The Distributional Consequences of Supply-Side Reforms in General Equilibrium*

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November 2, 2010

Abstract

This paper addresses the issue on whether tax reforms consistent with lower public debt-to-GDP in the long-run can lead to a more efficient and equitable economy. To this end we solve a heterogeneous agent model comprised of a government, a representative capitalist and representative skilled and unskilled workers, under both rational expectations and adaptive learning. Our main findings are that (i) reductions in capital taxation, while beneficial at the aggregate level, lead to increased inequality mainly due to the substitutability of unskilled labour and capital; (ii) a fall in taxation for skilled labour is Pareto improving, which is largely explained by its complementarity with the other factor inputs; (iii) all agents would prefer increasing the tax rate on capital to increasing the tax rate on skilled and unskilled labour since it leads to relatively lower welfare losses; and (iv) heterogeneity in initial beliefs under adaptive learning quantitatively matters for welfare.

Keywords: tax reform, structural heterogeneity, inequality, adaptive learning

JEL codes: E25, E62

*We would like to thank Apostolis Philippopoulos for helpful comments and suggestions. We are also grateful for financial support from the ESRC, Grant No. RES-062-23-2292, but the views expressed here are entirely our own. Address for correspondence: Jim Malley, Department of Economics, University of Glasgow, Adam Smith Building, Glasgow G12 8RT. E-mail jim.malley@glasgow.ac.uk.

1 Introduction

There now exists a significant and growing literature on tax reforms in dynamic general equilibrium (DGE) models, largely focusing on the aggregate welfare benefits and the distributional effects of permanent reductions in constant capital tax rates. At the aggregate level, studies within a representative agent framework suggest that tax reforms which reduce capital taxation will produce welfare gains for the society, even if the tax burden is concurrently shifted to labour (see e.g. Lucas (1990), Cooley and Hansen (1992) and Giannitsarou (2006).[1] The aggregate welfare benefits from tax reforms that reduce capital taxation are also confirmed in models with heterogeneous agents (see e.g. Garcia-Mila *et. al.* (2010)). However, at the same time, heterogeneous agents models make clear that such reforms will have large redistributive effects that will disadvantage different groups in the society (see e.g. Domeij and Heathcote (2004) and Garcia-Mila *et. al.* (2010)).[2] These results then suggest that even if they are beneficial at the aggregate level, tax reforms that reduce capital taxes are not likely to be supported by a majority of the population, unless a compensating mechanism is agreed. It would, of course, be preferable to find tax reforms that are Pareto improving since they are more likely to be implemented.

With the above background in mind, this paper aims to welfare-evaluate changes in income tax rates for different types of agents, to investigate which if any reforms are Pareto improving and if not, which are capable of generating enough gains to compensate potential losers. In most of the literature on tax reforms in heterogeneous agent models, agents differ proportionately with respect to their productivity and/or their asset holdings. However, the research by e.g. Stokey (1996) and Krusell *et al.* (2000) suggests that the roles performed by different types of labour can be structurally distinct in production. In particular, skilled labour complements capital, whereas unskilled labour can substitute for capital.[3]

[1] At the same time, at the aggregate level, there is also an important literature that examines optimal tax policy. The general message from Ramsey optimal taxation is that the tax rate on capital should be zero in the long-run (see e.g. Chamley (1986), Chari *et al.* (1994) and Chari and Kehoe (1999)). This result, however, does not necessarily hold in models incorporating market failures, see e.g. Guo and Lansing (1999), nor in models under time-consistent optimal taxation (see e.g. Klein *et. al.* (2008)).

[2] Studies that take into account the redistributive effects of capital taxation in designing optimal taxation in heterogeneous agent models are fewer. In Judd (1985), Ramsey-type optimal taxation leads to a zero tax on capital in the long run, but this is not a general result (see e.g. Lansing (1999) and Conesa *et. al.*, (2009)).

[3] See these papers and the references therein for empirical support regarding these complementarities/substitutabilities in factor inputs.

To achieve our objectives, we build on the research which allows for structural heterogeneity and construct a closed-economy DGE model comprised of three types of private agents and the government. In particular, the model contains a representative capitalist and representative skilled and unskilled workers who all consume output in the product market and supply labour in the factor market in return for labour income. Additionally, the first two income groups, subject to intermediation costs, allocate savings to physical capital and government bonds in return for capital income whereas unskilled workers do not save. The representative firm is owned by the capitalist who hires (skilled and unskilled) labour services and leases physical capital from the factor market for which it pays the competitive wage and interest rate respectively. Finally, the government taxes economic activity, provides public spending and issues debt to balance its budget.

We calibrate our model to the UK economy, with the aim of obtaining a realistic assessment of the likely costs and benefits of tax reforms for the different agents. Our modeling permits us to capture key features of heterogeneity. Following the literature on credit constraints and income inequality (see e.g. Galor and Zeira (1993), Benabou (1996) and Aghion and Howitt (1998)), financial intermediation costs allow our model to generate heterogeneity in savings. In particular, the heterogeneity in savings predicted by our model is consistent with the UK data, which suggest that about 30% of the households in the UK do not save and that the savings of high savers are about five times higher than those of low savers. In addition, we use a constant elasticity of substitution (CES) specification, following e.g. Stokey (1996) and Krusell *et al.* (2000), which assumes different roles in the production function for skilled and unskilled labour. This allows our calibrated model to produce factor input elasticities and a wage premium that are in line with empirical studies.

We also relax the assumption of rational expectations, to examine whether bounded rationality in the form of adaptive learning (see e.g. Evans and Honkapohja (2001)), can lead to different conclusions regarding the welfare costs of tax increases. For instance, Giannitsarou (2006), in a representative agent model, shows that learners can have lower welfare gains from tax reforms, compared to the case of rational expectations. In the case of bounded rationality, we also consider an additional source of heterogeneity, in the form of the initial beliefs of the agents regarding the equilibrium laws of motion (see e.g. Honkapohja and Mitra (2006), for heterogeneity in learning). In particular, we examine the case where the capitalists are able to predict the rational expectations solution in the post-reform economy and thus use this as their initial guess for the parameters of their policy functions, but the skilled workers cannot and thus use the pre-reform economy to determine their policy functions in the first period after the reform. In other words, for the capitalists the tax reform is perfectly anticipated, while for the skilled workers it is unanticipated. This would correspond to an unequal distribution of information regarding the reform in the economy. As far as we know, this type of heterogeneity has not been considered in the tax reform literature.

As discussed above, we also differ from the previous literature on tax reforms in heterogeneous agent models, as we model the complementarities and substitutabilities between the different types of agents. An additional difference is in the types of tax reform we consider. In particular, in the tax reform literature the main interest has been in policy reforms that lowers the capital tax rate, while increasing the labour tax rate at the same time, to keep the budget balanced (see e.g. Lucas (1990), Domeij and Heathcote (2004) and Garcia-Mila *et. al.* (2010) and, for the UK, Angelopoulos *et al.* (2008)). In a heterogeneous agent setup, this implies that cuts in capital taxes will directly hurt the share of the population whose income is primarily from labour, thus suggesting that a large part of the welfare losses of these agents will come from the increased labour taxation. To isolate the effects of changes in each tax rate on all agents, we consider changes in tax rates that are not followed by opposite movements in the other tax rates. Instead, the target for tax reforms is a lower steady-state debt-to-GDP ratio, holding government spending (excluding interest payment on the debt) fixed.

Our proposed analysis is particularly relevant in the current economic environment as economies struggle to deal with aftermath of the world financial crisis in 2007/08. Supply-side reforms, aimed at creating a tax system that is more efficient and more equitable, have been on the political and economic agenda for many years.[4] Given the burgeoning public debt which has accumulated since the onset of the economic crisis, a number of both demand and supply side measures are being implemented in countries across the world to reduce public debt as a share of national income.[5] Indeed in the UK, an increase in the VAT to 20% together with public spending cuts has been announced. With existing as well as new pressing policy objectives in mind, we employ our modelling framework to address the issue on whether supply-side reforms consistent with lower public debt-to-GDP in the long-run can lead to a more efficient and equitable economy.

Our main findings are that (i) reductions in capital taxation, while ben-

[4]See e.g. the research for the Mirrlees Review, at the Institute for Fiscal Studies for the UK and the discussion on actual tax reforms in Giannitsarou (2006) and Garcia-Mila *et. al.* (2010).

[5]According to data from the OECD for the G-7 (Economic Outlook 87 database), the UK, France and the US have experienced the largest percentage increases in debt-to-GDP of 86%, 68% and 58% respectively.

eficial at the aggregate level, lead to increased inequality mainly due to the substitutability of unskilled labour and capital; (ii) a fall in taxation for skilled labour is Pareto improving, which is largely explained by its complementarity with the other factor inputs; (iii) all agents would prefer increasing the tax rate on capital to increasing the tax rate on skilled and unskilled labour since it leads to relatively lower welfare losses; and (iv) heterogeneity in initial beliefs under adaptive learning quantitatively matters for welfare.

The rest of the paper is organised as follows. Sections 1 through 3 set out the model structure, calibration and steady-state and solution respectively. Section 4 contains the policy analysis and finally Section 5 concludes.

2 Model

The closed-economy setup which follows describes the interaction of three types of private agents and the government in final goods, labour and asset markets.

2.1 Population composition

The population size, N, is exogenous and constant. Among N, $N^c < N$ are identical capitalists, $N^s < N$ are identical skilled workers, and the rest, $N^u = N - N^c - N^s$, are identical unskilled workers. Capitalists are indexed by the subscript $c = 1, 2, ..., N^c$, skilled workers by $s = 1, 2, ..., N^s$ and unskilled workers by $u = 1, 2, ..., N^u$. There are also N^f firms, $f = 1, 2, ..., N^f$. We assume that the number of firms equals the number of capitalists, $N^c = N^f$, and that each capitalist owns one firm. It is useful, for what follows, to define $N^c/N = n^c$, $N^s/N = n^s$, $N^u/N = n^u = 1 - n^c - n^s$ and $N^f/N = n^f$.

