# Energy Balance Climate Models and the Spatial Structure of Optimal Mitigation Policies

Anastasios Xepapadeas

Athens University of Economics and Business and Beijer Fellow
Joint work with William Brock and Gustav Engstrom

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- The spatial dimension of damages from climate changer can be associated with two main factors:
  1. Natural mechanisms which produce a spatially *non-uniform* distribution of the surface temperature across the globe.
     1. Heat flux that balances incoming and outgoing radiation
     2. Differences among the local heat absorbing capacity - the local albedo - which is relatively higher in ice covered regions;
  2. Economic related forces which determine the damages that a regional (local) economy is expected to suffer from a given increase of the local temperature.
     1. Production characteristics (e.g. agriculture vs services) or
     2. local natural characteristics ( e.g. proximity to the sea and elevation from the sea level).
- The interactions between the spatially non-uniform temperature distribution and the spatially non uniform economic characteristics will finally shape the spatial distribution of damages.

- Existing literature and in particular the DICE/RICE models (e.g. Nordhaus and Boyer 2000, Nordhaus, 2007, 2010, 2011) provide a spatial distribution of damages where the relatively higher damages from climate change are concentrating in the zones around the equator.
- Integrated Assessment Models (IAMs) do not account the natural mechanism generating temperature distribution across the globe.
- IAMs with a carbon cycle are zero-dimensional models and they do not include spatial effects due to heat diffusion across space.
- Energy balance climate models (EBCMs) are one- or two-dimensional models developed by climate scientists which model heat diffusion across latitudes or across latitudes and longitudes ( e.g. Budyko 1969, Sellers 1969,1976, North 1975 a,b, North et al. 1981, Kim and North 1992, Wu and North 2007).

- One-dimensional EBCMs predict a concave temperature distribution across latitudes with the maximum temperature at the equator.
- This is important for understanding the so called "temperature anomaly" which is the difference between the temperature distribution at a given benchmark period and the current period and regional damages.
- The temperature anomaly is higher at the Poles relative to the equator (NASA data).
- Regional damages are obtained by mapping a given change in the temperature of a region relative to a benchmark period - the temperature anomaly - to the damages that this change is expected to bring given the characteristics of the region's economy.

- Zero-dimensional models (IAMs) assume a spatially homogeneous temperature anomaly, since climate change acts on the global average temperature which is spatially homogeneous.
- In the context of a one- or two-dimensional model climate change acts on the spatially non homogeneous temperature distribution.
- This results in a spatially non homogeneous distribution of the temperature anomaly which in turn will differentiate the distribution of damages from those implied by a zero dimensional model.

1. We study the economics of climate change by coupling spatial energy balance climate models (EBCM) in which heat diffuses across latitudes with economic growth models. This approach, integrates solution methods for spatial climate models, that may be new to economics, with methods of solving economic models, It provides new insights regrading the spatial distribution of temperature, damages relative to the more conventional integrated assessment models with carbon cycle which do not account heat transfer across latitudes.
2. The interactions between heat diffusion and economic variables however could be important in characterizing the spatial distribution of damages due to climate change as well as the spatial and temporal characteristics of mitigation policies

The main contribution of our paper is to couple spatial climate models, with economic models, and use these spatial climate models in order to achieve three objectives.

1. First Objective: To show how heat transport across latitudes matters regarding the prediction of the spatial distribution and the corresponding temporal evolution of temperature, damages and optimal mitigation efforts.
   - In pursuing this objective we endogenously derive temperature and damage distributions, climate response functions.
   - As far as we know, this is the first time that the spatial distribution of surface temperature and damages, and their temporal evolutions are determined endogenously in the conceptual framework of a coupled EBCM - economic growth model.

2. Second Objective: To provide insights regarding the optimal spatial and temporal profile for current and future mitigation, when thermal transport across latitudes is taken into account.
    a. Regarding the spatial profile of fossil fuel taxes our result suggest higher tax rates for wealthier geographical zones due to practical inability of implementing the international transfers needed to implement a competitive equilibrium with spatially uniform carbon taxes.

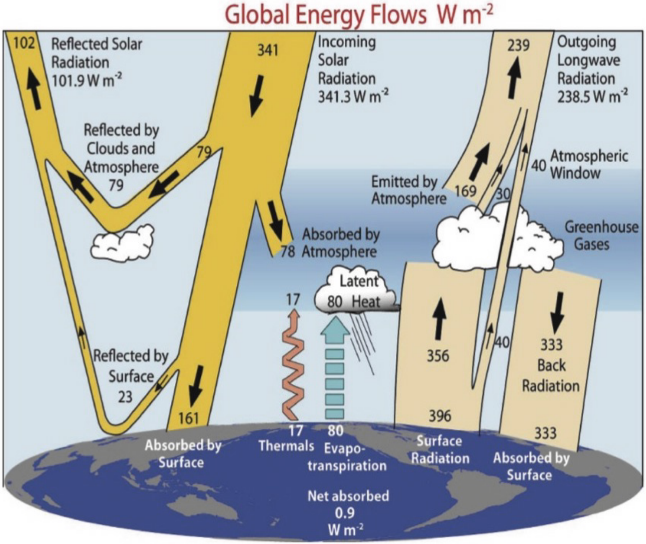
    - Our one-dimensional model allows us to show how heat diffusion across geographical zones impacts the size of the spatial differentiation of fossil fuel taxes between poor and wealthy regions.
    - Our result provides new insights to a result (non uniform optimal mitigation) that was first noted by Chichilnisky and Heal (1994) by characterizing the spatial distribution of fossil fuel taxes and linking the degree of spatial differentiation of optimal fossil fuel taxes to the diffusion of heat.

