Contagion & Correlation in Bank Credit Risk

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Correlated Risk

- Credit risk anticyclical
- Asymmetric information theory
- Bernanke & Gertler (1988): adverse selection, banks less choosy
- Kiyotaki & Moore (1997): moral hazard
- Carling et al (2007) micro data
- Bond defaults, credit ratings, corporate failure
- Chen (2001) macro feedback (credit crunch)

Financial Contagion

- Epidemiological models of counterparty risk
- Liquidity preference shocks: Allen & Gale (2000)
- Giesecke & Weber (2006): distribution of portfolio loss thru interacting particles
- Egloff et al (2007): random graph theory for counterparties
- Horst (2007): cascade model with contagion &correlation
- Proprietary data only
- No empirical research on contagion

Epidemiology

- Susceptibles (x), infectives (y), removals (z): $x_t + y_t + z_t = N$
- Infection rate: $\alpha = \beta[1 – (1 - \pi)^y]$
- π = probability of contact with infective
- $1 - \beta$ = probability of immunity
- $\Delta x_t/x_{t-1} = \alpha y_{t-1}$ $\Delta z_t/z_{t-1} = \delta y_{t-1}$
- Determinstic model: Kermack & McKendrick (1927)
- Stochastic model: Reed & Frost (Abbey 1952)

Economic Contagion

- Contagion is not just about counterparties
- Contagion thru trade
- Between firms
- Between industries (input – output)
- Rumors
- Rational v mechanical contagion theory
- “Keynesian” contagion
- Malign v benign contagion
- Central banks publish data on sectoral credit risk

This Paper

- Bank of Israel data on problem loans
- 1997 q1 - 2010 q3
- 12 sectors for banking system
- Data for individual banks (not by sector)
- Econometric identification of contagion
- Estimate contagion between sectors
- Model simulations

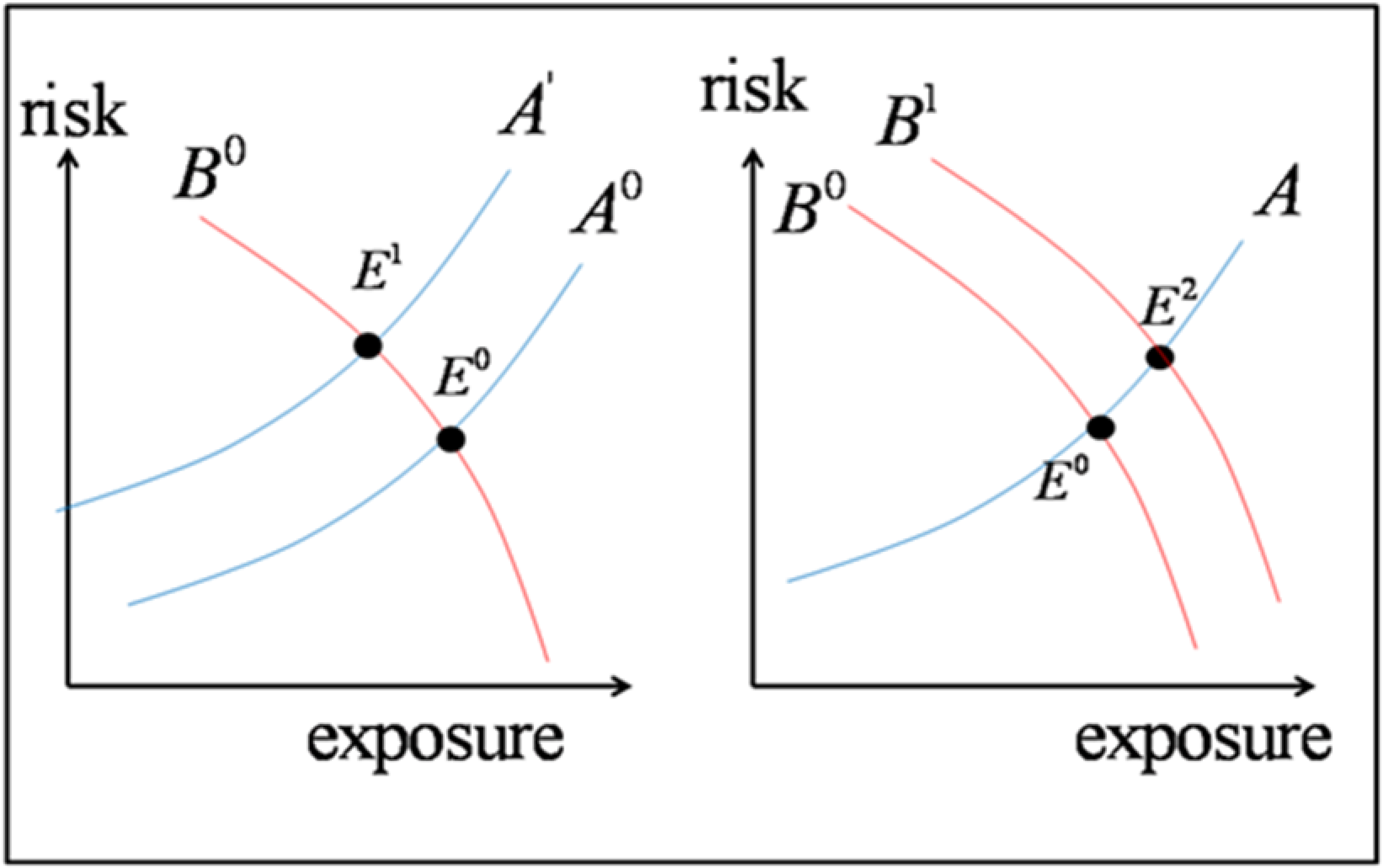
THEORY

Credit Risk Exposure

Endogenous Infection

Correlation & Contagion

Theory: Risk Exposure



Endogenous Contact

- $U_{ik} = F[c_{ik}, H(c_{ik}/Y_k = 1)] \quad c_{ik} = 1$
- U_{0ik}: $c_{ik} = 0$
- $c_{ik} = 1$: $d_{ik} = U_{ik} - U_{0ik} > 0$
- $P(c_{ik} = 1) = 1/[1+\exp(-d_{ik})]$
- $= f(q) = 1/q \qquad f` < 0$
- Endogenous resistance too
- Exposure avoidance
- Investment in resistance

$$Y_{At} = \alpha_A + \theta Y_{Bt-1} + \lambda Y_{At-1} + \phi z_t + u_{At}$$
$$Y_{Bt} = \alpha_B + \theta Y_{At-1} + \lambda Y_{Bt-1} + \phi z_t + u_{Bt}$$
$$r(u_A u_B) = \rho$$

Case	θ	λ	σ^2	ϕ	ρ	r	Var(Y)
1	0.1	0.3	1.5	1	0.2	0.567	2.847
2	0.2	0.3	1.5	1	0.2	0.614	3.065
3	0.2	0	1.5	1	0.2	0.520	2.604
4	0.2	0.3	1	1	0.2	0.681	2.478
5	0.2	0.3	1	1	0	0.597	2.446
6	0	0	1	1	0	0.500	2
7	0	0	1	0	0	0	1

