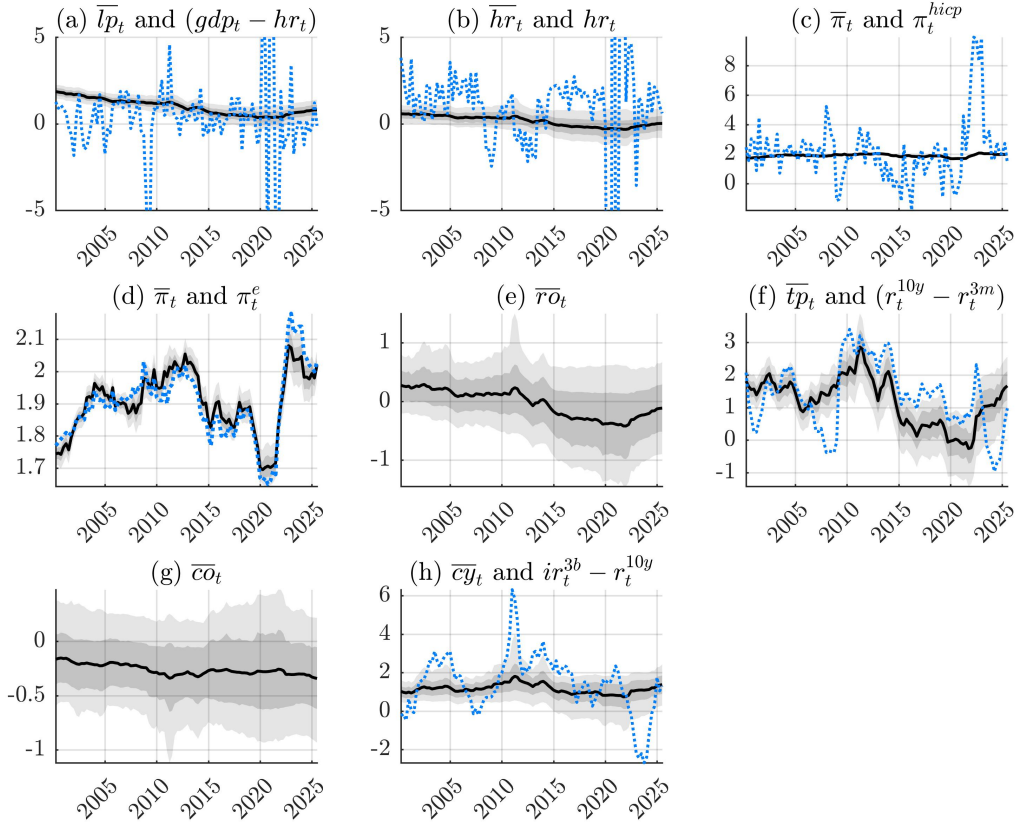


Figure K.43: **Results for:** $w_{\varepsilon_\mu} = 14$ based on the BM specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