2.

b Regarding the temporal profile of optimal mitigation, Among economists dealing with climate change on the mitigation side the debate has basically settled on whether to increase mitigation efforts that is, carbon taxes, gradually (e.g. Nordhaus 2007, 2010, 2011), or whether we should mitigate rapidly (e.g. Stern 2006, Weitzman 2009 a,b).

- We locate sufficient conditions for profit taxes on fossil fuel firms to be decreasing over time and for unit taxes on fossil fuels to grow over time less than the rate of return on capital. We also locate sufficient conditions for the tax schedule to be increasing according to the gradualist approach.

3. Third Objective: To introduce the economics profession to the spatial EBCMs with heat transport as a potentially useful alternative for studying the economics of climate change relative to the simple carbon cycle models.
   - By deriving the spatiotemporal profile for optimal taxes we show how the spatial EBCMs can contribute to the current debate regarding
     - how much to mitigate now,
     - whether mitigation policies should be spatially homogeneous or not, and
     - how to derive geographically specific information regarding damages and policy measures.


Global Energy Flows W m-2
102
Reflected Solar Radiation 101.9 W m-2
341
Incoming Solar Radiation 341.3 W m-2
239
Outgoing Longwave Radiation 238.5 W m-2
Reflected by Clouds and Atmosphere 79
79
40
Atmospheric Window
Emitted by Atmosphere
169
30
Greenhouse Gases
Absorbed by Atmosphere
78
Latent Heat
80
17
Reflected by Surface 23
356
40
333 Back Radiation
161
396
333
Absorbed by Surface
17 Thermals
80 Evapo-transpiration
Surface Radiation
Absorbed by Surface
Net absorbed 0.9 W m-2

1. Explicit incorporation of the spatial dimension into the climate model in the form of heat diffusion or transport across latitudes.
   - one, two-dimensional models
2. The presence of an endogenous ice line where latitudes north (south) of the ice line are solid ice and latitudes south (north) of the ice line are ice free.
3. The underlying latitude dependent temperature function.

The use of a spatial EBCMs model allows us to:

- Estimate a distribution of temperature anomaly across latitudes.
- Estimate the spatial effects of climate change by deriving a damage function that depends not on the average global temperature anomaly but on the distribution of temperature anomaly across latitudes.
- Introduce the concept "damage reservoirs" like ice-lines and permafrost as possible second component of damages in addition to the conventional components associated with conventional temperature increase. With damage reservoirs marginal damages are expected to be high initially and then decline as the ice-lines move to the Poles and permafrost disappears. Once the reservoir is exhausted there is no further damages

$$\begin{aligned}\frac{\partial I(x,t)}{\partial t} &= QS(x,t)\,\alpha(x,x_s(t)) - [I(x,t) - h(x,t)] + \\ & D\frac{\partial}{\partial x}\left[\left(1-x^2\right)\frac{\partial I(x.t)}{\partial x}\right] \\ I(x,t) &= A + BT(x,t),\ A = 201.4W/m^2,\ B = 1.45W/m^2\end{aligned}$$

where units of $x$ are chosen so that $x = 0$ denotes the Equator, $x = 1$ denotes the North Pole, and $x = -1$ denotes the South Pole; $Q$ is the solar constant divided by 2; $S(x)$ is the mean annual meridional distribution of solar radiation which is normalized so that its integral from -1 to 1 is unity; $\alpha(x, x_s(t))$ is the absorption coefficient which is one minus the albedo of the earth-atmosphere system, with $x_s(t)$ being the latitude of the ice line at time $t$; and $D$ is a thermal diffusion coefficient that has been computed as $D = 0.649Wm^{-2\circ}C^{-1}$
Outgoing radiation is reduced by $h(x,t)$: accumulated carbon dioxide.

The heat flux which is modelled by the term:
$D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial I(x,t)}{\partial x}\right]$
The ice line is determined dynamically by the condition :

$$\begin{array}{ll} T > -10°\text{C} & \text{no ice line present at latitude } x \\ T < -10°\text{C} & \text{ice present at latitude } x \end{array}$$

and the ice line absorption drops discontinuously because the albedo jumps discontinuously. North (1975a) specifies the co-albedo function as:

$$\alpha(x, x_s) = \begin{cases} \alpha_0 = 0.38 & |x| > x_s \\ \alpha_1 = 0.68 & |x| < x_s \end{cases}$$

- Human input: $h(x,t) = \sigma(x)\,\xi\left(1 + \ln\frac{M(t)}{M_0}\right)$ where $M_0$ denotes the preindustrial and $M(t)$ the time $t$ stock of carbon dioxide in the atmosphere, $\xi$ is a temperature-forcing parameter ($^\circ$C per $W$ per $m^2$), and $\sigma(x)$ is a weighting function that capture latitudinal differences in the impact of the stock of the atmospheric carbon dioxide on latitude $x$'s temperature.
- The stock of the atmospheric carbon dioxide:

$$\dot{M}(t) = \int_{x=-1}^{x=1} \beta(x,t)\, q(x,t)\, dx - mM(t)\,, \ \ M(0) = M_0$$

Emissions are proportional to the amount of fossil fuels used.