$\sigma_Z = 1$

Sources of Correlation

- Common risk factors (z)
- Correlated risk factors $r(x_k x_J) \neq 0$
- Correlated shocks $r(u_n u_m) \neq 0$
- Contagion $\theta \neq 0$

ECONOMETRICS

Reflection Problem
“Spatial” identification
Identification by IV (GMM)
Identification by weak exogeneity

Static Contagion
Cross – section data

$$y = \alpha Cy + \varepsilon \qquad N - vector$$

$$y_i = 1 \; if \;\; defaulted \qquad \& \;\; 0 \;\; otherwise$$

$$linear \quad probabilit \quad y \quad \mathrm{mod} \;\; el$$

$$C \;\; sparse \quad N \times N \;\; c_{ij} = 1 \;\; i \;\; \& \;\; j \, contact \qquad (0 \;\; otherwise \quad)$$

$$y = A\varepsilon \qquad A = \left[I_N - \alpha C \right]^{-1}$$

$$\frac{\partial y_i}{\partial \varepsilon_k} = a_{ik}$$

$$q_k = \sum_{i=1}^{N} a_{ik}$$

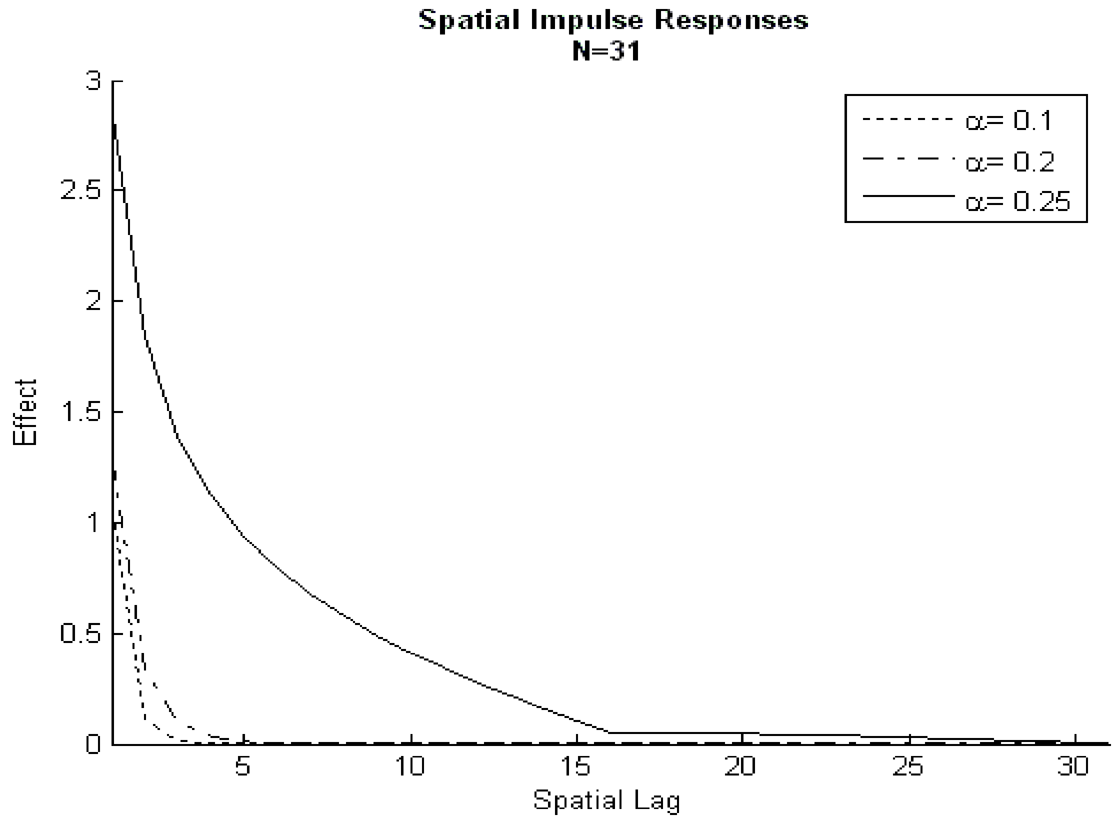
$$\hat{\pi}_{ML} \quad (SAR \;\;)$$

$$\varepsilon_i = e + \omega_i$$

Contagious Impulses

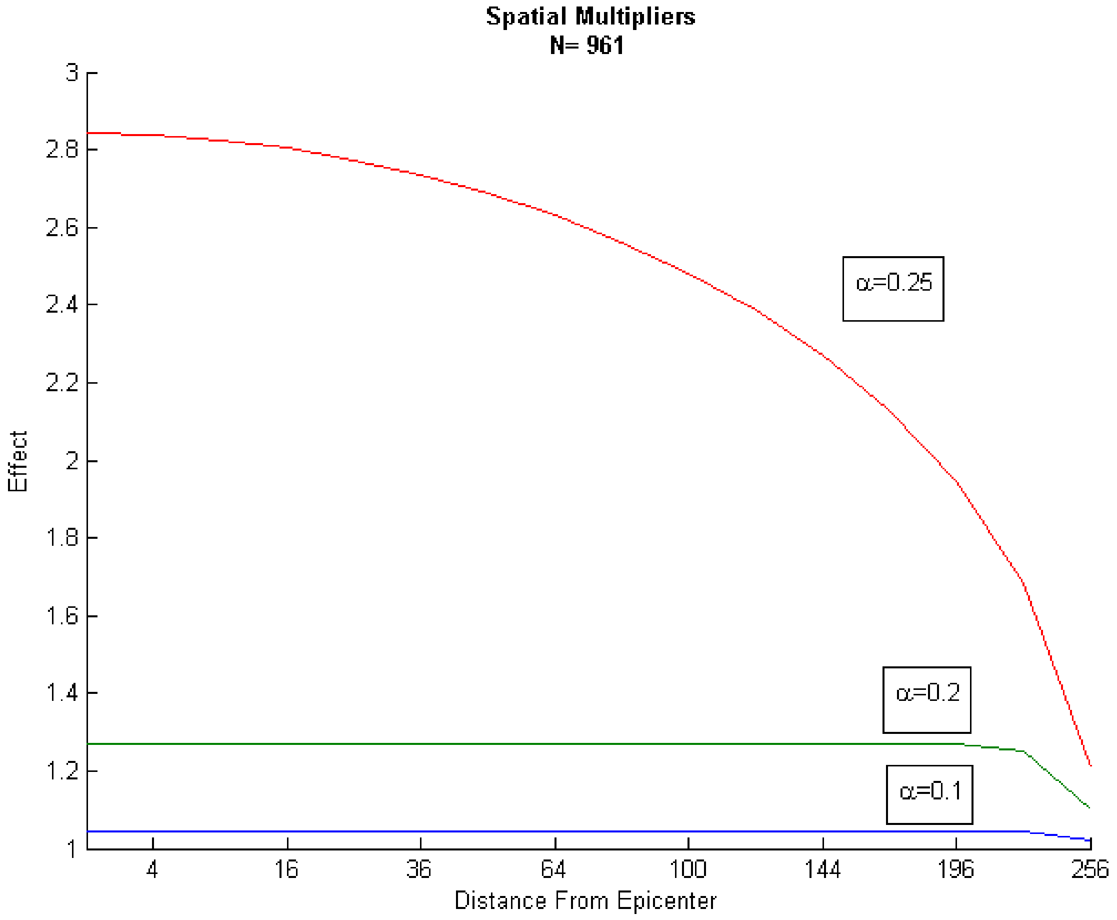
31 x 31 Rook Lattice

$Y = \alpha Cy + \varepsilon$



- Impulse responses (from epicenter) die away more slowly with larger α.
- Edge effects attenuate impulse responses when α=¼