2.2 Firms

Each firm produces a single output, Y_t^f, using physical capital, K_t^f, and labour services. There are two distinct types of labour that are used in the production process, unskilled labour, $h_{u,t}^f$, that can be substituted for capital, and skilled labour, h_t^f, which complements capital.[6] The production function is given by a constant returns to scale (CRS) technology assumed to take a constant elasticity of substitution (CES) specification following e.g. Stokey (1996) and Krusell *et al.* (2000):

$$Y_t^f = A_t \left[\rho K_t^f + (1-\rho) h_{u,t}^f\right]^{\alpha} \left[h_t^f\right]^{1-\alpha} \tag{1}$$

[6] Note that skilled labour also includes the labour services of capitalists.

where A_t is exogenous stochastic productivity; $0 < \alpha$, $(1-\alpha)$, are the productivity of weighted capital and unskilled labor and of skilled labour, respectively; $0 < \rho < 1$ measures the degree of substitutability between capital and unskilled labour.[7]

Each firm acts competitively, taking prices and policy variables as given, and maximises profits given by:

$$\Pi_t^f \equiv Y_t^f - r_t^k K_t^f - w_t h_t^f - w_{u,t} h_{u,t}^f \tag{2}$$

subject to the technology constraint given by (1); where w_t and $w_{u,t}$ are, respectively, the wage rates of skilled and unskilled labour and r_t^k is the interest rate on capital.[8] The different roles in the production function for skilled and unskilled labour imply that there will be a skill premium for the former, in the sense that the ratio of w_t to $w_{u,t}$ will be larger than unity. We will calibrate the production function so that the implied factor input elasticities and the resulting wage premium are in line with empirical studies.

2.3 Budget constraints of capitalists

The representative capitalist owns one firm and receives its profits. He also receives income from providing skilled labour services, $h_{c,t}$, to the labour market and income from interest on his accumulated stock of financial assets, in the form of capital, $K_{c,t}$, and government bonds, $B_{c,t}$. The interest rate on government bonds is given by r_t^b. All these sources of income are taxed. In particular, financial asset and profit income are taxed at the constant rate τ^k, while labour income is taxed at the constant rate τ^h.

We assume that those agents holding assets need to pay intermediation or transaction premia due to imperfections in capital markets. For instance, these premia can represent the costs of gathering extra information relating to legal issues, asset-specific government regulations, intermediation fees and so on. We follow Persson and Tabellini (1992) and assume a quadratic cost function such that the capitalist incurs a cost of $\varphi_c^k K_{c,t}^2$ for holding physical capital and of $\varphi_c^b B_{c,t}^2$ for holding government bonds, where $\varphi_c^b, \varphi_c^k > 0$ measures the size of the transaction costs.[9] The presence of this capital market imperfection and of the associated transaction costs help the model to capture a feature of realism. However, their main contribution here is that they

[7] Note that when $\rho = 1$, unskilled labour serves no purpose in production and the production function is of the standard Cobb-Douglas form in capital and skilled labour.

[8] Note that, in equilibrium, profits, Π_t^f, are driven to zero due to perfect competition.

[9] See also Benigno (2009) for intermediation or transaction costs of the form assumed here, in international financial markets.

will allow us, as we shall see below, to capture household heterogeneity in asset holdings.[10]

The capitalist uses his income for consumption, $C_{c,t}$, investment in capital, $I_{c,t}$, and investment in government bonds, $D_{c,t}$. He also receives average (per agent) transfers from the government, $\overline{G}_t \, (= G_t/N)$. Thus, his budget constraint is:

$$\begin{aligned} C_{c,t} + I_{c,t} + D_{c,t} &= \left(1-\tau^k\right)\left(r_t^k K_{c,t} + r_t^b B_{c,t}\right) + \\ &+ \left(1-\tau^k\right)\Pi_t^f + \left(1-\tau^h\right) w_t h_{c,t} + \overline{G_t} - \varphi_c^b B_{c,t}^2 - \varphi_c^k K_{c,t}^2 \end{aligned} \tag{3}$$

while the evolution of the stock of capital and government bonds, respectively, are given by:

$$K_{c,t+1} = (1-\delta) K_{c,t} + I_{c,t} \tag{4}$$

$$B_{c,t+1} = B_{c,t} + D_{c,t} \tag{5}$$

where $0 < \delta < 1$ is a depreciation rate and $K_{c,0}, B_{c,0} > 0$ are given.

2.4 Budget constraints of skilled workers

The problem of the skilled worker is similar to the capitalist's, in that he provides skilled labour to the factor market, invests the share of his income he does not consume in capital and government bonds, earns interest rate income on his financial stock and pays the same tax rates as the capitalist for these economic activities.

The skilled worker differs, however, in that he pays potentially different transaction costs, so that the capital market imperfections affect him to a greater extent.[11] In particular, we assume that firm ownership gives an insider advantage in financial transactions to the capitalist (due, for instance, to past experience, socioeconomic background, networks, etc.) and thus the size of the transaction costs is lower for the capitalist. The idea that capital market imperfections can explain heterogeneity has been extensively examined in the income inequality literature (see e.g. Galor and Zeira (1993), Benabou (1996) and Aghion and Howitt (1998)). Most of these models assume, for simplicity, that the intermediation cost is either infinite for some agents (and thus these agents are effectively excluded from the financial market) or zero. In this paper, we examine the case of non-zero, finite intermediation costs for both capitalists and skilled workers where $\varphi_c^b < \varphi_s^b$, $\varphi_c^k < \varphi_s^k$. We

[10] Transaction costs also helps to differentiate the Euler conditions for bonds and capital in the steady-state, thus allowing for a unique solution.

[11] The skilled worker also differs from the capitalist in that he does not appropriate the profits of the firm. Given that, in this model, these profits are zero, in equilibrium, this difference is of course trivial.

differentiate the skilled worker and capitalist even further by assuming that the former has lower initial holdings of capital and government bonds, i.e. $K_{s,0} < K_{c,0}$, $B_{s,0} < B_{c,0}$.[12]

Accordingly, the budget constraints and the evolution equations for capital and government bonds for the s^{th} skilled worker are:

$$\begin{aligned} C_{s,t} + I_{s,t} + D_{s,t} &= \left(1 - \tau^{k}\right)\left(r_{t}^{k} K_{s,t} + r_{t}^{b} B_{s,t}\right) + \\ &+ \left(1 - \tau^{h}\right) w_{t} h_{s,t} + \overline{G}_{t} - \varphi_{s}^{b} B_{s,t}^{2} - \varphi_{s}^{k} K_{s,t}^{2} \end{aligned} \tag{6}$$

$$I_{s,t} = K_{s,t+1} - (1-\delta) K_{s,t} \tag{7}$$

$$D_{s,t} = B_{s,t+1} - B_{s,t}. \tag{8}$$

2.5 Budget constraint of unskilled workers

Unskilled workers differ from capitalists and skilled workers in two important respects. First, they start with zero initial holdings of assets and capital market imperfections result in them being excluded from the financial markets as in the models of Benabou (1996) and Aghion and Howitt (1998). In the context of the formulation used above, this implies an infinite intermediation cost.[13] Second, we assume that exclusion from capital markets does not allow them to acquire the skills to provide skilled labour services, so that their labour effort differs, in nature, from the labour effort of the other two types of agents. Evidence from the UK, introduced later, suggests that skill acquisition, in the form of University education, is indeed related to socioeconomic income group.

Thus, the budget constraint of the u^{th} unskilled worker is:

$$C_{u,t} = (1 - \tau^{u}) w_{u,t} h_{u,t} + \overline{G}_{t} \tag{9}$$

where $0 \leq \tau^{u} < 1$ is the tax rate on unskilled labour, $h_{u,t}$ is the labour supply and $C_{u,t}$ is the consumption.

2.6 Utility function of agents

Each type of household $i = c, s, u$ maximises:

$$E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(C_{i,t}, h_{i,t}\right) \tag{10}$$

[12] For the policy experiments that we conduct, the initial conditions for each agent will effectively correspond to the steady-state of the pre-reform economy.

[13] See e.g. Aghion *et al.* (1999) for a microeconomic rationalisation of credit constraints that do not allow agents to participate in asset markets.

subject to the relevant budget constraints given above; where E_0 is the conditional expectations operator.

We use the instantaneous utility function:

$$u_{i,t} = (C_{i,t}, h_{j,t}) = \frac{\left[(C_{i,t})^{\mu}(1-h_{i,t})^{1-\mu}\right]^{1-\sigma}}{1-\sigma} \tag{11}$$

where $0 < \mu, 1-\mu < 0$ are the weights attached to each argument of the utility function and $\sigma > 1$ is the coefficient of relative risk aversion.

2.7 Government budget constraint

Following the literature on tax reforms (see e.g. Lucas (1990), Cooley and Hansen (1992), Giannitsarou (2006) and Garcia-Milà *et al.* (2010)), we do not model government spending.[14] Instead, government expenditure takes the form of transfers to the private agents, G_t. To finance these, it taxes income from labour and financial assets and issues government bonds, B_t. The budget constraint of the government is thus given by:

$$\begin{aligned} G_t + \left(1 + r_t^b\right) B_t = B_{t+1} + N^c[\tau^k\left(r_t^k K_{c,t} + r_t^b B_{c,t}\right) + \tau^h w_t h_{c,t}] + \\ + N^s[\tau^k\left(r_t^k K_{s,t} + r_t^b B_{s,t}\right) + \tau^h w_t h_{s,t}] + N^u[\tau^u w_{u,t} h_{u,t}]. \end{aligned} \tag{12}$$

2.8 Market-clearing conditions

The market clearing conditions for the capital, bond, skilled and unskilled labour and product markets respectively are:

$$N^f K_t^f = N^c K_{c,t} + N^s K_{s,t} \tag{13}$$

$$B_t = N^c B_{c,t} + N^s B_{s,t} \tag{14}$$

$$N^f h_t^f = N^c h_{c,t} + N^s h_{s,t} \tag{15}$$

$$N^f h_{u,t}^f = N^u h_{u,t} \tag{16}$$

$$\begin{aligned} N^f Y_t^f = N^c C_{c,t} + N^s C_{s,t} + N^u C_{u,t} + N^c\left[K_{c,t+1} - (1-\delta) K_{c,t}\right] + \\ + N^s\left[K_{s,t+1} - (1-\delta) K_{s,t}\right] + N^c\left(\varphi_c^b B_{c,t}^2 + \varphi_c^k K_{c,t}^2\right) + N^s\left(\varphi_s^b B_{s,t}^2 + \varphi_s^k K_{s,t}^2\right), \end{aligned} \tag{17}$$

where (17) gives the aggregate resource constraint of the economy.