- The total stock of fossil fuel available is fixed or,

$$\int_{x=-1}^{x=1} q(x,t)\, dx = q(t)\ \ , \ \ \int_0^\infty q(t) = R_0$$

where $q(t)$ is total fossil fuels used across all latitudes at time $t$, and $R_0$ is the total available amount of fossil fuels

$$\begin{aligned}\frac{\partial I(x,t)}{\partial t} &= QS(x,t)\,\alpha(x,x_s(t)) - [I(x,t) - h(x,t)] + \\ &\quad D\frac{\partial}{\partial x}\left[\left(1-x^2\right)\frac{\partial I(x.t)}{\partial x}\right] \\ I(x,t) &= A + BT(x,t),\ A = 201.4W/m^2,\ B = 1.45W/m^2\end{aligned}$$

$$h(x,t) = \sigma(x)\,\xi \ln\left(1 + \frac{M(t)}{M_0}\right)$$

$$\dot{M}(t) = \int_{x=-1}^{x=1} \beta(x,t)\,q(x,t)\,dx - mM(t)\,,\ M(0) = M_0$$

$$\int_{x=-1}^{x=1} q(x,t)\,dx = q(t)\ ,\ \int_0^{\infty} q(t) = R_0$$

$$B\frac{\partial T(x,t)}{\partial t} = QS(x)\alpha(x,x_s) - [(A + BT(x,t)) - h(x,t)] + DB\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T(x,t)}{\partial x}\right]$$

**Approximation**.
A satisfactory approximation of the solution for can be obtained by the so called two mode solution where $n = \{0, 2\}$.

$$\hat{T}(x,t) = \sum_{n \text{ even}} T_n(t)P_n(x)$$

$$\hat{T}(x,t) = T_0(t) + T_2(t)P_2(x)$$

$$B\frac{dT_0(t)}{dt} = -A - BT_0(t) + \int_{-1}^{1}\left[QS(x)\alpha(x,x_s) + \xi \ln\frac{M(t)}{M_0}\sigma(x)\right]dx$$

$$B\frac{dT_2(t)}{dt} = -B(1+6D)T_2(t) + \frac{5}{2}\int_{-1}^{1}\left[QS(x)\alpha(x,x_s) + \xi \ln\frac{M(t)}{M_0}\sigma(x)\right]P_2(x)$$

$$T_0(0) = T_{00},\ T_2(0) = T_{20},\ P_2(x) = \frac{(3x^2-1)}{2}$$

$$S(x) = 1 + S_2P_2(x)\ ,\ \ S_2 = -0.482$$

If $D \to \infty$, then the solution for $T_2(t)$ vanishes.
Thus for a given diffusion $D < \infty$ the relative contribution of $T_2(t)$ to the solution $\hat{T}(t)$ can be regraded as an a measure of whether the heat transport is important in the solution of the problem.
The ice line solves

$$T_0(t) + T_2(t; D) P_2(x_s(t)) = T_s, \; T_s = -10^{\circ}\mathrm{C}$$

and is given by a solution of the ice line condition above, i.e.

$$x_s(t) = P_+^{-1}\left(\frac{T_s - T_0(t)}{T_2(t; D)}\right)$$

$$\hat{T}(x,t) = T_0(t) + T_2(t;D)P_2(x)$$

$$B\frac{dT_0(t)}{dt} = -A - BT_0(t) +$$

$$\int_{-1}^{1}\left[QS(x)\alpha(x,\hat{T}(x,t)) + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\sigma(x)\right]dx$$

$$B\frac{dT_2(t)}{dt} = -B(1+6D)T_2(t) +$$

$$\frac{5}{2}\int_{-1}^{1}\left[QS(x)\alpha(x,\hat{T}(x,t)) + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\sigma(x)\right]P_2(x)d$$

$$T_0(t) + T_2(t;D)P_2(x_s(t)) = T_s, \quad T_s = -10°\mathrm{C}$$

$$h(x,t) = \sigma(x)\,\xi \ln\left(1 + \frac{M(t)}{M_0}\right)$$

$$\dot{M}(t) = \int_{x=-1}^{x=1} \beta(x,t)\, q(x,t)\, dx - mM(t)\,, \;\; M(0) = M_0$$

$$\int_{x=-1}^{x=1} q(x,t)\, dx = q(t) \;\;, \;\; \int_{0}^{\infty} q(t) = R_0$$

$a(x) = a_0 - a_1 P_2(x)$; $S(x) = 0.5\left[1 - s_0 P_2(x)\right]$, ($_0 = 0.681$, $a_1 = 0.202$, $s_0 = 0.477$). (North at al. 1981); $\sigma(x) = \sigma$

The two-mode approximating ODEs become

$$\frac{dT_0}{dt} = -\frac{A}{B} - T_0(t) + \frac{1}{B}\left[\langle QS(x)\alpha(x), 1\rangle + \xi \ln\left(1 + \frac{M(t)}{M_0}\right)\langle \sigma, 1\rangle\right]$$

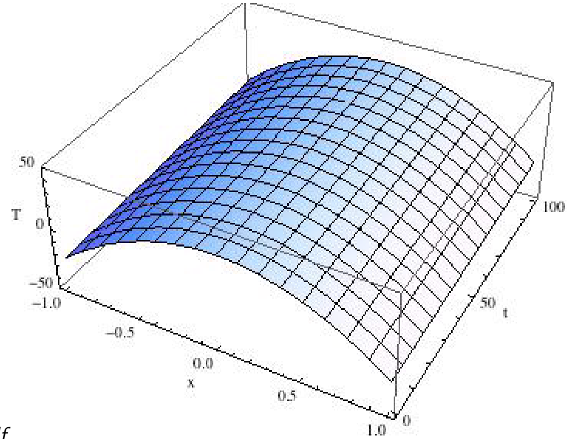
$$\frac{dT_2}{dt} = -(1+6D)\,T_2(t) + \frac{5}{2B}\langle QS(x)\alpha(x), P_2(x)\rangle$$