Local Impulses
31 x 31 rook lattice



Instrumental Variables

Cross – section data

$$y = \alpha C y + \beta z + \varepsilon$$

$$y = A(\beta z + \varepsilon)$$

$$A = I + \pi C + \pi^2 C^2 + \ldots$$

$$\tilde{z} = Cz\,, \quad \tilde{\tilde{z}} = C^2 z$$

$$\hat{y} = \beta[z + \pi \tilde{z} + \pi^2 \tilde{\tilde{z}} + ..]$$

$$y = \alpha C \hat{y} + \beta z + \varepsilon$$

Dynamic Contagion
Panel data

$$y_t = \alpha C y_{t-1} + \varepsilon_t$$

$$y_{t-1} = \alpha C y_{t-2} + \varepsilon_{t-1}$$

$$y_t = \alpha^2 C^2 y_{t-2} + \varepsilon_t + \alpha C \varepsilon_{t-1} = \varepsilon_t + \alpha C \varepsilon_{t-1} + \alpha^2 C^2 \varepsilon_{t-2} + \ldots$$

$$\frac{\partial y_{it}}{\partial \varepsilon_{kt-p}} = \alpha^p c_{ik}^p \quad (domino\ contagion)$$

$$\frac{dy_{it}}{d\varepsilon_{it-p}} = \alpha^p c_{ii}^p \quad (boomerang\ \ contagion)$$

$$p\lim \hat{\alpha} = \alpha \ \ if \ \ y_{t-1} \ weakly\ exogenous$$

Sectoral Var Model

x: sectoral factors

z: macroeconomic factors

u: innovations

Data: Israel, 7 sectors, 1997 Q1 – 2010 Q3

Not panel VAR!

Identification through weak exogeneity @ t-1

$$Y_{nt} = \alpha_n + \sum_{i=1}^{p} \lambda_{ni} Y_{nt-1} + \sum_{h=1}^{H} \sum_{i=0}^{b} \phi_{nhi} z_{ht-i} + \sum_{k=1}^{K} \sum_{i=0}^{d} \beta_{nki} x_{nkt-i} + \sum_{j \neq n}^{J} \sum_{i=1}^{c} \theta_{nji} Y_{jt-i} + u_{nt}$$

Weak Exogeneity

(Granger causality v weak exogeneity)

$$Y_t = \beta X_{t-1} + \lambda Y_{t-1} + u_t$$

$$u_t = \rho u_{t-1} + \delta v_t + \varepsilon_t$$

$$X_t = \theta Y_{t-1} + v_t$$

$$E(X_{t-1} u_t) = E[(\theta Y_{t-2} + v_{t-1})(\rho u_{t-1} + \delta v_t + \varepsilon_t)]$$

$$u_{t-1} = \rho u_{t-2} + \delta v_{t-1} + \varepsilon_{t-1}$$

$$E(X_{t-1} u_t) = \theta \rho^2 \sigma_u^2 + \rho \delta \sigma_v^2 + \delta E(v_t v_{t-1})$$

DATA

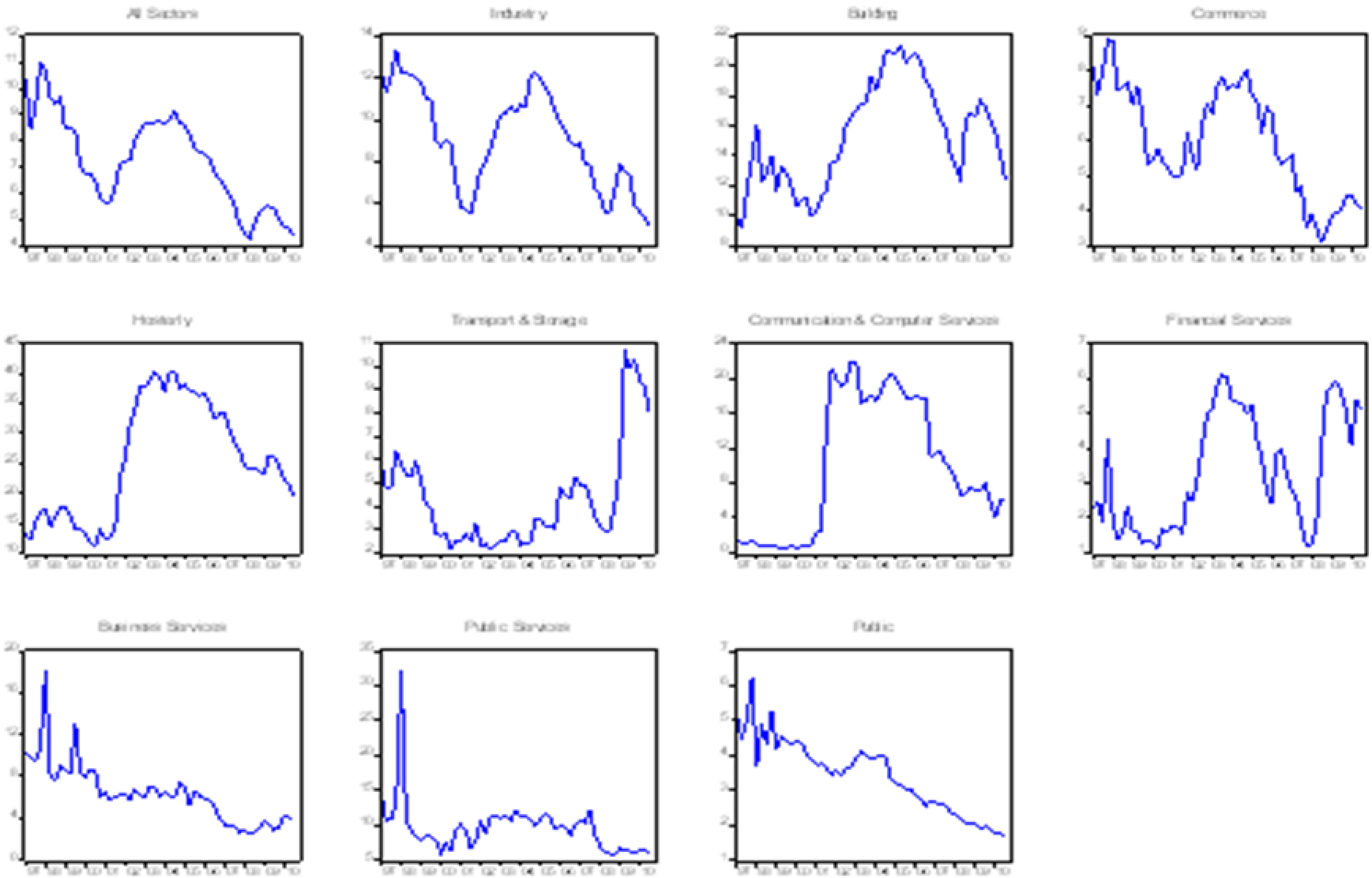
Israel 1997 Q1 – 2010 Q3

7 bank credit sectors

“Problem Loans”