[14] To address distributional effects of spending reforms, the possible benefits of public spending for different socioeconomic groups would need to be added to our model. Moreover, the potential distorting or crowding-out effects of public spending would also need to be more realistically incorporated to capture the effects of increased spending on the interest rate over and above changes to the marginal product of capital.

2.9 Decentralised competitive equilibrium

The decentralised competitive equilibrium (DCE) is defined when (i) households and firms optimize, taking prices and policy as given; (ii) all constraints are satisfied; and (iii) all markets clear. The optimality conditions for the heterogenous households include four from the capitalist and four from the skilled worker and two from the unskilled worker. In particular, each representative capitalist and skilled worker chooses $\{C_{c,t}, h_{c,t}, K_{c,t+1}, B_{c,t+1}\}_{t=0}^{\infty}$ to maximise discounted lifetime utility subject to $(3-5)$ and $\{C_{s,t}, h_{s,t}, K_{s,t+1}, B_{s,t+1}\}_{t=0}^{\infty}$ subject to $(6-8)$, respectively. Whereas the representative unskilled worker chooses $\{C_{u,t}, h_{u,t}\}_{t=0}^{\infty}$ subject to (9). Finally, each representative firm chooses $\left\{K_t^f, h_{u,t}^f, h_t^f\right\}_{t=0}^{\infty}$ to maximises profits subject to the technology constraint (1). In addition to these thirteen first-order conditions, the DCE also includes the production function (1), two of the three budget constraints from the households' problems, the government budget constraint (12), the market clearing conditions (13)-(16) and the aggregate resource constraint (17). We substitute out the Lagrangian multipliers and output, using the appropriate first-order conditions and the production function, respectively, and use the market clearing conditions (13)-(16) to substitute out $K_t^f, h_{u,t}^f, h_t^f$ and B_t. We can then summarise the DCE by a system of fourteen equations in the paths of the following variables: $(C_{c,t}, C_{s,t}, C_{u,t}, h_{c,t}, h_{s,t}, h_{u,t}, w_t, w_{u,t}, K_{c,t+1}, K_{s,t+1}, B_{c,t+1}, B_{s,t+1}, r_t^k, r_t^b)$ given the exogenously set stationary AR processes for technology and fiscal policy instruments which are discussed below.[15] We define the relevant aggregate, economy-wide quantities using a capital letter X_t, for $X_t = \{C_t, I_t, K_t, B_t, Y_t\}$.

2.10 The motion of productivity and fiscal policy instruments

2.10.1 Total factor productivity

Following the literature, we assume that the stochastic process determining A_t is an exponential first-order Markov process:

$$A_t = A_0^{(1-\rho^a)} A_{t-1}^{\rho^a} e^{\varepsilon_t} \tag{18}$$

where $A_0 > 0$ is a constant, $0 < \rho^a < 1$ is the autoregressive parameter and $\varepsilon_t \sim iid(0, \sigma^2)$ are random shocks to productivity.

[15] To save space we have not reported the 14-equation DCE system here but it will be provided on request.

2.10.2 Policy instruments

Given that we wish to analyse the welfare implications of permanent tax regime changes, all tax rates are treated as exogenous constants, $0 \leq \tau^k, \tau^h, \tau^u < 1$. In particular, in the policy reforms that we will examine, the economy will start from the steady-state and will be subjected to an exogenous, permanent change in one or more tax instruments, holding the other policy instruments, including G, constant at the pre-reform steady-state values. We examine economic outcomes and welfare in the new steady and during the transition period to the new steady-state. While we do not analyse the welfare implications of changes in government spending, the calibrated value of $\frac{G}{Y}$ is important in helping to obtain the current steady-state debt-to-output ratio for the UK.

3 Calibration and steady-state

In Table 1 below, we next calibrate the structural parameters of the model so that its steady-state solution reported in Table 2 reflects the main empirical characteristics of the UK economy. The calibration also provides empirical justification for the key modelling decisions made above.

3.1 Distinguishing agent types

3.1.1 Population shares

We first wish to map out agent heterogeneity and thus distinguish the three types of households by their differing shares in the population, n^i. According to the Family Resources Survey in 2008-2009, 28% of households do not have any savings, 53% have savings up to £20,000 and 19% have savings above £20,000.[16] In light of this, since we assume that unskilled workers do not have savings, we set n^u equal to 30%. At the other end of the distribution, since we model capitalists as the income group with the highest share of savings and assets, we set n^c to 20% implying that n^s is 50%. Other data providing an additional dimension by which unskilled workers differ from skilled workers and capitalists is that the former group offers a labour input that is lacking in skills. According to the Labour Force Survey, Office for National Statistics[17], in 2003, 28% of the working population was employed in semi-routine and routine occupations, whereas the remaining share worked

[16] The Survey is sponsored by the Department for Work and Pensions (see Table 4.9 for the information reported here).

[17] See `http://www.statistics.gov.uk/STATBASE/Expodata/Spreadsheets/D7665.xls`.

in supervisory, technical, professional and managerial occupations, which require an increasingly higher skilled labour input. Moreover, according to data from the Department for Education and Skills on the participation rates in higher education for different income groups, the participation ratio was about three times higher in the 1990s for the three highest, relative to the three lowest groups.[18] Thus, there appears to be adequate support for associating skill with income group.

Table 1: Parameter Values

parameter	value	definition
$0 < \alpha < 1$	0.590	productivity of composite input
$0 < \rho < 1$	0.400	capital weight in composite input share
$0 \leq \delta \leq 1$	0.100	depreciation rate on private capital
$\varphi_c^k, \varphi_c^b > 0$	0.001	transaction costs, capitalists
$\varphi_s^k, \varphi_s^b > 0$	0.005	transaction costs, skilled workers
$0 < \beta < 1$	0.976	rate of time preference
$0 < \mu < 1$	0.347	consumption weight in utility
$\sigma > 1$	2.000	coefficient of relative risk aversion
$0 < n^c < 1$	0.200	population share of capitalists
$0 < n^s < 1$	0.500	population share of skilled workers
$0 < G/Y < 1$	0.330	public spending share of output
$0 < \tau^h < 1$	0.300	labour tax rate skilled
$0 < \tau^k < 1$	0.442	capital tax rate
$0 < \tau^u < 1$	0.200	labour tax rate unskilled
$0 < \rho^a < 1$	0.920	AR(1) parameter productivity
$\sigma_a > 0$	0.030	std. dev. of productivity innovations

3.1.2 Asset holdings

We next turn to heterogeneity in asset holdings and returns to labour which governs the choice of the relevant production parameters. In particular, following e.g. Stokey (1996), we set the productivity of the composite input in the production function, α to 0.59 and ρ, the weight of capital in the composite input share to 0.4. This implies that, at the steady-state, the capital to skilled labour elasticity of substitution is 0.67, the elasticity of capital with respect to unskilled labour is 0.88 and the elasticity of skilled to unskilled labour is 1.30. These elasticities are well within the range of estimated elasticities of substitution in the literature (see e.g. Krusell *et al.* (2000) for a review of such studies). In turn, these parameters and the implied elasticities lead to a steady-state skill premium, defined as log

[18] See `www.statistics.gov.uk/STATBASE/Expodata/Spreadsheets/D7308.xls`.

differences between the wage rates for skilled and unskilled labour, of 24%. Also note that the ratio of the wage rates for skilled and unskilled labour is 1.28. These values are again broadly consistent with estimates for both the UK and the USA. For the UK, Walker and Zhu (2008) estimate a college premium (in log differences) of about 18% for males and 28% for females. Machin (1996) computes the ratio of wages between non-manual and manual jobs in manufacturing that ranges between 1.3 and 1.5, from 1970 to 1990. For the USA, Krusell *et al.* (2000) report a college premium, in terms of wage ratios, of about 1.18 in 1990.

3.1.3 Savings

Heterogeneity in savings is controlled for, as explained in the previous section, by the parameters that govern transaction costs in the financial markets. In particular, following the models in e.g. Galor and Zeira (1993), Benabou (1996) and Aghion and Howitt (1998), we set these costs to infinity for the unskilled workers, which implies that these agents do not have any savings. As said above, about 28% of the UK households do not save. Regarding the households with positive savings, data from the Family Resources Survey of 2008-2009 suggest that households in the highest saving bracket have five times higher savings than the other savers, on average. In terms of our model, this difference is applied to the representative capitalist and skilled worker by setting the transaction costs for the latter to be five times greater than the former. For simplicity, we set this cost in capital asset markets to be the same in the bond market. We chose the level of the transaction costs parameter, so that in combination with a usual annual depreciation rate, δ, of 10%, the total ratio of capital to GDP in the steady-state is about 2, which again coheres with UK data.

3.1.4 Effective tax rates

Effective average tax rates for capital and labour income are constructed by following the approach in Conesa *et al.* (2007).[19] We use data from the National Accounts and the Public Sector, Taxation and Market Regulation databases (available from OECD.Stat), to obtain the same series as in Conesa *et al.* (2007) for 1970-2005.[20] The average capital tax rate over the time period is $\tau^k = 0.442$, while the average labour income rate is 0.27. Using

[19] See their Appendix B for more details.

