Public debt

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The Crisis in the Euro-Area

A conference of the Bank of Greece

The close of the 20th century was a time of euphoria.

Advanced economies enjoyed comfortable rates of growth, low unemployment and booming stock markets; and many less advanced economies, large ones in particular, were set on fast tracks of growth and development.

In academic economics a similar euphoria prevailed.

Robert Lucas, in his presidential address to the American Economic Association, could proclaim that "...macroeconomics in the original sense has succeeded: its central problem of depression prevention has been solved for all practical purposes, and has in fact been solved for many decades."

It took a short decade into the 21st century for the euphoria to turn to gloom.

Poverty, inequality, and the depletion of natural resources have been long standing concerns policy makers.

But it it was the financial crisis that erupted in 2008 and the public debt crisis of more recent challenged both the faith in competitive markets and the prestige of academic economics.

Gideon Rachman in the Financial Times called for economists to be "swept off their throne."

The impression is widespread that, under risk, markets fail; and academic economics has hardly bothered with economic practice.

What is to be done?

One approach is to dismiss what has been accomplished in the past half century, the fascinating body of knowledge that has been developed, and to give up the “grammatical thinking” about the economy that, in the words of Frank Hahn, this work has made possible.

But this will not do.

In economies subject to aggregate as well as uninsurable idiosyncratic risk, competitive equilibrium allocations are constrained inefficient: reallocations of assets support Pareto superior allocations.

This is the case even if the asset market for the allocation of aggregate risks is complete.

It is often argued that the road to prosperity passes through the transfer of economic activity from the public to the private sector and the reduction of public expenditure and public debt.

I shall argue, through examples, and not only, that this can be questioned.

Idiosyncratic risk does not affect the allocation of resources at Pareto optimal allocations.

Competitive equilibria inherit this property if the asset market for the insurance of idiosyncratic risk is complete.

But if realizations of idiosyncratic shocks are publicly unobservable or unverifiable, idiosyncratic risks may well not be insurable; indeed, this is a standard assumption in macroeconomic theory.

The demonstration that competitive equilibria in economies with uninsurable idiosyncratic shocks are constrained suboptimal makes an important methodological point relevant for economic policy.

Intervention is often said to be counterproductive because competitive equilibrium cannot be Pareto improved; such a view is untenable.

Example: Underinvestment

A two-period setting, with a continuum of ex-ante identical individuals.

One commodity is available for exchange and consumption in date 0, and it is storable.

At date 1, there are two commodities: individuals trade and consume what they have stored of the first commodity, along with any further endowment they may have, and they also exchange and consume a second commodity.

The amount of commodity that is stored by each individual at date 0 is k, and e_2 is the endowment of commodity 2 they all receive at date 1.

The endowment of commodity 1 is subject to idiosyncratic risk: for an individual in personal state s, the endowment of commodity 1 is $e_{1,s}$.

The proportion of individuals in state s is $\pi_{i,s}$, and $e_1 = \mathbb{E}_\pi[e_{1,s}]$.

With Bernoulli utility indices v_s, the individual's von Neumann-Morgestern ex-ante utility is

$$u(k, x) = -k + \mathbb{E}_{\pi}[v_s(x_s)],$$

where $x_s = (x_{1,s}, x_{2,s})$ is the individuals' consumption at date 1 in state s.

Commodity 1 be numèraire at date 1, and prices are by $(1, p)$.

The wealth of an individual in personal state s is

$$e_{1,s} + k + pe_2$$

.

If λ_s is the marginal utility of income in personal state s, the first-order condition for optimization at date 0 is

$$\mathbb{E}_\pi[\lambda_s] = 1,$$

and, as a consequence, the ex-ante utility impact of an infinitesimal perturbation to the level of savings, dk, around its competitive equilibrium level, is

$$\begin{aligned} du &= -dk + \mathbb{E}_\pi\{\lambda_s[(-x_{2,s} + e_2)dp + dk]\} \\ &= \mathbb{E}_\pi[\lambda_s(-x_{2,s} + e_2)]dp. \end{aligned}$$

Market-clearing requires that

$$\mathbb{E}_\pi[(-x_{2,s} + e_2)] = 0.$$

Under certainty, it is impossible to improve ex-ante utility by the implementation of levels of savings different from the ones chosen under competitive equilibrium.

Under uncertainty, even idiosyncratic, the competitive equilibrium utility can be improved upon if

$$\mathbb{E}_\pi[\lambda_s(-x_{2,s} + e_2)] \neq 0.$$

A specific structure allows for a closed-form solution.