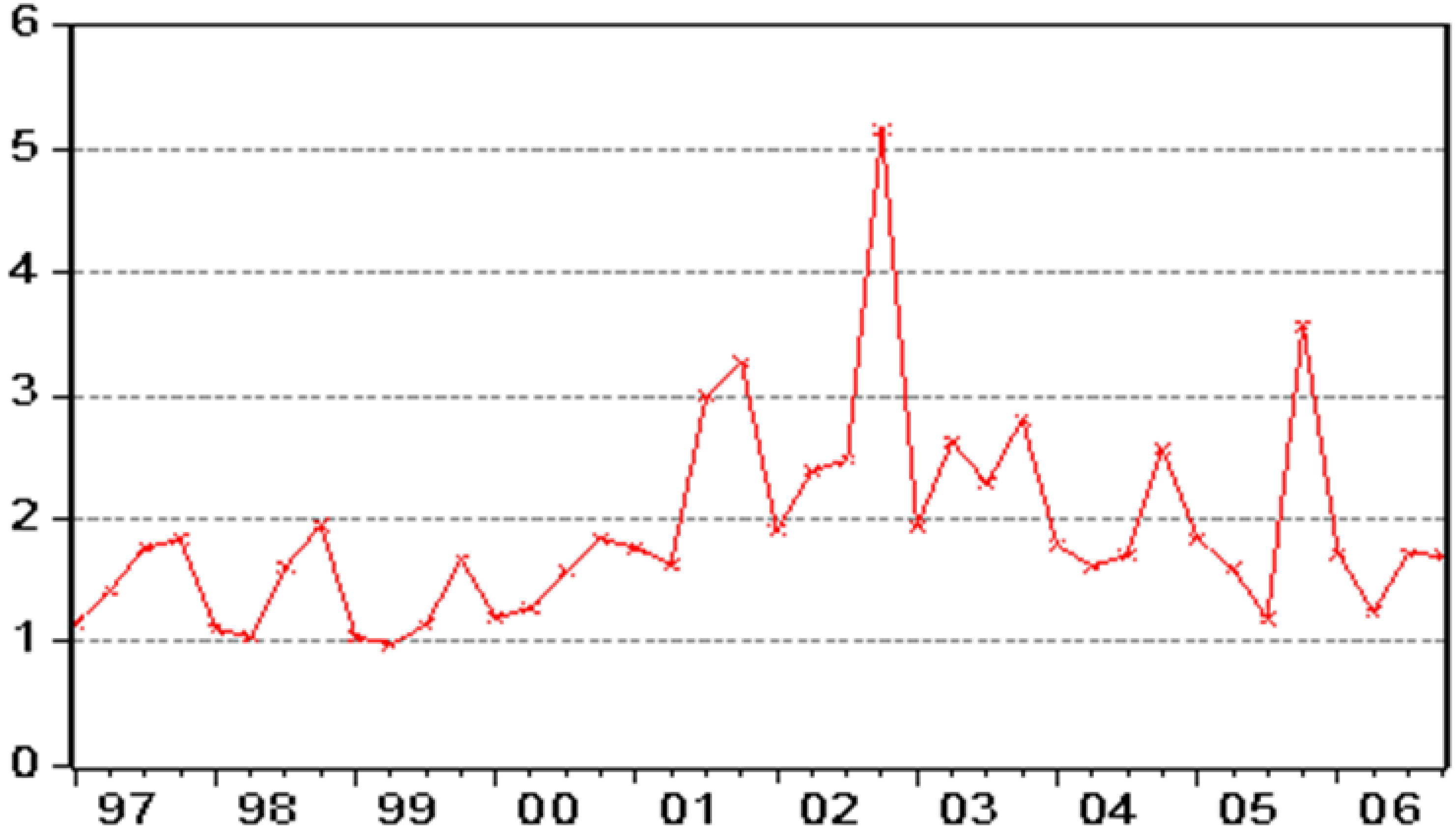
- Bank of Israel definition
- Includes loans under rescheduling
- Broader than FDIC definition
- Role of credit swaps