[20] As an alternative, we have also used the ECFIN effective average tax rates (see Martinez-Mongay (2000)). Both approaches give similar long-run averages of labour tax rates, while the ECFIN capital tax rate is higher.

data from Social Trends 38, Office for National Statistics, we are able to approximate the progressivity of the UK income tax system at about 1.6.[21] A ratio of $\tau^h/\tau^u = 1.6$, together with the requirement that the weighted average of the two tax rates equal the effective labour income tax rate, would imply that $\tau^h = 0.304$ and $\tau^u = 0.19$. However, the progressivity of income taxation probably overestimates the progressivity of labour income taxation, which is our interest here. This is because, in light of the data discussed, we would expect the higher income brackets to have more capital income compared to lower income brackets. On the other hand, the lower the progressivity ratio, the higher the implied value of τ^u. We thus use a progressivity ratio of $\tau^h/\tau^u = 1.5$ for the calibration, which guarantees that τ^u is equal to the base income tax rate. Accordingly, we approximate the lower tax rate, τ^u, at 20%, and the higher labour income tax rate, τ^h at 30%.

3.2 Parameters common to all agents

We next approximate the rate of time preference, β, so that $1/\beta$ is equal to 1 plus the *ex-post* real interest rate, where we use real interest rate data from OECD Main Economic Indicators, from 1970-2005. This gives a value 0.976 for β. Following Kydland (1995), we set μ, the weight given to consumption relative to leisure in the utility function, equal to the average value of work versus leisure time, which is obtained using data on hours worked from the OECD Economic Outlook database, from 1970-2005.[22] We also use a common value from the literature for the intertemporal elasticity of consumption, $1/\sigma = 0.5$ or $\sigma = 2$.

Given that we will evaluate policies that reduce the debt-to-GDP ratio below, we calibrate the share of government spending in GDP, G/Y, to obtain a B/Y ratio of 70% based on official forecasts for 2011-2013 (see e.g. the Pre-Budget Forecast, June 2010, Office for Budget Responsibility)[23]. Finally, the $AR(1)$ relation for the productivity process in (18) is estimated using TFP data from the Office for National Statistics, 1970-2005. The estimated values for ρ^a is 0.92 and is significant at less than the 1% level of significance. The standard deviation, σ_a, is 0.03.

[21] This is obtained by calculating the average income tax rate that applies approximately to the lower 30% and the upper 70% of the tax payers. We then add the national insurance contribution rate of 11% and calculate the ratio of these two effective average tax rates.

[22] To obtain this we divide total hours worked by total hours available for work or leisure, following Ho and Jorgenson (2001). They assume that there are 14 hours available for work or leisure per day with the remaining 10 hours accounted for by physiological needs.

[23] See `http://budgetresponsibility.independent.gov.uk.`

3.3 Steady-state

The steady-state solution of the model in this, pre-reform, high debt equilibrium, is given in Table 2 below in terms of the aggregate variables. In particular, we see that the capitalists consume in total 17.6% of total income (or about 22% of total consumption[24]), skilled workers consume in total 40.9% of total income (or around 51% of total consumption) and unskilled workers consume in total 21.9% of total income (or approximately 27% of total consumption). In addition, the capitalists in total have around 67% of total savings and own about 67% of the capital and government bonds in the economy. As said above, the ratio of savings, I_c/I_s, and assets, K_c/K_s and B_c/B_s, of the representative capitalist to the representative skilled worker, are equal to five. Note also that the net (after depreciation, tax and transaction costs) interest rates on capital and bonds, are given respectively by:

$$\widetilde{r}_k = r^k(1-\tau^k) - \delta - 2\varphi_c^k\left(\frac{n^c}{n^c+n^s}\right)K_c - 2\varphi_s^k\left(\frac{n^s}{n^c+n^s}\right)K_s \tag{19}$$

$$\widetilde{r}_b = r^b(1-\tau^k) - 2\varphi_c^b\left(\frac{n^c}{n^c+n^s}\right)B_c - 2\varphi_s^b\left(\frac{n^s}{n^c+n^s}\right)B_s \tag{20}$$

and are equal in the steady-state.

It is also worth noting that the labour supply elasticities of this model are more in line with microeconometric studies than in the standard aggregate RBC models. In particular, the Frisch (or λ-constant) labour supply elasticity (see e.g. Browning *et. al.*, 1999) is 3.41 for capitalists, 0.43 for skilled and 2.31 for unskilled workers. Microeconometric studies (see e.g. Browning *et. al.* (1999) for a review) generally suggest that the Frisch labour supply elasticity is less than one and the standard RBC model implies an elasticity around four (see e.g. King and Rebelo, (1999)). The elasticities reported here suggest that capitalists (skilled workers) are less (more) dependent on labour income respectively.

[24]This is calculated as $\frac{(N^c * C_c)/Y}{C/Y} = (N^c * C_c)/C$. The same formula is used below for similar quantities.

Table 2: Steady-state (pre reform)

variable	value	variable	value
$\frac{N^c * C_c}{Y}$	0.176	$\frac{N^c * K_c}{Y}$	1.292
$\frac{N^s * C_s}{Y}$	0.409	$\frac{N^s * K_s}{Y}$	0.646
$\frac{N^u * C_u}{Y}$	0.219	$\frac{K}{Y}$	1.938
$\frac{C}{Y}$	0.804	$\frac{N^c * B_c}{Y}$	0.467
$\frac{N^c * I_c}{Y}$	0.129	$\frac{N^s * B_s}{Y}$	0.233
$\frac{N^s * I_s}{Y}$	0.065	$\frac{B}{Y}$	0.700
$\frac{I}{Y}$	0.194	$\widetilde{r}_k$	0.025
w	0.434	$\widetilde{r}_b$	0.025
w_u	0.340	U_c	-93.925
h_c	0.165	U_s	-101.051
h_s	0.224	U_u	-105.267
h_u	0.226	U_a	-100.891

The skill premium, measured as the log differences in the wage rates is about 24% as noted earlier. In the steady-state, capitalists work considerably less than skilled and unskilled workers, who work more or less the same time (see the h_i figures in Table 2). Also note that in the steady-state $C_c = 0.1349$, $C_s = 0.1253$ and $C_u = 0.1120$. Thus in terms of welfare, U_i, higher consumption and lower work effort make the capitalists better off, followed by the skilled and unskilled workers respectively. The weighted average measure of aggregate or Benthamite lifetime utility, U_a, is also reported in Table 2.[25]

4 Model Solution

To solve the model, we start by taking the first-order Taylor series expansion of the DCE and exogenous process for productivity around their respective steady-states. For any variable X_t, these values are denoted $\widehat{X}_t = \log X_t - \log X$. We next re-express the model in matrix form as second-order difference equation system:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{M}_1 \mathbf{E}_t \mathbf{x}_{t+1} + \mathbf{M}_2 \mathbf{x}_{t-1} + \mathbf{M}_3 \mathbf{z}_t \\ \mathbf{y}_t &= \mathbf{N}_1 \mathbf{x}_t + \mathbf{N}_2 \mathbf{x}_{t-1} + \mathbf{N}_3 \mathbf{z}_t + \mathbf{N}_4 \mathbf{E}_t \mathbf{x}_{t+1} \\ \mathbf{z}_t &= \boldsymbol{\rho} \mathbf{z}_{t-1} + \mathbf{u}_t. \end{aligned} \tag{21}$$

[25] The lifetime utility of agent i in the steady-state is given by $U_i = \frac{(1-\beta^T)}{1-\beta}\overline{u}_i$, for $i = c, s, u$, where $\overline{u}_i$ is the welfare of i calculated at the steady state using (11) and $T = 1000$.

where $\mathbf{x}_t = \left[\hat{B}_{c,t+1}, \hat{K}_{c,t+1}, \hat{B}_{s,t+1}, \hat{K}_{s,t+1}\right]'$ contains the endogenous state variables; $\mathbf{y}_t = \left[\hat{C}_{c,t}, \hat{C}_{s,t}, \hat{C}_{u,t}, \hat{h}_{c,t}, \hat{h}_{s,t}, \hat{h}_{u,t}, \hat{r}_t^b, \hat{r}_t^k, \hat{w}_t, \hat{w}_{u,t}\right]'$ the endogenous control variables; and $\mathbf{z}_t = [\hat{a}_{t+1}]$ the exogenous state variables.[26] The various $\mathbf{M}$ and $\mathbf{N}$ matrices contain convolutions of the structural parameters calibrated in Table 1. Finally, since we only have one exogenous state variable, $\boldsymbol{\rho} = \rho^a$ and $\mathbf{u}_t = \varepsilon_{t+1}$.

In what follows we use (21) to briefly describe how we obtain both the rational expectations (RE) and adaptive learning (AL) solutions of the log-linearised model. Since these methods are well known, our purpose is merely to set out notation and variable definitions which will be used in the results and analysis below.