Set $\frac{dT_0}{dt} = \frac{dT_2}{dt} = 0$. Then

$$T(x, t; D) = C_0 + C_1 \ln\frac{M(t)}{M_0} - \frac{C_2}{(1+6D)} P_2(x)\ ,\ C_0, C_1, C_2 > 0$$

Exogenous emissions growth at 1,026% per year (IPCC-A1F1 scenario) and
$A = 203.4; B = 2.09; Q = 310; \sigma = 0.7; \xi = 5,35,$
$D = 0.649,\ M_0 = 583$GTC. Current average temperature:
$T(0,0) = 24°\text{C},\ T(1,0) = T(-1,0) = -25°\text{C}$

1

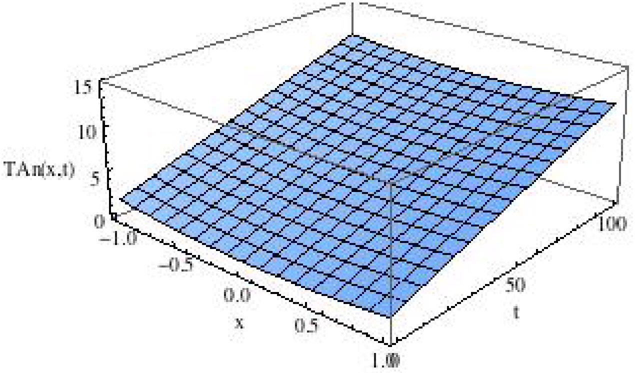



2.pdf

- Similar temperature functions have been derived by climate scientists (e.g. Sellers 1969, 1976), but without the impact of human activities on climate. I
- n our case this impact is realized by the increase in the concentration of atmospheric carbon dioxide.
- When $D \longrightarrow \infty$ the temperature function is spatially homogeneous or 'flat' across latitudes
- The distinction between a latitude dependent and a flat temperature field provides a first sign of the impact of thermal transport on the estimation of the temperature function.

$T^{+}(x,t;D) = \hat{T}(x,t;D) - T_0(x,t)$, where $T_0(x,t)$ is distribution of temperature across latitudes as implied by existing data (NASA) for a benchmark period (1880-1900)

2



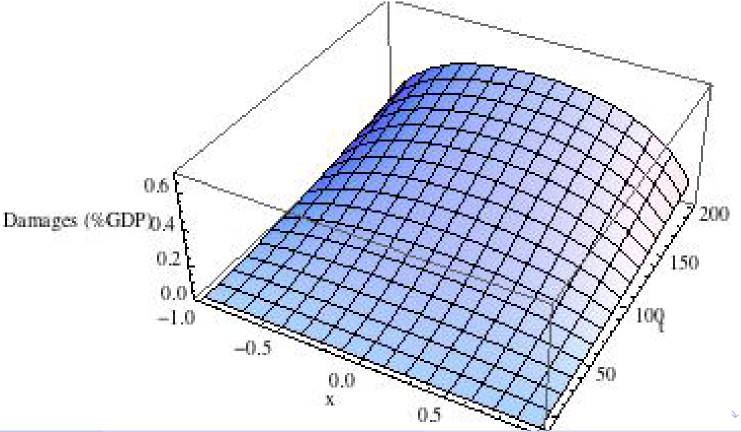

$_3$.*pdf*

Temperature anomaly between the average of 1890-1900 and the current period suggests 0.8°C for the equator and 1.7°C for the Poles.

$$\Omega\left(T^{+}(x,t;D)\right)=\frac{1}{1+\omega(x)\,\theta_1 T^{+}+\omega(x)\,\theta_2\left(T^{+}\right)^2}$$

$$1-\Omega\left(T^{+}(x,t;D)\right)$$

3



$$
\begin{aligned}
dT_0(t) &= (\partial_{T_0,M})\, dM(t), \;\; dT_2(t) = (\partial_{T_2,M})\, dM(t) \\
dT(t,x) &= dT_0(t) + P_2(x)\, dT_2(t) = \\
&\quad [(\partial_{T_0,M}) + (\partial_{T_2,M})\, P_2(x)]\, dM(t)
\end{aligned}
$$

The impact on damages will then be determined as:

$$
\begin{aligned}
d\Omega(T(x,t)) &= \Omega'_T\,[dT_0(t) + P_2(x)\, dT_2(t)] = \\
&\quad \Omega'_T\,[(\partial_{T_0,M}) + (\partial_{T_2,M})\, P_2(x)]\, dM(t)
\end{aligned}
$$

$$
\begin{aligned}
Y(t,x) &= A(x,t)\Omega(T(x,t))F(K(x,t),L(x,t),q(x,t)) \\
&\equiv e^{(a+n\alpha_L)t}\Psi(x,T(x,t))K(x,t)^{\alpha_K}q(x,t)^{\alpha_q}
\end{aligned}
$$

$$
\begin{aligned}
& F_{total}(K(t),q(t),\{T(x,t)\}_{x=-1}^{x=1};x,t) \\
= \; & F_{total}(K(t),q(t),T;t) \\
& C(t)+\dot{K}(t)+\delta K(t) = F_{total}(K(t),q(t),T;t), \\
& j(t) = \int_{x=-1}^{x=1} j(x,t)\,dx, j = C,K,q \\
& F_{total}(K(t),q(t),T;t) = \left[e^{(a+\alpha_L n)t}K(t)^{\alpha_K}q(t)^{\alpha_q}\right]J(t;D) \\
& J(x,t;D) = \frac{\Psi(x,T(x,t))^{1/\alpha_L}}{\left[\int_{x'}\Psi(x',T(x',t)^{1/a_L}dx'\right]^{a_K+a_q}} \\
& J\left(\{T(x,t)\}_{x=-1}^{x=1}\right) = J(t;D) \equiv \int_x J(x,t;D)\,dx
\end{aligned}
$$

- The Cobb-Douglas specification allows the "separation" of the climate damage effects on production across latitudes, as the "index" $J(t; D)$, which depends on thermal diffusion coefficient $D$, that multiplies a production function that is independent of $x$.
- Thus population growth and technical change affect the "macrogrowth component" $e^{(a+\alpha_L n)t} K(t)^{\alpha_K} q(t)^{\alpha_q}$, while changes in the size of $D$ have a direct effect on the "climate component".
- The combination of the macrogrowth and the climate component determine the potential world input.