Rate of Problem Loans

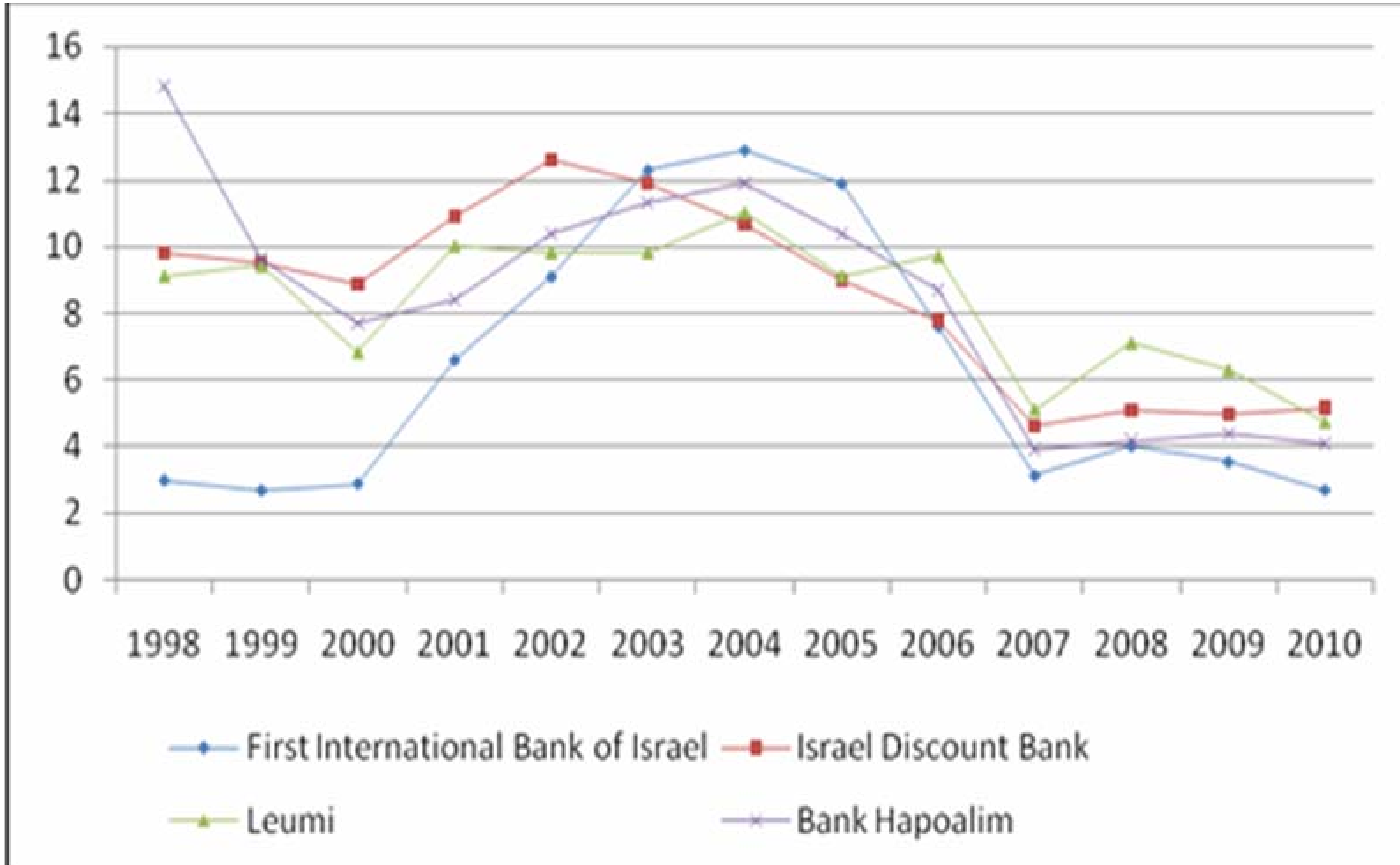


Loan Loss Provisions

(% of Problem Loans)



Problem Credit Rates



Unit Root Tests

	Industry	Building	Commerce	Hostelry	Transport & Storage	Communications &Computer services	Financial services	Business services	Persons
ADF	-1.47	-1.22	-2.07	-1.90	-1.72	-1.67	-2.08	-3.67	0.36
PP	-1.28	-1.90	-1.42	-1.47	-1.55	-1.49	-1.90	-2.77	-1.27
KPSS	0.47	0.54	0.62	0.49	0.38	0.39	0.44	1.02	1.13

Notes: ADF is the 4th order augmented Dickey-Fuller statistic and PP is the Phillips-Perron statistic with bandwidth 1-4 with critical value -2.9 at $p = 0.05$. KPSS is the 5th order KPSS statistic with critical value 0.463.

Correlation Matrix
Problem Credit Rate

	Industry	Building	Commerce	Hostelry	Transport & Storage	Communications &Computer services	Financial services	Business services
Building	0.23							
Commerce	0.90	0.15						
Hostelry	0.17	0.89	0.16					
Transport & Storage	-0.16	0.00	-0.24	-0.20				
Communications &Computer services	0.05	0.74	0.13	0.89	-0.37			
Financial services	0.01	0.65	0.03	0.65	0.25	0.53		
Business services	0.66	-0.27	0.74	-0.36	-0.15	-0.32	-0.32	
Persons	0.70	-0.37	0.77	-0.31	-0.38	-0.24	-0.34	0.74

RESULTS

Sectoral Models
Drivers of credit risk
Coefficients of inertia
Coefficients of contagion
Misspecification tests

Table 5.1 Factor Models: Industry

	Coefficient	D	s	Lag order	Standard error
Industrial production[a]	-6.76	1	2	1	1.81
Share of electrical equipment in industrial production	-5.28	0	0	4	0.73

^ Logged and HP filtered.

Table 5.2 Factor Model: Building

	Coefficient	d	s	Lag order	Standard error
Construction: gross output[a]	-31.45	1	1	1	4.49
Exchange rate[b]	-9.87	0	0	1	2.58
Consumption per head	-19.05	1	4	0	4.91
Public sector investment	-5.90	1	1	3	1.66

Notes. b: logs

Table 5.3 Factor Model: Commerce

	Coefficient	d	s	Lag order	Standard error
Exchange rate (USD)	3.44	1	1	2	1.55
Gross investment	-1.84	1	1	3	0.80
Inflation	-0.79	1	2	0	0.22
Unemployment rate	0.16	0	0	4	0.05

Table 5.4 Factor Model: Hostelry

	Coefficient	d	s	Lag order	Standard error
Foreign tourists[b]	-3.25	0	0	2	0.91
Domestic tourism[b]	-25.86	0	0	2	6.17
Deaths due to terror	0.06	0	0	1	0.01
Real wages[c]	-48.19	1	1	0	9.62

Notes. : HP filtered.

Table 5.5 Factor Model: Transport & Storage

	Coefficient	d	s	Lag order	Standard error
Price of diesel fuel	0.47	0	0	4	0.10
Employment in sector[c]	-0.21	1	0	0	0.04
Wages in sector[c]	0.00	1	3	1	0.00
Gross output in sector[c]	-0.04	0	0	1	0.01
YTM indexed bonds	0.54	1	1	2	0.13
Exports	-3.41	1	3	0	0.79

Table 5.6 Factor Model: Financial Services

	Coefficient	d	s	Lag order	Standard error
Employment in sector[a]	-25.69	1	1	2	5.82
TASE 100[c]	-2.57	0	0	0	0.52

Table 5.7 Factor Model: Persons

	Coefficient	d	s	Lag order	Standard error
Interest rate: Bank of Israel	0.18	0	0	2	0.02
Unemployment rate	0.14	0	0	1	0.04
Inflation	0.74 0.69	0	0	1 2	0.22 0.23

Notes. The dependent variable is expressed as log[RPC/(1-RPC)].

Coefficients of Inertia

	Persons	Industry	Building	Commerce	Hostelry	Transport-Storage	Financial services
Coefficient	0.22	0.51	0.88	0.79	0.72	0.83	0.74
Lag order	2	1	1	1	1	1	1

Notes. P-value of coefficients of inertia < 0.025.

Coefficients of Contagion

	Commerce	Industry	Transport Storage	Building	Hostelry	Financial Services	Persons
Industry			-1.17 Δ(2)	0.19 (1)			
Building		-0.68 Δ(3)	-0.45(4)			-0.32 Δ_2(1)	
Commerce				0.19 Δ(3)			0.31 Δ(1)
Hostelry				0.56 (2)			1.29 Δ_2(1)
Transport – Storage	-0.35 Δ(2)						0.56 (1)
Financial Services		-0.11 (3)	-0.18 Δ(3)	0.07 (4)			
Persons			-0.06 (1)	-0.13 Δ(2)		0.14 (2)	

Notes. The table reads horizontally. P-value of contagion coefficients < 0.025. Δ_s indicates that contagion is in seasonal difference and (p) denotes that lag order of contagion.