4.1 Rational expectations

Employing the undetermined coefficients method, agents first guess that the equilibrium laws of motion for the state variables under RE have the following linear form:

$$\mathbf{x}_t = \boldsymbol{\gamma}_{\mathbf{x}}\mathbf{x}_{t-1} + \boldsymbol{\gamma}_{\mathbf{z}}\mathbf{z}_t \tag{22}$$

where $\boldsymbol{\gamma}_{\mathbf{x}}$ and $\boldsymbol{\gamma}_{\mathbf{z}}$ are coefficient matrices. Substituting for $\mathbf{z}_t$ using the last equation in (21) gives:

$$\mathbf{x}_t = \boldsymbol{\phi}_{\mathbf{x}}\mathbf{x}_{t-1} + \boldsymbol{\phi}_{\mathbf{z}}\mathbf{z}_{t-1} + \boldsymbol{\phi}_{\mathbf{z}}\boldsymbol{\rho}^{-1}\mathbf{u}_t \tag{23}$$

where $\boldsymbol{\phi}_{\mathbf{x}} = \boldsymbol{\gamma}_{\mathbf{x}}$ and $\boldsymbol{\phi}_{\mathbf{z}} = \boldsymbol{\gamma}_{\mathbf{z}}\boldsymbol{\rho}$. Leading (23) by one-period and taking expectations of both sides yields:

$$\mathbf{E}_t\mathbf{x}_{t+1} = \boldsymbol{\phi}_{\mathbf{x}}\mathbf{x}_t + \boldsymbol{\phi}_{\mathbf{z}}\boldsymbol{\rho}\mathbf{z}_t \tag{24}$$

since $\boldsymbol{\phi}_{\mathbf{z}}\boldsymbol{\rho}^{-1}\mathbf{E}_t\left[\mathbf{u}_{t+1}\right] = 0$. Substituting (24) and (23) into the first equation of (21) gives:

$$\begin{aligned} \mathbf{x}_t \;=\; & \left[(\mathbf{I} - \mathbf{M}_1\boldsymbol{\phi}_{\mathbf{x}})^{-1}\mathbf{M}_2\right]\mathbf{x}_{t-1} + \\ & + \left[(\mathbf{I} - \mathbf{M}_1\boldsymbol{\phi}_{\mathbf{x}})^{-1}(\mathbf{M}_1\boldsymbol{\phi}_{\mathbf{z}} + \mathbf{M}_3)\right]\left(\boldsymbol{\rho}\mathbf{z}_{t-1} + \mathbf{u}_t\right). \end{aligned} \tag{25}$$

Comparing (25) with (23) implies that the unique RE solution of the reduced-form model is given by the two parameter matrices, hereafter denoted by $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$, that satisfy the following two equations:

$$\begin{aligned} \boldsymbol{\phi}_{\mathbf{x}} &= (\mathbf{I} - \mathbf{M}_1\boldsymbol{\phi}_{\mathbf{x}})^{-1}\mathbf{M}_2 \\ \boldsymbol{\phi}_{\mathbf{z}} &= (\mathbf{I} - \mathbf{M}_1\boldsymbol{\phi}_{\mathbf{x}})^{-1}(\mathbf{M}_1\boldsymbol{\phi}_{\mathbf{z}} + \mathbf{M}_3)\,\boldsymbol{\rho}. \end{aligned} \tag{26}$$

[26] For examples of others papers in the literature using this particular reduced form, see Evans and Hokhaponja (2001), Kasa (2004), Giannitsarou (2006) and Carceles Poveda and Giannitsarou (2008).

Assuming $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$ exist, the solution for the model's state variables under RE is:[27]

$$\mathbf{x}_t = \bar{\boldsymbol{\phi}}_{\mathbf{x}}\mathbf{x}_{t-1} + \boldsymbol{\rho}^{-1}\bar{\boldsymbol{\phi}}_{\mathbf{z}}\mathbf{z}_t. \tag{27}$$

Substituting (27) and the expected value of its lead into the second equation of (21) gives the RE solution for the model's control variables:

$$\begin{aligned}\mathbf{y}_t \;=\; & \left[\mathbf{N}_1\bar{\boldsymbol{\phi}}_{\mathbf{x}} + \mathbf{N}_2 + \mathbf{N}_4\bar{\boldsymbol{\phi}}_{\mathbf{x}}^2\right]\mathbf{x}_{t-1} + \\ & + \left[\mathbf{N}_1\boldsymbol{\rho}^{-1}\bar{\boldsymbol{\phi}}_{\mathbf{z}} + \mathbf{N}_3 + \mathbf{N}_4\left(\bar{\boldsymbol{\phi}}_{\mathbf{z}} + \bar{\boldsymbol{\phi}}_{\mathbf{x}}\boldsymbol{\rho}^{-1}\bar{\boldsymbol{\phi}}_{\mathbf{z}}\right)\right]\mathbf{z}_t. \end{aligned} \tag{28}$$

4.2 Adaptive learning

Under the AL hypothesis, it is also assumed that private agents can correctly guess the form of the equilibrium policy functions of the state variables given by (22). However, in contrast to the RE solution, it is assumed that they do not know the time-invariant parameter values given by $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$, which ultimately govern the dynamics of the economy.[28] Therefore, they must rely on past data and a recursive learning algorithm to estimate these parameters to produce forecasts of the endogenous state variables for the next period. As new data become available in each period, they revise their parameter estimates so that their forecasting errors are corrected gradually.

More formally, agents' expectations are assumed to follow a so-called perceived law of motion (PLM) of the form:

$$\mathbf{E}_t^*\mathbf{x}_{t+1} = \tilde{\boldsymbol{\phi}}_{\mathbf{x},t-1}\mathbf{x}_t + \tilde{\boldsymbol{\phi}}_{\mathbf{z},t-1}\mathbf{z}_t \tag{29}$$

where parameters $\tilde{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\tilde{\boldsymbol{\phi}}_{\mathbf{z}}$ are the estimates of $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$ coming from a recursive least-squares regression and $\mathbf{E}^*$ denotes that expectations do not follow the RE hypothesis.[29]

Following a similar procedure as under RE, we substitute (29) into the first equation of (21) to obtain:

$$\mathbf{x}_t = \mathbf{P}_1\mathbf{x}_{t-1} + \boldsymbol{\rho}^{-1}\mathbf{P}_2\mathbf{z}_t \tag{30}$$

[27] The two solution matrices $\bar{\boldsymbol{\phi}}_x$ and $\bar{\boldsymbol{\phi}}_z$, were obtained applying the method proposed by Klein (2000).

[28] Adaptive learning has its foundations in the work of Marcet and Sargent (1989) and Evans and Honkapohja (2001).

[29] Note, we follow the common assumption (see, e.g. Evans and Honkaphoja (2001) and Carceles-Poveda and Giannitsarou (2007)) that at period t agents form expectations for $\mathbf{x}_{t+1}$ using their estimates from the previous period, $\tilde{\boldsymbol{\phi}}_{\mathbf{x},t-1}$ and $\tilde{\boldsymbol{\phi}}_{\mathbf{z},t-1}$, which allows us to avoid a problem of simultaneity in the learning process.

where

$$\begin{aligned}\mathbf{P}_1 &= (\mathbf{I}-\mathbf{M}_1\tilde{\boldsymbol{\phi}}_{\mathbf{x},t-1})^{-1}\mathbf{M}_2\\ \mathbf{P}_2 &= (\mathbf{I}-\mathbf{M}_1\tilde{\boldsymbol{\phi}}_{\mathbf{x},t-1})^{-1}\left(\mathbf{M}_1\tilde{\boldsymbol{\phi}}_{\mathbf{z},t-1}+\mathbf{M}_3\right)\boldsymbol{\rho}.\end{aligned} \tag{31}$$

Equation (30) is referred to as the actual law of motion (ALM) since every new observed value of $\mathbf{x}_t$ depends on the deep parameters of the model economy but also on the agents' forecasts given by the PLM (29).

The actual laws of motion for the control variables under learning are found by substituting (30) for $\mathbf{x}_t$ and (29) for $\mathbf{E}_t\mathbf{x}_{t+1}$ in the second equation of (21) giving:

$$\begin{aligned}\mathbf{y}_t &= \left[\mathbf{N}_1\mathbf{P}_1+\mathbf{N}_2+\mathbf{N}_4\tilde{\boldsymbol{\phi}}_{\mathbf{x},t-1}\mathbf{P}_1\right]\mathbf{x}_{t-1}+\\ &\quad+\left[\mathbf{N}_1\boldsymbol{\rho}^{-1}\mathbf{P}_2+\mathbf{N}_3+\mathbf{N}_4\left(\tilde{\boldsymbol{\phi}}_{\mathbf{x},t-1}\boldsymbol{\rho}^{-1}\mathbf{P}_2+\tilde{\boldsymbol{\phi}}_{\mathbf{z},t-1}\right)\right]\mathbf{z}_t.\end{aligned} \tag{32}$$

4.2.1 Recursive least-squares learning

We next focus on how the estimates $\tilde{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\tilde{\boldsymbol{\phi}}_{\mathbf{z}}$ in (29) are obtained. We first define a vector $\mathbf{w}_t$ containing the observed values of all the state variables (including the exogenous process) and another vector $\tilde{\boldsymbol{\phi}}_t = [\tilde{\boldsymbol{\phi}}_{\mathbf{x},t}, \tilde{\boldsymbol{\phi}}_{\mathbf{z},t}]'$ containing the parameter estimates obtained in each period.[30] In what follows, we will assume that all agents who form expectations, namely capitalists and skilled workers, follow a recursive least-squares (RLS) learning algorithm. According to this algorithm (see e.g. Evans and Honkapohja (2001) and Carceles-Poveda and Giannitsarou (2007)), in each period agents estimate the parameter matrices $\tilde{\boldsymbol{\phi}}_t = [\tilde{\boldsymbol{\phi}}_{\mathbf{x},t}, \tilde{\boldsymbol{\phi}}_{\mathbf{z},t}]'$ as they try to find their corresponding true values $\bar{\boldsymbol{\phi}} = [\bar{\boldsymbol{\phi}}_{\mathbf{x}}, \bar{\boldsymbol{\phi}}_{\mathbf{z}}]'$ which come from the RE solution. For this purpose they make use of all the available data up to that period in a least-squares regression:

$$\mathbf{x_t} = \tilde{\boldsymbol{\phi}}'\mathbf{w}_{t-1} + \widetilde{\xi}_t \tag{33}$$

to get a new estimate $\tilde{\phi}_t$, where $\widetilde{\xi}_t$ is the forecast error. According to the least-squares method, $\tilde{\phi}_t$ will be the coefficient vector which minimizes $\sum_{t=1}^{T}\widetilde{\xi}_t^2$ and is given by:

$$\tilde{\boldsymbol{\phi}}_t = \left(\sum_{i=1}^{T}\mathbf{w}_{i-1}\mathbf{w}_{i-1}'\right)^{-1}\sum_{i=1}^{T}\mathbf{w}_{i-1}\mathbf{x}_i' \tag{34}$$

[30]Note that under our model setup we have $\mathbf{w}_t = [\hat{B}_{c,t+1}, \hat{K}_{c,t+1}, \hat{B}_{s,t+1}, \hat{K}_{s,t+1}, \hat{a}_{t+1}]'$.