## The Problem of the Social Planner

$$\max \int_0^{\infty} e^{-\rho t} \int_X \upsilon (x) \, L(x,t) \left[ U \left( \frac{C(x,t)}{L(x,t)} \right) - \Omega_C (T(x,t)) \right] dxdt$$

subject to:

- Climate dynamics
- Resource constraint for the economy
- Total consumption and total fossil fuel constraints,
- States: $\mathbf{v} = (K(t), R(t), M(t), T(t,x))$,
- Controls: $\mathbf{u} = (C(t), C(x,t), q(t), q(x,t))$

$$
\begin{aligned}
\mathcal{H}= & \qquad\qquad (1)\\
& \int_X \upsilon(x) L(x,t) \left[ U\left( \frac{C(x,t)}{L(x,t)} \right) - \Omega_C(T(x,t)) \right] dx + \\
& \lambda_K(t) \left[ F_{total}(K(t), q(t), T; t) - C(t) - \delta K(t) \right] \\
& + \mu_R \left[ R_0 - \int_0^\infty q(t) \right] + \mu_q(t) \left[ q(t) - \int_X q(x,t) dx \right] \\
& \lambda_M(t) \left[ \int_{-1}^{1} \beta(t) q(x,t) dx - mM(t) \right] \\
& + \lambda_T(t,x) \left[ \frac{1}{B} \left[ QS(x) \alpha(x, T(x,t)) - (A + BT(x,t)) \right.\right. \\
& \left.\left. + \sigma(x) \xi \ln\left( 1 + \frac{M(t)}{M_0} \right) + DB \frac{\partial}{\partial x} \left[ (1 - x^2) \frac{\partial T(x,t)}{\partial x} \right] \right] \right] \\
& + \mu_C(t) \left[ C(t) - \int_X C(x,t) dx \right] +
\end{aligned}
$$

$$C(t), C(x,t) \quad : \quad \lambda_K(t) = \mu_C(t) = v(x)\, U'\left(\frac{C(x,t)}{L(x,t)}\right) \tag{2}$$

$$q(t) \quad : \quad \lambda_K(t)\, F'_{total,q} = \lambda_R(t) - \mu_q(t) \tag{3}$$

$$q(x,t) \quad : \quad \lambda_M(t)\, \beta(t) = \mu_q(t) \tag{4}$$

$$\text{or } F'_{total,q} = \frac{\lambda_R(t) - \lambda_M(t)\, \beta}{\lambda_K(t)}, \tag{5}$$

For weights independent of $x$ (2) implies that per capital consumption should be equated across locations.

$$\dot{\lambda}_K(t) = \left[\rho + \delta - F'_{total,K}(K(t), q(t), T; t)\right] \lambda_K(t)$$

$$\dot{\lambda}_M(t) = (\rho + m)\lambda_M(t) - \frac{\xi}{BM(t)} \int_{-1}^{1} \sigma(x)\lambda_T(t,x)$$

$$\dot{\lambda}_T(t,x) = (\rho + 1)\lambda_T(t,x) +$$

$$\upsilon(x) L(t,x) \Omega'_{c,T}(T(t,x)) -$$

$$\lambda_K(t) F'_{total,T}(K(t), q(t), T; t)$$

$$-QS(x)\frac{\lambda_T(t,x)}{B}\frac{\partial \alpha(x, T(x,t))}{\partial T}$$

$$-D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial \lambda_T(x,t)}{\partial x}\right]$$

- A solution of the welfare maximization problem, provided it exists and satisfies the desirable stability properties, will determine the optimal temporal and latitudinal paths for the states, the controls and the costates.
- The optimal time paths will be dependent on the thermal diffusion coefficient $D$. Denoting optimality by a $(^*)$ these paths can be written as:

$$\{K^*(t;D), K^*(t,x;D)\, R^*(t;D),$$
$$M^*(t;D), T^*(t,x;D)_{x=-1}^{x=1}\}$$
$$\{C^*(t;D), C^*(x,t;D), q^*(t;D), q^*(x,t;D)\}_{x=-1}^{x=1}$$
$$\{\lambda_K^*(t;D), \lambda_R^*(t;D), \lambda_M^*(t;D), \lambda_T^*(t,x;D)\}_{x=-1}^{x=1}$$

- We consider a global market economy with each latitude $x$ considered as a country.
- In each country the representative consumer maximizes utility subject to a permanent income constraint by considering as parametric damages due to climate change,
- The representative firm maximizes profits by considering as parametric fossil fuel prices and taxes on fossil fuel use.
- Fossil fuel firms maximize profits by considering as parametric taxes on their profits. The social planner, given the optimizing choices of firms and consumers, chooses the amount of fossil fuel $q(x, \cdot)$ to be used in each location, by taking onto account climate change damages and climate dynamics.
- The choice of fossil fuel allocation determines optimal taxes.