Long-run Residuals

$$u_{jt}^{*} = Y_{jt} - \alpha_{j}^{*} - \sum_{k=1}^{K} \beta_{kj}^{*} z_{kt} - \sum_{n=1}^{N} \gamma_{nj}^{*} x_{nt} - \sum_{h \neq j}^{J} \theta_{hj}^{*} Y_{ht-1}$$

$$\beta_{kj}^{*} = \frac{\sum_{i=0}^{b} \beta_{kji}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \quad \gamma_{nj}^{*} = \frac{\sum_{i=0}^{d} \gamma_{jni}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \quad \theta_{hj}^{*} = \frac{\sum_{i=1}^{c} \theta_{hji}}{1 - \sum_{i=1}^{p} \lambda_{ji}}$$

Goodness of Fit & Specification Tests

	Adjusted R^2	Standard error	CV	LM1	LM2	Forecast test	ADF long run residuals	KPSS dynamic simulation residuals
	0.95	0.54	0.06	0.36	0.61	0.67	-6.75	0.11
	0.95	0.76	0.05	0.25	0.08	0.96	-8.37	0.23
	0.94	0.38	0.06	0.59	0.62	0.90	-7.58	0.15
	0.98	1.31	0.05	0.32	0.60	0.36	-8.91	0.23
age	0.95	0.53	0.12	0.08	0.07	0.06	-8.95	0.09
ces	0.92	0.48	0.14	0.30	0.22	0.40	-6.38	0.06
	0.93	0.27	0.08	0.30	0.20	0.74	-9.99	0.11

Innovation Correlation Matrix

	Industry	Building	Commerce	Hostelry	Transport-Storage	Financial services
	0.039					
:e	0.347	0.021				
	0.208	0.030	0.092			
t-Storage	0.174	0.184	0.036	0.187		
services	-0.053	0.079	-0.181	0.216	0.245	
	0.139	-0.165	0.025	-0.043	-0.290	0.020

Notes. Bartlett test of sphericity = 26.9 P-value for zero correlations df = 21 is 0.172.

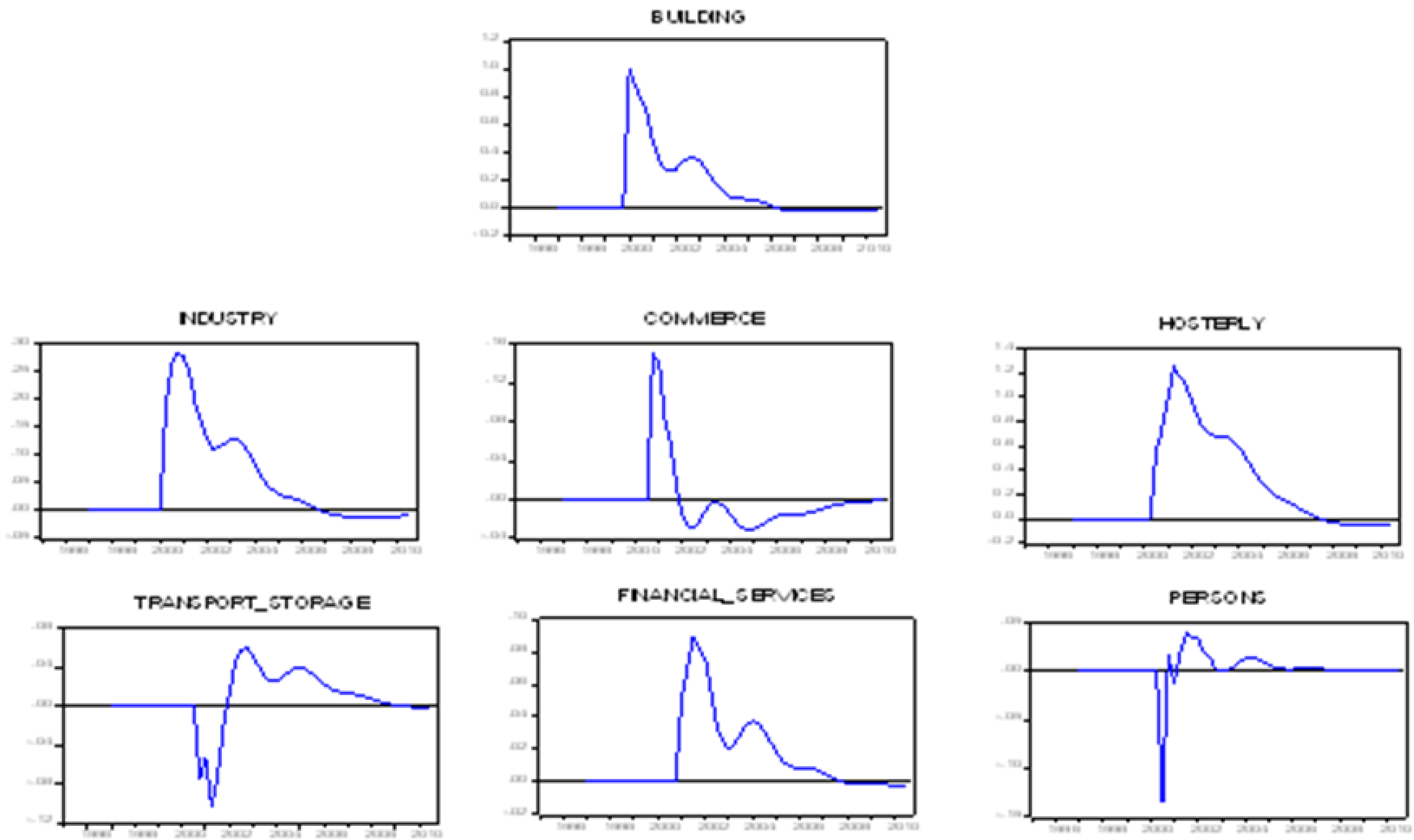
Model Properties

Impulse responses
Counterfactual simulation
Variance decomposition
(Contagion induced correlation)