The estimator above can be written in a recursive fashion for $t = 1, 2, 3...$, as follows:

$$\begin{aligned} \tilde{\boldsymbol{\phi}}_t &= \tilde{\boldsymbol{\phi}}_{t-1} + g_t \mathbf{R}_t^{-1} \mathbf{w}_{t-1} (\mathbf{x_t} - \tilde{\boldsymbol{\phi}}'_{\mathbf{t-1}} \mathbf{w}_{t-1})' \\ \mathbf{R}_t &= \mathbf{R}_{t-1} + g_t \left(\mathbf{w}_{t-1} \mathbf{w}'_{t-1} - \mathbf{R}_{t-1} \right) \end{aligned} \tag{35}$$

where $\mathbf{R}_t$ is a matrix with the second moments of the regressors included in $\mathbf{w}_t$; $(\mathbf{x_t} - \tilde{\boldsymbol{\phi}}'_{\mathbf{t-1}} \mathbf{w}_{t-1})$ is the latest forecast error that will be used to correct the current estimates; and $g_t = 1/t$ is the gain sequence implying that as t increases, every new forecast error will have a lower relative importance.[31]

4.2.2 Stability and convergence

An important issue is whether the learning algorithm will converge to the RE solution. We first consider the so-called expectational stability or E-stability of the model under AL (see the Appendix for further details). Intuitively, the RE solution $\bar{\boldsymbol{\phi}} = [\bar{\boldsymbol{\phi}}_{\mathbf{x}}, \bar{\boldsymbol{\phi}}_{\mathbf{z}}]'$ will be E-stable under learning if small deviations from it return to $\bar{\boldsymbol{\phi}} = [\bar{\boldsymbol{\phi}}_{\mathbf{x}}, \bar{\boldsymbol{\phi}}_{\mathbf{z}}]'$ under the chosen learning rule. For our model, the E-stability condition is met for the base calibration and all policy experiments considered below.

A second condition for convergence is the stationarity of the RE solution. This requires that the eigenvalues of $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ have real parts less than one, ensuring that the part of the RE solution associated with the lags of the state variables do not have an explosive path. The stationarity condition is also met for the base calibration and all policy experiments considered below.

Evans and Honkapohja (2001) show that if the E-stability and stationarity conditions are satisfied, the RLS algorithm converges locally to $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$ and thus the model at hand is learnable. Honkapohja and Mitra (2006) show that the above conditions also guarantee local convergence in models with structural heterogeneity and heterogeneous initial beliefs. However, these two conditions cannot ensure global convergence of the learning algorithm to the RE solution. As we shall see below, for two of our experiments, where we initialise learning from a point that is far from the RE solution, the learning algorithm breaks down as it implies that the adaptive learners make such big mistakes that a well defined solution to the problem cannot be obtained. In other words, the RE solution is not learnable in these cases.

[31] We make use of Matlab functions made available by Carceles-Poveda and Giannitsarou (2007) to solve the model using AL.

5 Policy analysis

5.1 Laffer curves in tax revenue and debt

Our aim is to evaluate the effects of tax reforms that are consistent with a particular long-run debt-to-output target, both at the aggregate level but also for each heterogeneous group of agents in the population. To this end, we first calculate the change in a tax rate (or a combination of tax rates) required to bring the debt-to-GDP ratio to a desired level at the steady-state, leaving all the other tax rates and parameters in the model constant. In particular, we start from the current (or pre-reform) steady-state in the economy and calculate the new (or post-reform) steady-state, where the debt-to-GDP ratio is given at the 60% target and a tax rate (or a combination of tax rates) is chosen to satisfy the government budget constraint.

The relationship between the tax revenue from a particular tax base and the associated tax rate is, in general, given by a Laffer curve (see e.g. Schmitt-Grohé and Uribe (1997) for a discussion of Laffer curves in tax revenue in a DGE context). Thus, in our model, an increase (decrease) in a tax rate can lead to either increases (decreases) or decreases (increases) in the tax revenue collected from this tax base, depending on whether the economy is on the upward or downward slopping part of the curve, respectively. In a heterogenous agent context, the complementarity/substitutability of the factors in the production function implies that a tax rate change will have spillover effects to the tax revenue collected from the other tax bases. For instance, an increase in the capital tax rate will decrease the capital supply, but will tend to increase the supply of unskilled labour, which can substitute for capital in production. Thus, the tax revenue collected from the tax base of unskilled labour income is likely to increase after an increase in the capital tax. Hence, these spillover effects, due to the form of technology employed, imply that the relationship between total tax revenue and the tax rates can be different from the relationship between tax revenue from a particular tax base and the tax rate. In particular, the economy can be on one side of the Laffer curve of tax revenue from a particular tax base but on either side the total revenue Laffer curve.

A lower level of debt in the steady-state implies that there will also be a reduction in interest payments on debt and thus in total government spending, assuming, as we do here, that the remaining components of government spending do not change. Hence, tax reforms consistent with reducing steady-state debt will need to generate a lower level of total tax revenue.[32] As long

[32] This is different from the analysis in e.g. Lucas (1990), Angelopoulos *et al.* (2008) and Garcia-Mila *et al.* (2010), where the tax reforms considered, in the form of decreasing

as a Laffer curve exists in tax revenue with respect to the tax rates, these tax changes can take the form of either decreases or increases in the tax rates since this relationship suggests that a particular tax revenue target can be achieved by two tax rates. We would then expect that there should be a Laffer curve characterising the relationship between debt and debt-to-GDP with respect to the tax rates. We plot and discuss these relationships below.[33]

The above discussion implies that, consistent with the analysis in Schmitt-Grohé and Uribe (1997), for a given level of debt, when a tax rate is the variable that is chosen to satisfy the government budget constraint, it can have two long-run solutions.[34] A critical condition for this is that there is a Laffer curve with respect to total tax revenue. As will be discussed below, the role of the complementarity/substitutability in the capital and labour supply policies of the heterogeneous agents is important in obtaining this Laffer curve and thus in making both long-run solutions empirically plausible. In addition, this complementarity/substitutability implies that the optimal reactions to tax reforms will have distributional effects that might differ between the two long-run solutions, across the different policy reforms. To rank order these two equilibria at both the aggregate level and for each individual type of agent, across different tax reforms we undertake a welfare analysis.

5.2 Alternative tax regimes

Our analysis concentrates on examining tax policies to reduce the debt-to-GDP ratio to the EU target of 60%.[35] Therefore, starting from the current

the capital tax, leave the total tax revenue unchanged. This is achieved in these models by an increase in the labour tax. Giannitsarou (2006), on the other hand, considers the case where the capital tax rate is decreased, while the labour tax rate is kept fixed and government spending is endogenously chosen to close the model in the post-reform economy.

[33] Note that in a general equilibrium framework, for a given level of spending other than interest payments, a higher level of debt can only be sustained, in the steady-state, by higher tax revenues. This is because the interest payments on the debt, which are part of government expenditure, increase with the level of debt. Thus, there should be a positive relationship in the long-run between debt and tax revenue, for a given level of government spending other than interest payments.

[34] Note that Schmitt-Grohé, and Uribe (1997), also discuss the parameter range under which some of these *equilibria* can be indeterminate. For our model and the calibrated parameters for the UK, all solutions obtained below are saddle-path stable.

[35] Note that our results are qualitatively robust to tax reforms to achieve lower debt targets. Quantitatively, the welfare effects are naturally bigger under such targets because the required changes in the tax rates are also bigger. Moreover, the costs associated with AL are also larger in this case.

state of the economy summarised in Table 2, in the experiments that follow, the government increases or decreases the various tax rates at its disposal with the aim of reducing the steady-state debt-to-GDP ratio. Recall that in the baseline pre-reform economy, $\tau^u = 0.2$, $\tau^h = 0.3$, and $\tau^k = 0.442$. Given that we seek to evaluate the distributional effects of tax reforms to meet a particular debt target and not the optimal size of the government or government debt, we take this debt target as given. Hence, we do not evaluate the potential welfare benefits from reducing the debt-to-GDP ratio, in the form of, for instance, lowering the cost of borrowing for the government and reassuring financial markets that there is no risk of default.[36]

We examine four different tax reforms to meet the debt target. First, the scenario where the government, *ceteris paribus*, increases/decreases the tax rate on skilled labour, τ^h, implying that the progressivity of labour income taxation has risen/fallen. We then examine the case where the government increases/decreases the effective average labour tax rate, i.e. it increases/decreases τ^h and τ^u proportionately, so that the progressivity in the labour income taxation remains unchanged. We do not examine increases in τ^u only, as this would require a reversal of the tax system from progressive to regressive, which we consider not to be socio-politically feasible. Next, we evaluate the distributional effects of increases/decreases in the capital income tax rate, τ^k, holding all other rates constant and finally, the effects of increasing/decreases all tax rates proportionately.

For each tax regime considered, we first find the steady-state tax rate(s) required to reduce the debt-to-GDP ratio to 60%, by working as described in the previous sub-section. We also discuss the Laffer curves in tax revenue and debt associated with each tax change in the long-run. We then assume that the economy is in the current, pre-reform steady-state when the government implements the tax reform as a permanent change in a tax rate (or a combination of tax rates) and simulate the response of the model economy, as described in Section 2, to this change until it reaches the new steady-state. Hence, the only change in policy in the post-reform economy is the higher/lower tax rate(s).