$$\max_{\{C(x,t)\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} L(x,t) U\left(\frac{C(x,t)}{L(x,t)}\right) - \Omega_C(T(x,t;D)) dt \right\}$$

subject to

$$C(x,t) + \dot{K}(x,t) + \dot{B}(x,t) =$$
$$r(t)(K(x,t) + B(x,t)) + I(x,t)$$
$$\text{or } \int_{t=0}^{\infty} e^{-R(t)} C(x,t) dt = K_0(x) + \int_{t=0}^{\infty} e^{-R(t)} I(x,t) dt$$
$$B(x,0) = 0, K(x,0) = K_0(x)$$
$$I(x,t) \equiv$$
$$w(x,t)L(x,t) + s_{FF}(x,t)\pi_{FF}(t) + s_{Tax}(x,t)Tax(t)$$

FONC

$$U'\left(\frac{C(x,t))}{L(x,t}\right)=\Lambda(x)\exp(\rho t-\int_{s=0}^{t}r(s)ds)$$

$\Lambda(x)$ the marginal utility of capitalized income at location $x$.
$\Lambda(x)=\Lambda(x')$ endowment reshuffling 2nd Welfare Theorem.
Furthermore

$$\dot{\lambda}_K(x,t)/\lambda_K(x,t)=\rho-r(t)=\dot{\lambda}_K(x',t)/\lambda_K(x',t) \tag{6}$$

for each $x,x'$ for all $t$, where $\lambda_K(x,t)$ is the current shadow value of capital at location $x$.For $\lambda_K(x,t)=\lambda_K(x',t)$ for each $x,x'$, we need intertemporal endowment flows are adjusted.

$$\max \; \{\mathbb{A}(x,t)\Omega(T(x,t))F(K(x,t),L(x,t),q(x,t))-$$
$$(r(t)+\delta)K(x,t)-w(x,t)L(x,t)-(p(x,t)+\tau(x,t))\,q(x,$$
$$-(p(x,t)+\tau(x,t))\,q(x,t)\}$$

Optimality conditions:

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F'_K(K(x,t),L(x,t),q(x,t)) = r(t)+\delta$$

$$\mathbb{A}(x,t)\Omega(T(x,t;D))F'_q(K(x,t),L(x,t),q(x,t)) = p(x,t)+\tau(x,t)$$

In market equilibrium

$$\dot{\lambda}_K(x,t) = \left(\rho+\delta-F'_K\right)\lambda_K(x,t)$$

$$\max_{q(x,t)} \int_{t=0}^{\infty} \exp\left[-\int_{s=0}^{t} r(s)ds\right] (p(x,t)q(t)(1-\theta(t))]dt,$$

$$\text{subject to} \quad \int_{t=0}^{\infty} q(x,t)dt \leq R_0$$

FONC

$$\begin{aligned} p(x,t)(1-\theta(t)) &= \mu_0 \exp(\int_{s=0}^{t} r(s)ds), \text{ or} \\ \left[\mathbb{A}\Omega F_q' - \tau(x,t)\right](1-\theta(t)) &= \mu_0 \exp(\int_{s=0}^{t} r(s)ds) \end{aligned}$$

In any decentralized problem consumption goods firms at latitude $x$ will choose demands $K(x,t)$ and $q(x,t)$ to set

$$r(t) + \delta = \mathbb{A}\Omega F_K', \;\; p(x,t) + \tau(x,t) = \mathbb{A}\Omega F_q'$$

For a multiplier value $\bar{\mu}_0$ that exhaust the fossil fuels reserves and parametric temperature and taxes we have:

$$\left\{C^e\left(x,t;D,\tau,\theta,p\right), K^e\left(x,t;T,\tau,\theta,p\right), q^e\left(x,t;T,\tau,\theta,p\right)\right\}_{x=-1}^{x=1}$$

- Spatial profile - Temporal profile.
- If there is no climate externality, $\beta(x,t) = 0$ for all $x$ and $t$, the solution of the social planners problem provides an externality free Pareto optimal allocation for any set of appropriate welfare weights $\upsilon(x)$.
- Without externality $\tau(x,t) = 0$ for all $x$ and $t$ and an externality free competitive equilibrium will be Pareto efficient.
- The efficient allocation corresponding to the externality free competitive equilibrium will maximize the sum of utilities across locations for a choice of welfare weights $\upsilon^*(x)$.
- Consider now the problem of a social planner that uses the welfare weights $\upsilon^*(x)$ to solve the welfare maximization problem under the climate externality, that is $\beta(x,t) \geq 0$.
- The Pareto optimal allocation obtained using $\upsilon^*(x)$ can be regarded as "natural" or "just" in some ethical sense. It is a "benchmark" Pareto optimum, PO$(\upsilon^*)$.

- Following the theory of the Second Welfare Theorem, endowments across locations are shuffled to implement $\text{PO}(\upsilon^*)$
- The planner can carry out without cost the necessary adjustments of intertemporal endowment flows across locations so that $\Lambda(x) = \Lambda(x')$ for all $x, x\prime$.Then per capita consumptions will be equated across latitudes
- $\lambda_K(x,t;D) = \lambda_K(x',t;D) \equiv \lambda_K(t;D)$.
- Implementation of $\text{PO}(\upsilon^*)$ requires that social and private marginal products for $K$ and $q$ are equated. Therefore:

$$\upsilon^*(x)\Lambda(x)e^{(\rho t - R(t))} = \lambda_K^*(t;D)$$

$$\tau^*(x,t) = \frac{\mu_R - \beta\lambda_M^*(t;D)}{\lambda_K^*(t;D)} - p(t) = \tau^*(t)$$

$$p(t) = \frac{\mu_0 e^{\Gamma(t)}}{1-\theta^*(t)}$$

Transfers across locations are costly. The planners resource constraint can be written as

$$C(t) + \dot{K}(t) + \delta K(t) = F_{total}(K(t), q(t), T; t) - \frac{C_0}{2}\Theta(t)$$

$$\Theta(t) = \int_X [y(t,x) - C(t,x)]^2 \, dx,$$

$$y(t,x) = Y(t,x) - \delta K(t,x) - u(t,x)$$

$$\dot{K}(t,x) = u(t,x),$$

$$Y(t,x) = \mathbb{A}(x,t)\Omega(T(x,t))F(K(x,t), L(x,t), q(x,t))$$

## The planner's problem: two-mode approximation

$\hat{T}(x,t) = T_0(T) + T_2(t,D)\, P_2(x)$.