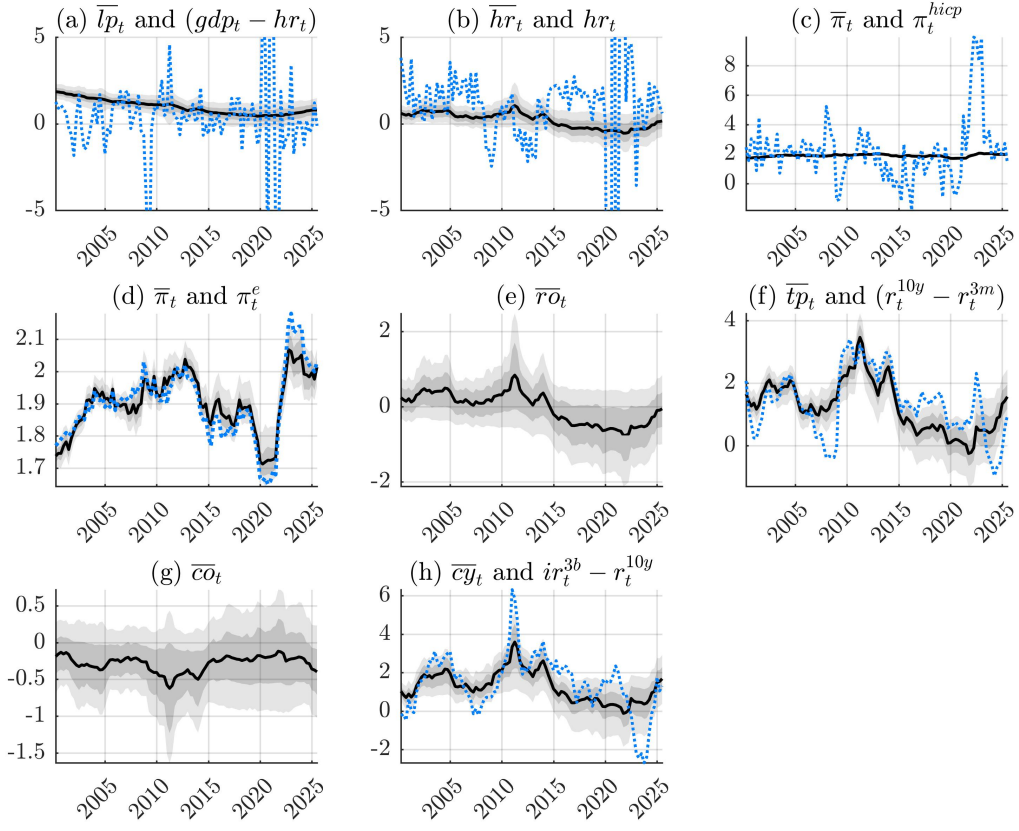


Figure K.44: **Results for:** $w_{\varepsilon_\mu} = 9$ based on the BM specification. Black lines denote the posterior mean of the elements of $\boldsymbol{\mu}_t$. The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

K.6.2 LSM-t-sv model

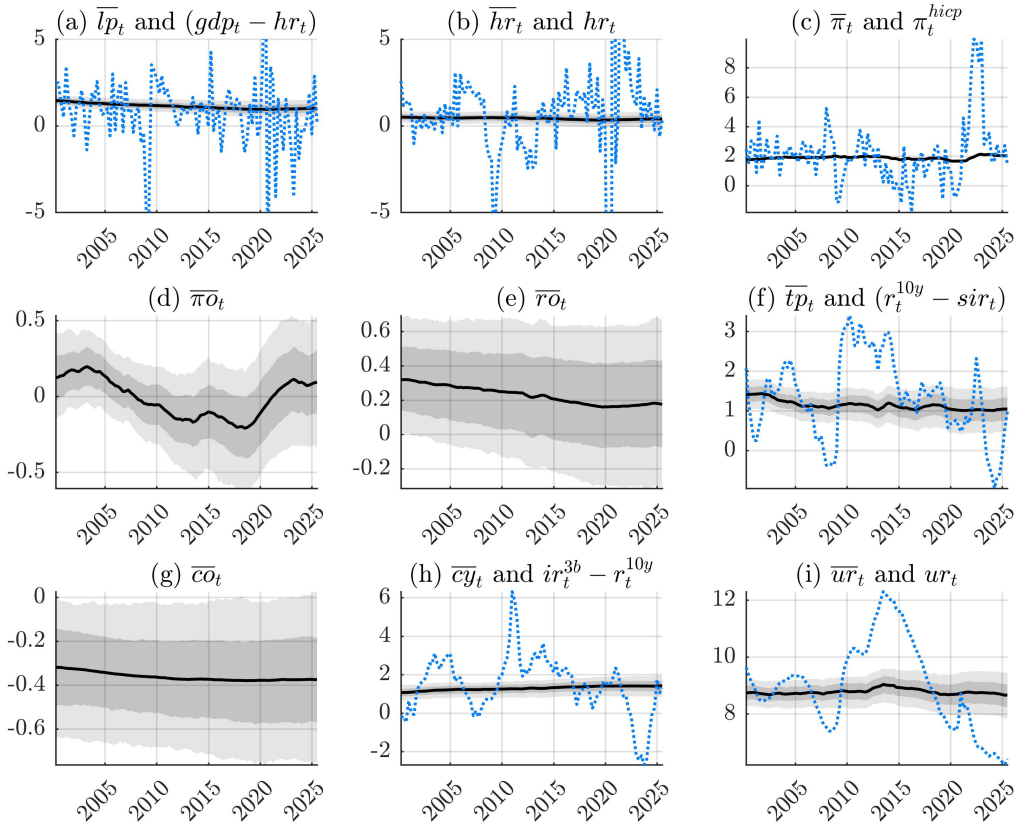


Figure K.45: **Results for:** $w_{\varepsilon_\mu} = 200$. Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