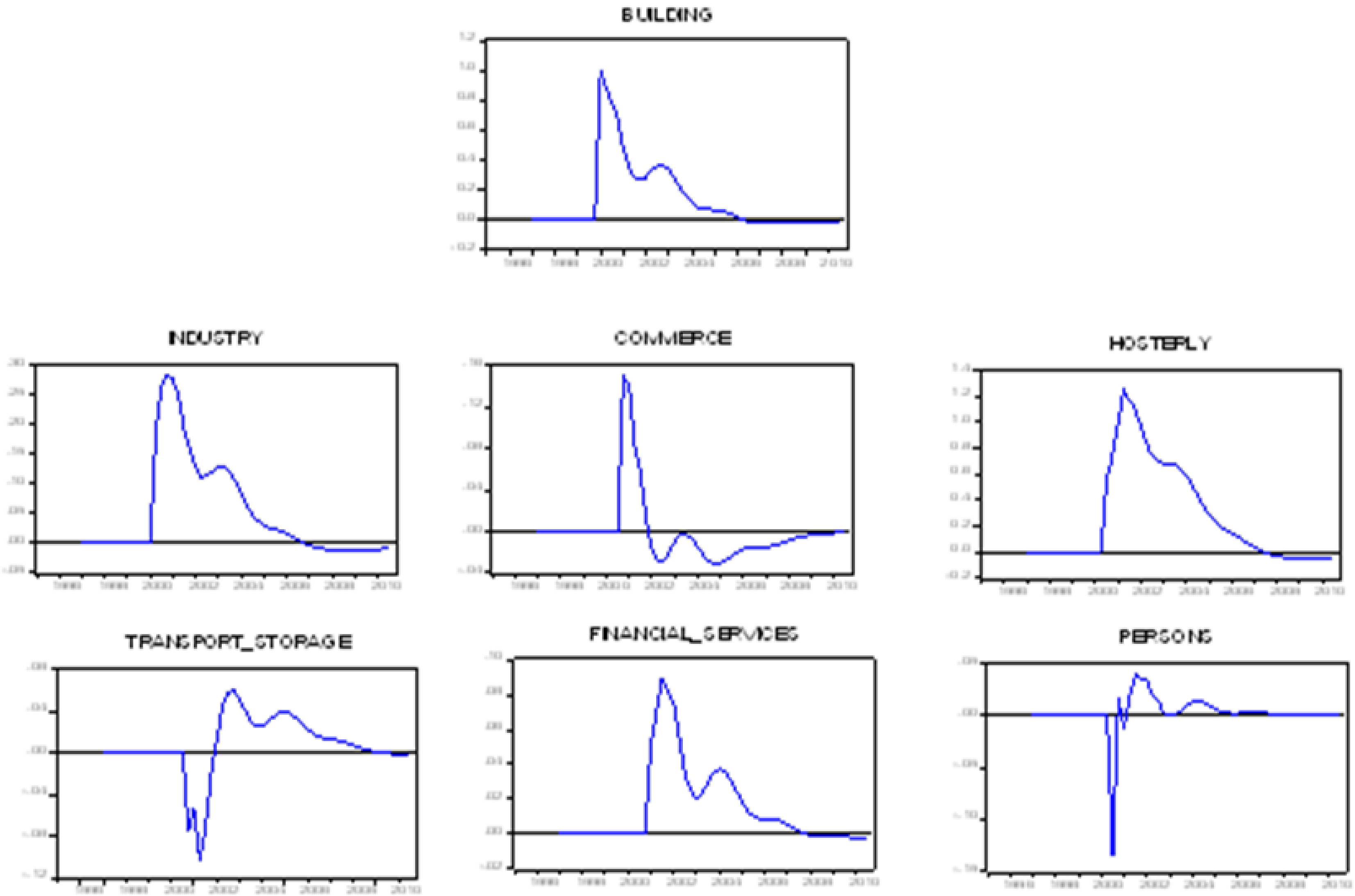
Technical Details

- $x_{it} = a_i + b_i C_t + d_i x_{it-1} + e_i C_{t-1} + v_{it}$
- Full dynamic simulation: 2000 Q1 – 2010 Q3
- Perturb C in 2000 Q1
- Impulse = Y_{nt}(peturbed) – Y_{nt} (FDS)
- Perturb u_n in 2000 Q1
- Fix x's from 2000 Q1
- Fix C from 2000 Q1
- Use model estimates of u_{nt}
- Set contagion coefficients (θ) = 0

Impulse Response: Shock to Credit Risk in Building



Impulse Response: Cyclical Shock



Variance Decomposition of Credit Risk

	Industry	Building	Commerce	Hostelry	Transport	Financial Services	Persons
Standard Deviation	2.11	3.39	1.44	9.24	7.01	1.66	0.83
Cycle	0.78	2.50	1.09	4.15	1.04	1.45	0.39
Factors	1.88	2.15	0.86	8.16	6.91	0.65	0.14
Residual	0.54	0.76	0.38	1.31	0.53	0.48	0.27

Epilogue

- Why don’t regulators publish sectoral credit risk data for banks?
- Why aren’t banks required to publish these data?
- If they did, maybe banks would be more cautious

Epidemiology

- $N = x_t + y_t + z_t$
- x: susceptibles
- y: infectives
- z: removals (deaths, immunes)
- π: probability of contact with an infective
- $(1-\pi)^y$: probability of no contact with infectives
- $1 - \beta$: probability of immunity
- $\alpha = \beta[1-(1-\pi)^y]$ probability of infection

Bernoulli (1760)

$$N_t = x_t + z_t$$

$$\dot{x}_t = -(\alpha + \delta_t)x_t$$

$$\dot{z}_t = -\delta_t z_t + (1-\beta)\alpha x_t$$

$$N_t = N_0 e^{-\delta t}\left(1-\beta+\beta e^{-\alpha t}\right) \quad (\delta_t = \delta)$$

$$\alpha = \beta = 0.125 \ (smallpox)$$

Verhulst (1838)

$$N = x_t + y_t$$

$$\frac{\dot{y}_t}{y_t} = \alpha x_t$$

$$y_t = \frac{N y_0}{y_0 + (N - y_0)e^{-\alpha t}} \xrightarrow{t \to \infty} N$$

Kermack-McKendrick (1927)

$$N = x_t + y_t + z_t$$

$$\frac{\dot{x}_t}{x_t} = -\alpha y_t$$

$$\dot{z} = \delta y_t$$

$$x_t \xrightarrow{t \to \infty} < N$$

Reed-Frost Model
Abbey (1952)

$$x_0 + y_0 = N + 1 \qquad y_0 = 1$$

$$\alpha_t = \beta\left[1 - (1-\pi)^{y_{t-1}}\right]$$

$$f(\Delta y_1) = \frac{x_0!}{\Delta y_1!(x_0 - \Delta y_1)!}\alpha_1^{x_0}(1-\alpha_1)^{x_0 - \Delta y_1} \qquad \alpha_1 = \beta\pi$$

$$f(\Delta y_t) = \frac{x_{t-1}!}{\Delta y_t!(x_{t-1} - \Delta y_t)!}\alpha_t^{x_{t-1}}(1-\alpha_t)^{x_{t-1} - \Delta y_t}$$

$$x_{t-1} = N + 1 - y_{t-1}$$

$$1st\ order\ Binomial\ Markov\ Chain$$

$$E(y_t) \xrightarrow{t\to\infty} < N+1$$

Criticisms

- π exogenous: no precaution
- π should vary inversely with y
- β exogenous: no investment in resistance
- β should vary inversely with y
- Infection rate (α) varies inversely with y
- Mechanical not behavioral
- Heterogeneity in susceptibility √