The Bernoulli indices are state-independent and

$$v(x_s) = \ln x_{1,s} + \ln x_{2,s}.$$

By direct computation, given savings of k, equilibrium prices are $p = (e_1 + k)/e_2$, an expression that depends positively on k. Also, $\lambda_s = 2/(e_{1,s} + e_1 + 2k)$ and

$$e_2 - x_{2,s} = \frac{e_2}{2}\left(\frac{e_1 - e_{1,s}}{e_1 + k}\right).$$

If, further, there are only two personal states, $e_{1,s} = e_1 \pm \varepsilon$, that occur with equal probability, then

$$\mathbb{E}_\pi[\lambda_s(e_2 - x_{2,s})] = \frac{1}{e_1 + k}\left(\frac{\varepsilon^2}{4(e_1 + k)^2 - \varepsilon^2}\right).$$

At the equilibrium level of savings, $\mathbb{E}_\pi[\lambda_s] = 1.$ By direct computation,

$$\mathbb{E}_\pi[\lambda_s(e_s - x_{2,s})] = \frac{\varepsilon^2}{4(e_1 + k)^2} > 0.$$

Individuals *underinvest* at competitive equilibrium.

Example: Social security

First, again, a two-period setting with a continuum of mass 1 of identical individuals.

A single commodity is available in the first period, and it can be either consumed or invested: the amount of this commodity that is saved, k, becomes the endowment for the second period.

In the second period, this level k of capital is combined with a second production factor, labor, to produce a consumption good; the amounts of labor used is l and c is the consumption good produced.

In the first period, individuals only consume and save. In the second period, they are endowed with $\bar{l}$ units of labor that they supply inelastically, together with their savings k, in exchange for the consumption good.

Under certainty, all individuals are endowed with $\bar{l}$ units of labor at date 1.

The over-all utility of the individuals is given by

$$u(k, c) = -k + v(c),$$

where v is the utility for consumption in the second period.

The consumption good is numèraire in the second period, the interest factor is $1 + r$ and the price of labor as w; the wealth of the individuals is

$$\tau(k) = (1 + r)k + w\bar{l},$$

which they use to consume $c = \tau(k)$.

The technology of production is

$$y = f(k, l) + (1 - \delta)k,$$

where the function f exhibits constant returns to scale.

It follows from the maximization of profits, that

$$(1 + r) = f_k + (1 - \delta) \quad \text{and} \quad w = f_l,$$

while

$$\frac{\partial r}{\partial k} = f_{kk} \quad \text{and} \quad \frac{\partial w}{\partial k} = f_{lk} = f_{kl}.$$

From the optimization of the individuals, if λ is the marginal utility of consumption at date 1,

$$\lambda = \frac{1}{1+r}.$$

This implies that[1]

$$du = -dk + \lambda(kdr + \bar{l}dw + (1+r)dk) = \frac{kdr + \bar{l}dw}{1+r} = \frac{kf_{kk} + \bar{l}f_{kl}}{1+r}dk.$$

Since production displays constant returns to scale, $du = 0$: the competitively determined level of investment cannot be improved upon.

[1] With consumption c, endowment e and prices p,

$$du = -dk + \lambda(-cdp + edp + (1+r)dk).$$

Under idiosyncratic uncertainty, the endowment of labor in the second period is $\bar{l}_s = \bar{l} + \varepsilon_s$ in personal state s, which occurs with probability π_s. As before, $\mathbb{E}_\pi[\varepsilon_s] = 0$, and

$$\tau_s(k) = (1 + r)k + w\bar{l}_s$$

is the individual's wealth in personal state s.

With ex-ante preferences

$$u(k, c) = -k + \mathbb{E}_\pi[v_s(c_s)],$$

if λ_s is the marginal utility of revenue in state s, the first-order condition of the individuals at date 0 is

$$\mathbb{E}_\pi[\lambda_s] = \frac{1}{1 + r}.$$

As a consequence,

$$du = \mathbb{E}_\pi[\lambda_s(kf_{kk} + \bar{l}_s f_{kl})]dk,$$

which becomes simply

$$du = \mathbb{E}_\pi[\lambda_s \varepsilon_s] f_{kl} dk.$$

The equilibrium allocation is constrained suboptimal, as long as

$$\mathbb{E}_\pi[\lambda_s \varepsilon_s] \neq 0.$$

Once again, for a closed-form solution, the Bernoulli indices are

$$v_s(c_s) = \frac{\beta}{\gamma} c_s^\gamma, \quad \beta > 0, \quad \gamma < 1.$$

The marginal utilities of income are given by

$$\lambda_s = \beta c_s^{\gamma-1} = \beta(y + \varepsilon_s f_l)^{\gamma-1},$$

and the first-order condition of the individuals' optimization problem is

$$\frac{1}{1+r} = \beta \mathbb{E}_\pi[(y + \varepsilon_s f_l)^{\gamma-1}]$$

while

$$du = \beta \mathbb{E}_\pi[(y + \varepsilon_s f_l)^{\gamma-1} \varepsilon_s] f_{kl} dk.$$

If, further, as before, there are two equally probable personal states, with $\varepsilon_s = \pm\varepsilon$, then the first order condition becomes

$$\frac{1}{2}\beta(y + \varepsilon f_l)^{\gamma-1} + \frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1} = \frac{1}{1+r}$$

which implies, since $\varepsilon > 0$ and $\gamma < 1$, that

$$\frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1} > \frac{1}{1+r}.$$

By substitution,

$$\mathbb{E}_\pi[\lambda_s \varepsilon_s] = \frac{1}{2}\beta(y + \varepsilon f_l)^{\gamma-1}(\varepsilon) + \frac{1}{2}\beta(y - \varepsilon f_l)^{\gamma-1}(-\varepsilon) =$$

$$\beta\varepsilon(1 - (y - \varepsilon f_l)^{\gamma-1}) < 0,$$

Since $f_{kl} > 0$, at a competitive equilibrium, individuals overinvest at date 0: if $dk < 0$, then $du > 0$.