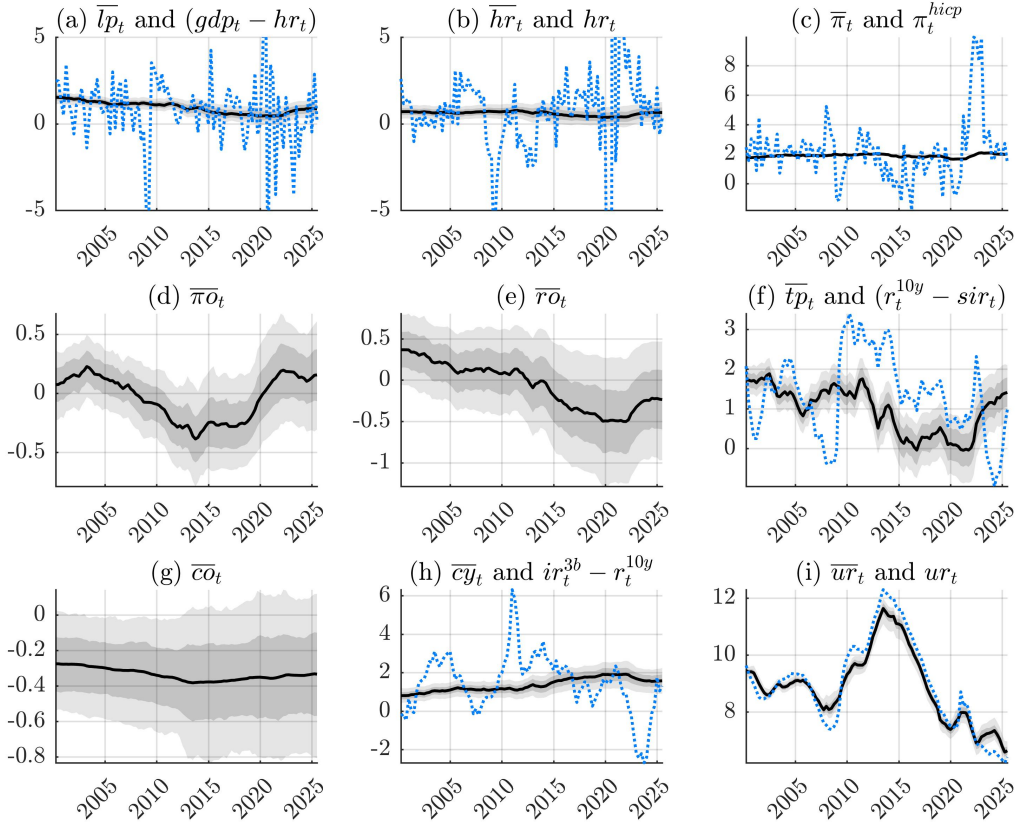


Figure K.46: **Results for:** $w_{\varepsilon_\mu} = 50$. Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