5.2.1 Welfare comparisons

In each case, we present the conditional welfare costs or benefits for each agent and the aggregate economy for two different assumptions regarding expectations. First, we evaluate the lifetime welfare of all agents over time,

[36] It would, of course, be very interesting to examine whether the benefits from reducing the debt-to-GDP ratio in this fashion can be distributed differentially in the population, but this is outside the scope of our analysis.

as they converge to the post-reform steady-state starting from the current economy, assuming rational expectations. Then, we evaluate welfare over this transition path, assuming that agents have fully learned the pre-reform rational expectations solution and form expectations in the post-reform economy under bounded rationality. In particular, we assume that the agents learn the coefficients of their reduced form policy functions in the post-reform economy by using the adaptive learning algorithm described above.

5.2.2 Initial conditions for learning

For the second case above involving bounded rationality, we examine two scenarios for the initial conditions. In the first, which serves to contextualise our results relative to the literature, we follow e.g. Giannitsarou (2006) and Evans *et. al.* (2009) and assume that the agents start learning using the reduced form coefficients that correspond to the pre-reform economy. This would represent the case where the tax change is not anticipated. In the second, we assume that there is heterogeneity in the initial conditions used for learning.

In particular, we assume that the skilled workers "guess" that the coefficients remain the same and thus use the coefficients that correspond to the pre-reform economy in their policy functions for the initial period. In contrast, we assume that the capitalists are able to predict the post-reform RE steady-state and their optimal reduced form coefficients for their policy functions in this equilibrium, so that their "guess" for their initial coefficients correspond to the post-reform RE solution. Therefore, in the initial period the capitalists make their choices as if they were in the post-reform RE economy, while the skilled workers make their choices as if they were in the pre-reform RE economy. This heterogeneity in beliefs implies that the initial guesses for both agents are incorrect, as the actual economy, as determined by the interaction of their choices, is neither in the pre- nor in the post-reform RE equilibrium. Given the gap between the expected and actual outcomes, both agents use thereafter recursive least-squares to learn the coefficients.

More formally, to represent the importance of initial beliefs for the solution of the model, define $\bar{\boldsymbol{\phi}}_{pre} = \left[\bar{\boldsymbol{\phi}}_{\mathbf{x},pre}, \bar{\boldsymbol{\phi}}_{\mathbf{z},pre}\right]'$ and $\bar{\boldsymbol{\phi}}_{post} = \left[\bar{\boldsymbol{\phi}}_{\mathbf{x},post}, \bar{\boldsymbol{\phi}}_{\mathbf{z},post}\right]'$ as the RE solution matrices for the pre-reform and post-reform economies, respectively, and $\tilde{\boldsymbol{\phi}}_0 = \left[\tilde{\boldsymbol{\phi}}_{\mathbf{x},0}, \tilde{\boldsymbol{\phi}}_{\mathbf{z},0}\right]'$ as the matrix containing the starting values of the learning algorithm. To obtain the rational expectations solution, we assume that:

$$\tilde{\boldsymbol{\phi}}_0 = \left[\bar{\boldsymbol{\phi}}_{\mathbf{x},post}, \bar{\boldsymbol{\phi}}_{\mathbf{z},post}\right]' \tag{36}$$

where $\mathbf{R}_0$ is the covariance matrix associated with the values of the endogenous state variables as predicted by their corresponding policy functions under the post-reform RE solution $\bar{\boldsymbol{\phi}}_{post}$.[37]

For the case of homogeneous learning, we assume, as in Giannitsarou (2006) that:

$$\tilde{\boldsymbol{\phi}}_0 = \left[\bar{\boldsymbol{\phi}}_{\mathbf{x},pre}, \bar{\boldsymbol{\phi}}_{\mathbf{z},pre}\right]' \tag{37}$$

where the covariance matrix $\mathbf{R}_0$ is computed as described above, using (37) instead of (36).

For the case of heterogeneous learning, let $\bar{\boldsymbol{\phi}}^c_{\mathbf{x},post}$ and $\bar{\boldsymbol{\phi}}^c_{\mathbf{z},post}$ be a (4×2) and (1×2) sub-matrices of $\bar{\boldsymbol{\phi}}_{\mathbf{x},post}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z},post}$, respectively, containing the two columns of $\bar{\boldsymbol{\phi}}_{\mathbf{x},post}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z},post}$ that correspond to the policy functions of the capitalists. Similarly, let $\bar{\boldsymbol{\phi}}^s_{\mathbf{x},pre}$ and $\bar{\boldsymbol{\phi}}^s_{\mathbf{z},pre}$ be a (4×2) and (1×2) sub-matrices of $\bar{\boldsymbol{\phi}}_{\mathbf{x},pre}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z},pre}$, respectively, containing the two columns of $\bar{\boldsymbol{\phi}}_{\mathbf{x},pre}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z},pre}$ that correspond to the policy functions of the skilled workers. Hence, $\tilde{\boldsymbol{\phi}}_0$ is constructed as:

$$\tilde{\boldsymbol{\phi}}_0 = \begin{bmatrix} \left[\bar{\boldsymbol{\phi}}^c_{\mathbf{x},post}\right]_{4\times 2} & \left[\bar{\boldsymbol{\phi}}^s_{\mathbf{x},pre}\right]_{4\times 2} \\ \left[\bar{\boldsymbol{\phi}}^c_{\mathbf{z},post}\right]_{1\times 2} & \left[\bar{\boldsymbol{\phi}}^s_{\mathbf{z},pre}\right]_{1\times 2} \end{bmatrix} \tag{38}$$

while, for consistency, $\mathbf{R}_0$ is now computed as above but using (38) instead.

Note that for all the post-reform scenarios considered, $\tilde{\boldsymbol{\phi}}_0$ always satisfies the stationarity condition that the real parts of all the eigenvalues of $\tilde{\boldsymbol{\phi}}_{\mathbf{x},0}$ must lie inside the unit circle, while $\mathbf{R}_0$ is always an invertible matrix. These two conditions ensure the algorithm is adequately initialised, see, e.g. Carceles-Poveda and Giannitsarou (2007).

5.2.3 Measuring conditional welfare

We calculate conditional welfare or discounted lifetime utility using equation (10) and a time horizon of 1000 periods, by conditioning on the initial guesses for the cases under learning. When the economy is at the steady-state, this is equivalent to welfare reported in Table 2. To calculate the welfare losses for all agents between the post- and pre-reform economy, we follow e.g. Lucas

[37] To obtain $\mathbf{R}_0$ we make use of a numerical approximation involving the following steps: (i) simulate a series of $N(0, \sigma_a)$ random shocks for the exogenous state variable a_t, for $T_{num} = 100,000$ periods; (ii) using (i), simulate the values for the endogenous state variables as predicted by their corresponding policy functions under the post reform RE solution ($\bar{\boldsymbol{\phi}}_{post}$) for T_{num}; (iii) construct $\mathbf{w}_{(5\times T_{num})}$ including the time series of the simulated values for the five states ($B_{c,t}$, $K_{c,t}$, $B_{s,t}$, $K_{s,t}$, a_t); and (iv) compute the covariance matrix in a recursive fashion according to the second equation of (35), where the starting values $\mathbf{R}_0$ and $\mathbf{w}_0$ are given by two zero matrices.

(1990), Cooley and Hansen (1992), Giannitsarou (2006) and Malley *et al.* (2009) by computing the percentage extra consumption that an individual would require so as to be equally well off between the two regimes. This is defined as $\xi_{i,j}$

$$\xi_{i,j} = \left(\frac{U_{i,j}^{post}}{U_{i,ss}^{pre}} \right)^{\frac{1}{\mu(1-\sigma)}-1} \tag{39}$$

for each agent $i = c, s, u, a$ and each case $j = re, al$, where ss denotes welfare calculated in the steady-state, re welfare under the rational expectations solution and al, welfare under adaptive heterogeneous learning. For all the cases below, in the case of homogeneous learning, the welfare effects of tax reforms for all agents are effectively the same as under the RE solution as in Giannitsarou (2006).[38] Hence, to save on space, we do not discuss results from this solution further. To quantify the importance of bounded rationality with heterogeneous initial beliefs for welfare, we also calculate the cost of the heterogeneous learning, compared to the rational expectations solution. This is defined as $\zeta_{i,k}$

$$\zeta_{i} = \left(\frac{U_{i,re}^{post}}{U_{i,al}^{post}} \right)^{\frac{1}{\mu(1-\sigma)}-1} \tag{40}$$

where $i = c, s, u, a$.

In Tables 3 - 6 below we report the welfare losses or gains for all agents under the different scenarios and policy regimes, calculated using the formulas above. In Figures 1 - 4, we plot the tax revenue and debt Laffer curves, the pre-reform steady-state in percent deviations from the post-reform steady-state and the transition paths of the four state variables as well as consumption and labour hours for each agent, under rational expectations and learning. Each Figure corresponds to a particular tax reform case. The paths of consumption and hours are important as these will ultimately determine welfare for each agent.

5.3 Laffer curves in τ^h

The Laffer curves associated with changes in τ^h are shown in Figure 1a. These show the tax revenue collected and debt relative to the current steady-state, which is normalised to 100 for ease of presentation. For B/Y, we plot debt as a share of GDP. The pre-reform steady-state is also given on each Laffer

[38] Note it is only when the tax reform was accompanied by a positive or negative shock to TFP that the rational expectations and learning transition paths more substantially differed in Giannitsarou (2006).

curve in Figure 1a. We present those for a range of labour income tax rates that are consistent with a well-defined solution to the model.

As can be seen from the debt-to-GDP Laffer curve (lower right panel), the B/Y target of 60% can be achieved by either increasing τ^h to 47.7%, or by decreasing it to 27.1% from its pre-reform rate of 30%. The required increase in τ^h is large and essentially implies increasing the effective average tax rate for the upper 70% of the income distribution to the effective income tax that currently applies to highest income brackets (i.e. those with annual incomes above £200,000), when the National Insurance contributions are accounted for as well (see the average tax rates per income group in Social Trends 38, Office for National Statistics). This implies an increase in the progressivity of the effective labour income tax system to 2.39, from less than 1.6 as it currently stands. In contrast, the required decreases in τ^h are small, and would result in decreasing the labour income progressivity ratio to 1.36.