$$
\begin{aligned}
H = & \int_X \left\{ \upsilon^*(x)\, L(x,t) \left[ U\left( \frac{C(x,t)}{L(x,t)} \right) - \Omega_C(\hat{T}(x,t)) \right] \right. \\
& \left. + \lambda_K(t,x)\, u(x,t)\, dx \right\} \\
& + \lambda_K(t) \left[ F_{total}(K(t), q(t), \hat{T}(x,t)) - \int_X u(x,t)\, dx \right. \\
& \left. - C(t) - \delta K(t) - \frac{C_0}{2} \Theta(t) \right] \\
& + \lambda_M(t) \left[ -mM(t) + \beta \int_X q(x,t)\, dx \right] \\
& + \lambda_{T_0}(t) \left[ -T_0 + Z_1 \ln\left( \frac{M(t)}{M_0} \right) + Z_0 \right] + \\
& \mu_R \left[ R_0 - \int_0^\infty \int_X q(x,t)\, dx dt \right], \quad q(t) = \int_X q(t,x)\, dx \\
& K(t) = \int_X K(t,x)\, dx, \quad C(t) = \int_X C(t,x)\, dx ,
\end{aligned}
$$

$$\upsilon^*(x)\,U'\left(\frac{C(x,t)}{L(x,t)}\right) = \lambda_K(t)\left[1 - C_0\left[y(t,x) - C(t,x)\right]\right]$$

$$\mathbb{A}\Omega F'_q = \frac{\mu_R - \beta\lambda_M}{\lambda_K(t)\left[1 - C_0\left[y(t,x) - C(t,x)\right]\right]}$$

$$\lambda_K(x,t) = \lambda_K(t)\left[1 - C_0\left[y(t,x) - C(t,x)\right]\right]$$

$$\dot{\lambda}_K(x,t) = \rho\lambda_K(x,t) - \frac{\partial H}{\partial K(x,t)}$$

$$\mathbb{A}\Omega F'_K = \rho + \delta - \frac{\dot{\lambda}_K(t)}{\lambda_K(t)} + C_0\frac{d\left[y(x,t) - C(y,t)\right]/dt}{\left[1 - C_0\left[y(t,x) - C(t,x)\right]\right]}$$

$$\dot{\lambda}_M(t) = (\rho + m)\,\lambda_M(t) - \lambda_{T_0}(t)\,\frac{Z_1}{M(t)}$$

$$\dot{\lambda}_{T_0}(t) = (\rho + 1)\,\lambda_{T_0}(t) + \upsilon^*(x)\,L(x,t)\Omega'_{C,T_0}$$
$$-\lambda_K(t)\left[1 - C_0\left[y(t,x) - C(t,x)\right]\right]\mathbb{A}\Omega'_{T_0}F$$

For $d\left[y\left(x,t\right)-C\left(y,t\right)\right]/dt\simeq 0$.
Equalization of returns on capital across locations

$$\mathbb{A}\Omega F_K' = \rho+\delta-\frac{\dot{\lambda}_K\left(t\right)}{\lambda_K\left(t\right)} = r\left(x,t\right) = r\left(t\right)$$

Fossil fuel prices at location $x$.

$$\frac{dp(t)/dt}{p(t)} = r\left(t\right)\ ,\ p\left(x,t\right) = p\left(x,0\right)\exp\left(\int_{s=0}^{t} r(s)ds\right)$$

Optimal spatially non uniform tax:

$$p\left(x,t\right)+\hat{\tau}\left(x,t\right) = \frac{\mu_R-\beta\lambda_M^*\left(t;D\right)}{\lambda_K^*\left(t\right)\left[1-C_0\left[y^*\left(x,t\right)-C^*\left(x,t\right)\right]\right]}$$

## Proposition

*Let $p(x,t) + \hat{\tau}(x,t)$ stand for the full price of fossil fuels at location $x$ and time $t$, and assume that $p(x,t)$ is approximately uniform across locations. if $[y^*(x,t) - C^*(x,t)] > [y^*(0,t) - C^*(0,t)]$ then $p(x,t) + \hat{\tau}(x,t) > p(0,t) + \hat{\tau}(0,t)$.*

- Since locations around the equator are poor relative to higher latitude locations $[y^*(x,t) - C^*(x,t)] > [y^*(0,t) - C^*(0,t)], x \gg 0$ is expected. Therefore the proposition suggest that poor location should pay a smaller full fuel price relative to rich locations.
- The spatially differentiated tax depends on the thermal diffusion coefficient follows since the damage functions depends on $D$ through the temperature field $T(x,t) = T_0(T) + T_2(t,D) P_2(x)$.