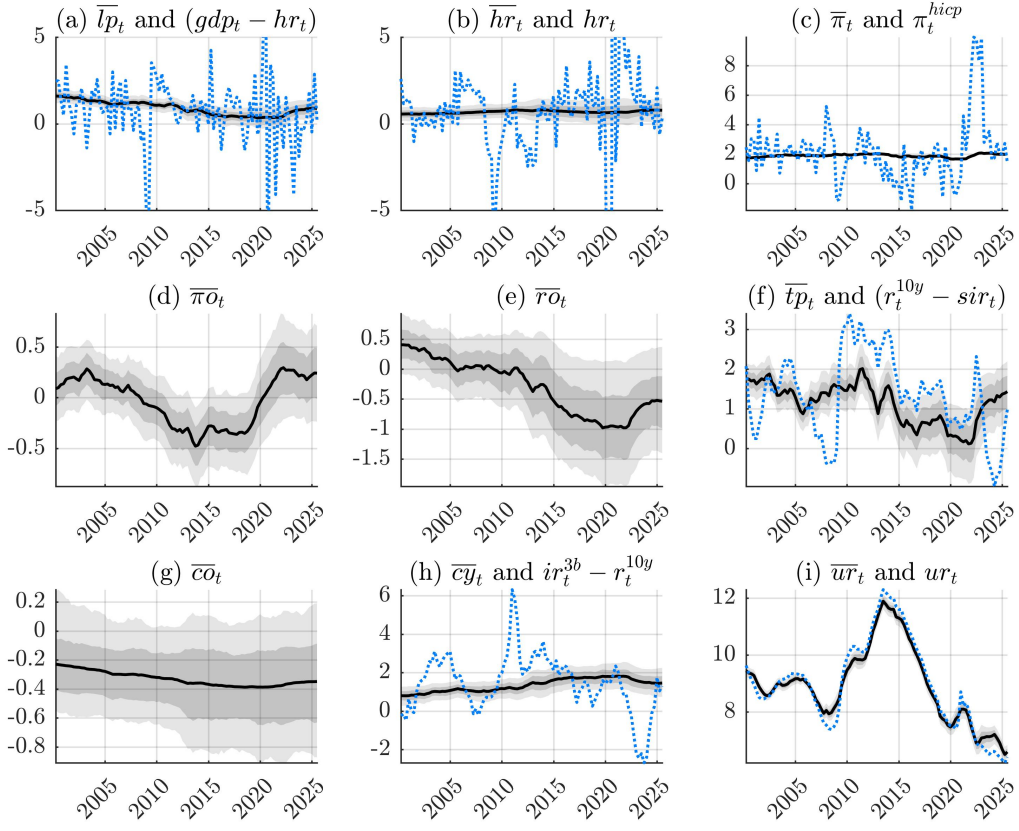


Figure K.47: **Results for:** $w_{\varepsilon_\mu} = 29$. Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

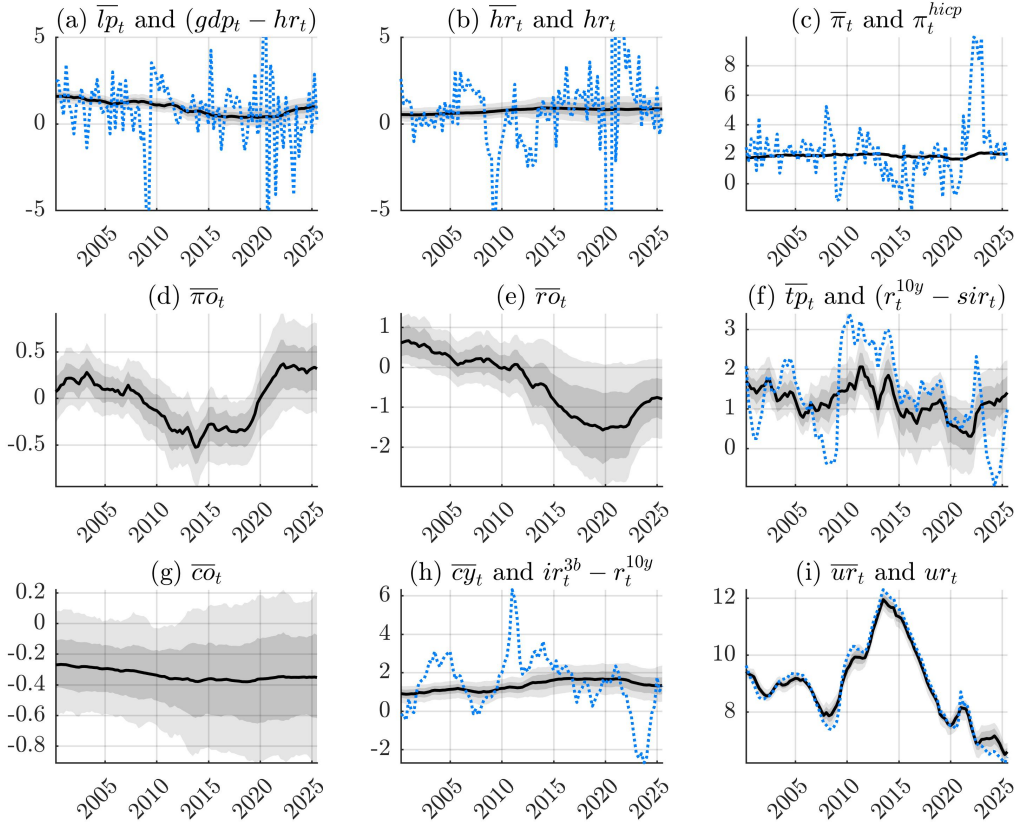


Figure K.48: **Results for:** $w_{\varepsilon_\mu} = 24$. Estimates of μ_t based on the LSM-t-sv specification. Black lines denote the posterior mean of the elements of μ_t . The dark (light) shaded areas show the 68% (95%) posterior credible interval. Light-blue dotted lines show the observables (wherever it is applicable).

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