Verhulst Model with Endogenous Infection

$$y_t + x_t = N$$

$$\frac{\Delta y_t}{y_{t-1}} = \alpha_t x_{t-1}$$

$$\alpha_t = \overline{\alpha} y_{t-1}^{-\theta}$$

$$\Delta \ln y_t \cong \ln \overline{\alpha} - \theta \ln y_{t-1} + \ln(N - y_{t-1})$$

$$y_t \xrightarrow{t\to\infty} < N$$

Definitions

Population of N lives in square lattice

Individuals have n 1st order neighbors

n = 4 in chessboard lattice

C: NxN sparse contact matrix

c_{ik} = 1 if i and k are neighbors

c_{ii} = 0

α probability of contamination on contact

Y_i = 1 if i is contaminated = contagious

q: number contaminated

Static Contagion

$$y = \alpha C y + \varepsilon \qquad N - vector$$

$$y = A\varepsilon \qquad A = \left[I_N - \alpha C \right]^{-1}$$

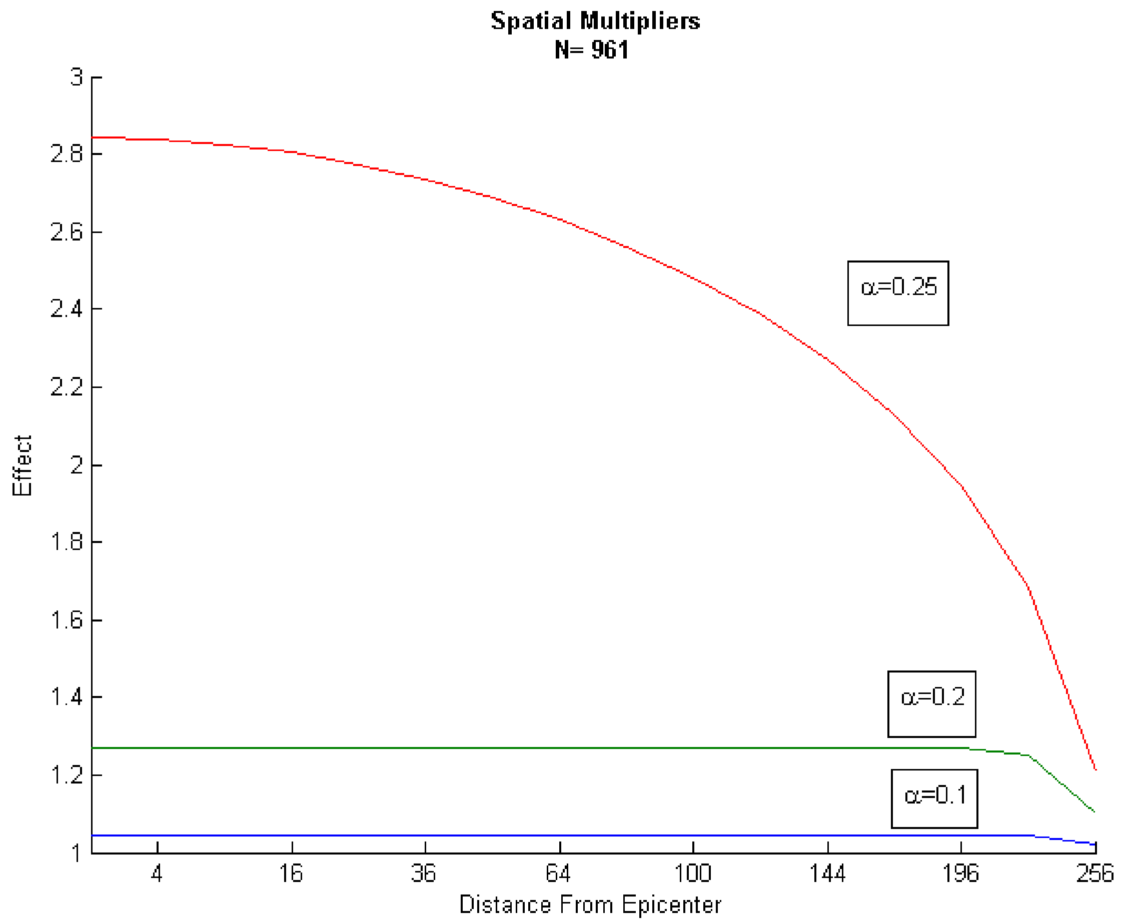
$$\frac{\partial y_i}{\partial \varepsilon_k} = a_{ik}$$

$$q_k = \sum_{i=1}^{N} a_{ik}$$

$$\hat{\pi}_{ML} \quad (SAR)$$

$$\varepsilon_i = e + \omega_i$$

Impulse Responses



Instrumental Variables

$$y = \alpha C y + \beta z + \varepsilon$$

$$y = A(\beta z + \varepsilon)$$

$$A = I + \pi C + \pi^2 C^2 + \ldots$$

$$\tilde{z} = Cz, \quad \tilde{\tilde{z}} = C^2 z$$

$$\hat{y} = \beta[z + \pi\tilde{z} + \pi^2 \tilde{\tilde{z}} + ..]$$

$$y = \alpha C \hat{y} + \beta z + \varepsilon$$

Contagion v Correlation

- Y_1 and Y_k are correlated
- Via α: Contagion
- Via e: Correlated shocks
- Contagion is identified

Dynamic Contagion

$$y_t = \alpha C y_{t-1} + \varepsilon_t$$

$$y_{t-1} = \alpha C y_{t-2} + \varepsilon_{t-1}$$

$$y_t = \alpha^2 C^2 y_{t-2} + \varepsilon_t + \alpha C \varepsilon_{t-1} = \varepsilon_t + \alpha C \varepsilon_{t-1} + \alpha^2 C^2 \varepsilon_{t-2} + \ldots$$

$$\frac{\partial y_{it}}{\partial \varepsilon_{kt-p}} = \alpha^p c_{ik}^p \quad (domino\ contagion)$$

$$\frac{dy_{it}}{d\varepsilon_{it-p}} = \alpha^p c_{ii}^p \quad (boomerang\ contagion)$$

$$p\lim \hat{\alpha} = \alpha \ \ if \ \ y_{t-1} \ weakly\ exogenous$$

Diffusion

$$q_0 = 1$$

$$E(q_1) = q_0 + \alpha n$$

$$E(q_2) = q_1 + \tfrac{1}{2}\alpha(n-1) + \tfrac{1}{2}\alpha(n-2)$$

$$E_{t-1}(q_t) = q_{t-1} + \alpha n - \tfrac{3}{2}\alpha$$

$$E(q) \Rightarrow N$$

Endogenous Contact

- $U_{ik} = F[c_{ik}, H(c_{ik}/Y_k = 1)]$ $c_{ik} = 1$
- U_{0ik}: $c_{ik} = 0$
- $c_{ik} = 1$: $d_{ik} = U_{ik} - U_{0ik} > 0$
- $P(c_{ik} = 1) = 1/[1+\exp(-d_{ik})]$
- $= f(q) = 1/q$ $f` < 0$
- Endogenous resistance too
- Exposure avoidance
- Investment in resistance

Contagion with Endogenous Exposure

$$y_t = \frac{\alpha C}{q_{t-1}} y_{t-1} + \varepsilon_t$$

$$E(q) \Rightarrow < N$$