It is a general result, in [Geanakoplos and Polemarchakis, 1986], that, when asset markets are imperfect, competitive equilibrium allocations are constrained inefficient. The result extends, in [Carvajal and Polemarchakis, 2011], to economies in it is only idiosyncratic risk that cannot be (fully) insured.

Here, just by trading the risk-free asset differently, all types of individuals in the society could be made ex-ante strictly better off.

Finally, the economy is one of overlapping generations, a la Diamond (1965), where each generation lives for two periods, and the population grows at a constant rate $n \geq 0$.

Only, individuals are endowed with $\bar{l}$ units of labor in each of the two periods they live, which they supply inelastically.

Under certainty, their ex-ante utility is given by

$$u(k) = \bar{l}w - k + v(\bar{l}w + (1+r)k),$$

and the effects of a perturbation are

$$du = \bar{l}dw - dk + \lambda(kdr + \bar{l}dw + (1+r)dk).$$

In this case, the first-order conditions of individual optimization require that

$$\lambda = \frac{1}{1+r},$$

and, by direct substitution,

$$du = \frac{(2+r)\bar{l}dw + kdr}{1+r}.$$

Since the total supply labor is $(2 + n)\bar{l}$ and the production technology is of constant returns to scale,

$$(2 + n)\bar{l}dw + kdr = 0,$$

and

$$du = \frac{r - n}{1 + r}\bar{l}f_{lk}dk,$$

which establishes the *Golden Rule* criterion: if the interest rate is above (resp. below) the rate of population growth, in equilibrium the economy underinvests (resp. overinvests).

Now, the endowment of labor in the second period is subject to idiosyncratic risk, and is $\bar{l}_s = \bar{l} + \varepsilon_s$ with probability π_s, where $\mathbb{E}_\pi[\varepsilon_s] = 0$. As before, letting λ_s be the marginal utility of income in state s, we have that the first-order condition for optimization is

$$\mathbb{E}_\pi[\lambda_s] = \frac{1}{1+r},$$

and, hence, the welfare effects of a perturbation are

$$\begin{aligned} du &= \bar{l}dw - dk + \mathbb{E}_\pi[\lambda_s(kdr + \bar{l}_s dw + (1+r)dk)] \\ &= \left(\frac{r-n}{1+r}\bar{l} + \mathbb{E}_\pi[\lambda_s \varepsilon_s]\right) f_{lk} dk. \end{aligned}$$

If

$$v(c) = \frac{\beta}{\gamma} c^{\gamma},$$

as in the previous example, then

$$\lambda_s = \beta c_s^{\gamma - 1} = \beta(y - (1 + n)\bar{l}w + \varepsilon_s w)^{\gamma - 1}.$$

If, further, $\varepsilon_s = \pm\varepsilon$ with probability $1/2$, then the first-order condition becomes

$$\mathbb{E}_\pi[\lambda_s] =$$

$$\tfrac{1}{2}\beta(y - (1+n)\bar{l}w + \varepsilon w)^{\gamma-1} + \tfrac{1}{2}\beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1} =$$

$$\tfrac{1}{1+r},$$

which implies that

$$\frac{1}{2}\beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1} > \frac{1}{1+r}.$$

By substitution,

$$\mathbb{E}_\pi[\lambda_s \varepsilon_s] =$$

$$\frac{1}{2}\beta(y - (1+n)\bar{l}w + \varepsilon w)^{\gamma-1}\varepsilon - \frac{1}{2}\beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1}\varepsilon =$$

$$\varepsilon\left(\frac{1}{1+r} - \beta(y - (1+n)\bar{l}w - \varepsilon w)^{\gamma-1}\right) <$$

$$0.$$

When the interest rate is below the rate of population growth the competitive equilibrium implies overinvestment (as in the case of certainty).

In the presence of idiosyncratic risk, the second prescription of the Golden Rule *may* fail: and in an economy where the interest rate is higher than the growth of population, it may be that a Pareto improvement requires for every generation to save less.

The identification of Pareto improving asset reallocations from market data remains an issue.

Because of the particular aggregation structure of idiosyncratic shocks, results in the literature, [Geanakoplos and Polemarchakis, 1990]or [Kubler et al., 2002], do not survive.

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