- Our result in the context of an one-dimensional EBCM, suggest an explicit dependency on the thermal diffusion coefficient $D$ through the second mode.
- An important policy issue is the size of bias introduced on optimal taxes when heat transportation is ignored. The bias can be defined as

$$\left|\tau^*(t;D) - \tau^*(t;D\to\infty)\right|$$
$$\left|\hat{\tau}(x,t;D) - \hat{\tau}(x,t;D\to\infty)\right|$$

since when $D \to \infty$ the second and all higher modes vanish and it is the average global temperature and not the distribution of temperature across latitudes that determined damages.

Hotelling's rule:

$$\frac{d\left[p(t)(1-\theta^*(t))\right]/dt}{p(t)(1-\theta^*(t))} = r(t) = \mathbb{A}\Omega F'_K - \delta$$

$$\frac{(\dot{p}(t)-\dot{\tau}^*(t))}{(p(t)-\tau^*(t))} = r(t) \text{ for } \theta^*(t) = 0$$

The policy ramp under the gradualist approach suggests that $\dot{\tau}^*(t) > 0, \dot{\theta}^*(t) > 0$.
To have a declining tax schedule through time:

$$r(t) - \frac{\dot{p}(t)}{p(t)} > 0,$$

## Lemma

$\zeta(t) \equiv \int_x \sigma \lambda_T^*(t,x;D)dx < 0$, $\lambda_M^*(t;D) < 0$.

- Thus $\zeta(t)$ is the global shadow cost of temperature at time $t$ across all latitudes
- $\lambda_M^*(t;D) < 0$ means that an increase in atmospheric accumulation of $CO_2$ at any time $t$ will reduce welfare.

## Proposition

*If $m < \delta$, then the optimal profit tax decreases through time, or $\dot{\theta}^*(t) < 0$. Furthermore, the optimal unit tax on fossil fuels grows at a rate less than the rate of interest, or $\frac{\dot{\tau}^*(t)}{\tau^*(t)} < r^*(t)$.*

## Proposition

*If $m > \delta$ and $\lambda_M^*(m-\delta) - \left(\frac{\xi}{BM(t)}\right)\int_X \sigma\lambda_T^* dx > 0$ then $\dot{\theta}^*(t) > 0$ and $\frac{\dot{\tau}^*(t)}{\tau^*(t)} = \frac{\dot{p}^*(t) - r^*(t)p^*(t)}{\tau^*(t)} + r^*(t) > r^*(t)$.*

- Thus, we have sufficient conditions for rapid ramp-up of profit taxes and for unit carbon to rise at a rate less than the net of depreciation rate of return $r^*(t)$ on capital
- The gradualist tax schedule requires rapid decay of the atmospheric carbon dioxide, and a relatively small global shadow cost of temperature at time $t$ across all latitudes.

**The discounting function effect**

$\int_x \sigma\lambda_T(t,x)\,dx \equiv \zeta$ the global shadow cost of temperature across latitudes: It holds $\dot{\zeta} = \upsilon\zeta - \Xi(t)$ , where

$$\upsilon \equiv \rho + 1 - \frac{Q}{B}\left(\frac{\int_x \sigma\lambda_T(x,t;D)\,S(x)\,(\partial a/\partial T)\,dx}{\int_x \sigma\lambda_T(x,t;D)\,dx}\right)$$

Since $\partial a/\partial T > 0$ the discounting function $\upsilon$ falls. Forward discounted costs of climate change higher than when for the co-albedo function $\partial a/\partial T = 0$.

**The damage effect**

$$J(t;\infty) = \Omega(T_0(t;\infty)) \int_x \left\{ \frac{\int_x [A(x,0)^{1/\alpha_L} L(x,0)]}{\int_{x'} A(x',0)^{1/\alpha_L} L(x',0)\,dx']^{\alpha_K+\alpha_q}} \right\} dx$$

Impact of $D$ on the estimation of damages (damage bias) $=$ $|J(t;\infty) - J(t;D)|$

- In our economy the use of fossil fuels in production generates emissions which accumulate in the atmosphere and block outgoing solar radiation, increasing thus the temperature. In our one-dimensional model heat diffuses across latitudes.
- We derive latitude dependent temperature, damage and climate response functions, as well as optimal mitigation policies which are all determined endogenously through the interaction of climate dynamics with optimizing forward looking economic agents.
- Our results suggest that if the international transfers required to attain a globally Pareto optimal solution cannot be implemented, then taxes on fossil fuels should be lower in relatively poorer geographical zones.
- Carbon taxes depends on the heat diffusion across latitudes. Zero-dimensional IAMs may introduce bias.

- If the elasticity of substitution between material consumption and environment is less than one, it will not be possible in the long run for wealthy latitudes to compensate damaged latitudes. Without appropriate implementation of international transfers carbon taxes should be latitude specific and their sizes should depend on the heat transfer across locations.
- If the decay of atmospheric $CO_2$ is lower than the depreciation of capital then profit taxes on fossil fuel firms will decline over time and unit taxes on fossil fuels will grow at a rate less then the rate of interest.
- These results, which can be contrasted with the gradually increasing policy ramps derived by IAM models like DICE or RICE indicate that mitigation policies should be stronger now relative to the future.
- Increasing policy ramps require rapid decay of the atmospheric carbon dioxide, and a relatively small global shadow cost of temperature increase.

- A world with two goods, consumption goods and environmental services, where utility is modeled by a CES function.
- Augmented our EBCM with a deep ocean component that redistributes vertically the heat energy via uniform vertical diffusion.
- Use the one-dimensional EBCM with spatially dependent co-albedo to introduce latitude depended damage reservoirs like endogenous ice-lines and permafrost.
- Since reservoir damages are expected to arrive relatively early and diminish in the distant future - because the reservoir will be exhausted - the temporal profile of the policy ramp could be declining, enforcing the result obtained for profit taxes, or even U-shaped