[Figure 1a about here]

The Laffer curve for labour income from skilled labour in Figure 1a (upper left panel) indicates that the economy is always on the upward slopping part of this curve, at least for values of τ^h that give a well-defined solution, so that increases (decreases) in τ^h increase (decrease) tax revenue. The pre- and post-reform steady-states for the increase in τ^h are shown in Figure 1b. As can be seen, the resulting fall in skilled labour supply causes a decrease in the supply of capital and unskilled labour, because of the complementarities in production. Given that the tax rates on capital and unskilled labour have not changed, the tax revenue collected from these two sources decreases, which is depicted in Figure 1a (upper right and upper middle panels respectively). These imply that the increases in total revenue when τ^h increases are smaller, as shown in the total revenue Laffer curve in Figure 1a (lower left panel), so that, for instance, an increase in τ^h from 0.3 to 0.4 increases tax revenue from skilled labour by about 20%, while leaving total tax revenue practically unchanged. Therefore, the complementarity between capital and labour implies that there is a Laffer curve between total tax revenue and τ^h, the peak of which is obtained for a value of τ^h about 35% despite that fact that we only obtained the upward slopping part of the skilled labour tax revenue Laffer curve.

[Figure 1b about here]

Of course, the effects on tax revenue are reversed for decreases in τ^h. As can be seen when comparing the pre- and post-reform steady-states for falls in τ^h in Figure 1c, when τ^h (and thus also tax revenue from skilled labour) decreases, there is an increase in skilled labour supply, which causes an increase in the supply of capital and unskilled labour, due to the complementarities in production. Given that the tax rates on capital and unskilled labour have not changed, the tax revenue collected from these two sources increases so that the total tax revenue falls by less. For instance, when τ^h decreases from 0.3 to 0.2, tax revenue from skilled labour falls by about 40%, while total tax revenue falls by approximately 10%.

[Figure 1c about here]

It is interesting to note that although the complementarity between capital and labour also exists in a standard Cobb-Douglas production function within a representative agent framework, the quantitative importance of the complementarities between the three different factor inputs is different in the heterogeneous agents setup, in light of the role of unskilled labour. Given that, as discussed in the calibration section, the elasticities of substitution between all three factor inputs considered here are well within those found in micro-econometric studies, the quantitative implications of these complementarities is not without empirical foundation. As said above, it is essentially because of these complementarities that we obtain the Laffer curve in total tax revenue, which, in turn, implies that there will be a Laffer curve with respect to debt and thus two long-run solutions for reducing the debt by changing τ^h.

5.3.1 Transition dynamics and welfare effects

In Figure 1b we plot the dynamic transition from pre- to the post-reform steady-state for an increase in τ^h and in Figure 1c for a fall in τ^h. In Table 3 we calculate the welfare losses and gains for all agents in both cases.

The first thing to note in Figure 1b is that the post-reform economy implies less consumption and less work time, for all agents, as a result of higher taxation. An increase in τ^h hurts the capitalist and the skilled worker directly, but indirectly it also negatively affects the unskilled worker. This happens because skilled labour complements unskilled, so that a decline in the former reduces the productivity of the latter. This trade-off implies a fall in welfare for all agents in the post-reform economy, as can seen by the ξ_{re} measure in the first column of Table 3. Given that the effects on the capitalist and the skilled worker are direct, they are hurt more by this change and hence suffer very big losses in welfare.

Table 3: Welfare costs/gains of reducing B/Y to 0.6

	$\tau^h = 0.477$			$\tau^h = 0.271$		
	ξ_{re}	ξ_{al}	ζ	ξ_{re}	ξ_{al}	ζ
Capitalist	-0.0802	-0.0873	0.0078	0.0034	0.0052	-0.0018
Skilled	-0.1164	-0.1200	0.0040	0.0173	0.0183	-0.0010
Unskilled	-0.0166	-0.0148	-0.0019	0.0024	0.0019	5.1e-04
Aggregate	-0.0798	-0.0825	0.0029	0.0100	0.0107	-7.0e-04

Adaptive learning increases the welfare costs, if we assume that capitalists and skilled workers form different initial beliefs about the economy and their policy functions. As can be seen in Figure 1b, the paths under heterogeneous learning deviate substantially and actually over- and under-shoot the rational expectations solution, as the agents learn and update their forecasts over time. In particular, the capitalist and the skilled worker overshoot in their decrease in consumption and undershoot the reduction in labour hours for long time periods. Both movements tend to reduce their welfare, compared to rational expectations. The additional losses, as reported in Table 3, are not trivial and amount to 0.78% of consumption for the capitalist and 0.4% for the skilled worker. The unskilled workers, forming no expectations, are not affected as much by learning. They are actually better off under learning, as their consumption is not reduced by as much, which more than compensates for the smaller increase in leisure time.

The results are reversed when τ^h is decreased. As discussed above, the increase in the labour supply of skilled labour crowds-in capital and unskilled labour, increasing the tax revenue collected and also consumption for all agents, as can be seen in Figure 1c. The trade-off is now in favour of all agents, who are now better off in the post-reform economy, as can be seen in Table 3. Hence, all agents in the economy would support a decrease in τ^h relative to the current steady-state. In addition, adaptive learning with heterogeneous initial beliefs implies, in fact, small welfare benefits for the agents. Under heterogeneous learning, they overshoot their adjustment to the post-reform steady-state and hence they reap more quickly the benefits associated with the post-reform economy.

5.4 Laffer curves in τ^u and τ^h

When both labour income tax rates are increased proportionately, they both need to increase significantly, to 0.365 for τ^u and 0.548 for τ^h. On the contrary, small decreases, $\tau^u = 0.189$ and $\tau^h = 0.283$ would suffice to reduce

B/Y to 0.6. We plot the Laffer curves for this case in Figure 2a, working as in Figure 1a. Note that we plot tax revenue from skilled labour against τ^h and all other cases for tax revenue and debt against τ^u, since $\tau^h = 1.5 * \tau^u$. As can be seen, only the upward slopping part of the Laffer curve between tax revenue from labour income and the labour income tax rates can be obtained for tax rates that give well-defined solutions. However, as above, the complementarities between labour and capital imply that both sides of the total revenue and the debt Laffer curves are plausible.

[Figure 2a about here]

It is worth noting that the required increase in τ^h in this case is higher than the increase above, when only τ^h is adjusted for the government to meet the debt target. As can be seen in Figure 2a, this is because tax revenue from unskilled labour increases with tax revenue from skilled labour, so that the effect of the decrease of capital tax revenue in total tax revenue is smaller. This implies that the peak of the Laffer curve for total tax revenue, and thus of the Laffer curve for debt as well, is now further to the right. For symmetrically opposite reasons, the required fall in τ^h is smaller in this case.

[Figures 2b and 2c about here]

5.4.1 Transition dynamics and welfare effects

The welfare gains/losses are reported in Table 4 and the transition paths from the pre- to the post-reform steady-state are plotted in Figures 2b and 2c. We first consider the case of tax increases. Given the higher tax rates compared to the case where only τ^h is increased, the welfare losses, reported in Table 4, are bigger for all agents under this policy regime, with unskilled workers being the most hurt. This is because now unskilled labour income is directly taxed. It is important to note, however, that these results indicate that it is in the interest of skilled labour (i.e. skilled workers and capitalists) not to increase the taxation of unskilled labour, if the policy changes take the form of increases in labour tax rates. Both the capitalist and the skilled worker are better off in the case where only skilled labour income is taxed. The dynamic transitions from the pre- to the post-reform steady-state are very similar to the previous case, where τ^h increased only. Consistent with the previous discussion, there are important welfare losses for the capitalist and the skilled workers under heterogeneous learning, and a small welfare gain for the unskilled worker in this scenario.

Table 4: Welfare costs/gains of reducing B/Y to 0.6

	$\tau^u = 0.365$, $\tau^h = 0.548$			$\tau^u = 0.189$, $\tau^h = 0.283$		
	ξ_{re}	ξ_{al}	ζ	ξ_{re}	ξ_{al}	ζ
Capitalist	-0.1143	-0.1221	0.0089	-0.0020	-7.6e-04	-0.0012
Skilled	-0.1634	-0.1670	0.0044	0.0092	0.0099	-6.6e-04
Unskilled	-0.1274	-0.1259	-0.0018	0.0094	0.0090	3.3e-04
Aggregate	-0.1433	-0.1461	0.0033	0.0072	0.0076	-4.5e-04

We next consider the case of decreases in the tax rates. For similar reasons with the case of decreases in τ^h, all agents are better off with tax cuts, compared to tax increases. As expected, unskilled labour prefers this scenario to decreases in τ^h only, while skilled labour (i.e. capitalists and skilled workers) prefer the tax cuts in τ^h only. What is more interesting is that the capitalists are worse off, relative to the current steady-state, by a decrease in all labour taxes (whereas they were better off when τ^h increased only). By comparing Figures 2c to 1c, it is evident that this happens because consumption has not increased by as much when all tax rates increase, and the fall in leisure dominates. The reason is that the fall in τ^u has increased the unskilled labour supply and this has substituted for capital in the production, so that the income of the capitalist under the new regime has not increased sufficiently. In addition, consistent with the previous discussion, where only τ^h decreased, adaptive learning improves outcomes for the capitalist and skilled worker, although the effects are now smaller.

In the case of proportional changes in labour taxes, labour heterogeneity and the substitutability of capital with unskilled labour in the production function result in one income group - the capitalist - opposing a fall in labour taxes. However, since aggregate welfare increases after the decrease in labour taxes, the welfare gains for skilled and unskilled workers are enough to compensate the capitalists to obtain their consent for this tax change.[39]

5.5 Laffer curves in τ^k

In Figure 3a we plot the Laffer curves for tax revenue and debt, with respect to the capital income tax rate. The relationship between tax revenue from capital and the capital tax rate shows that the economy is on the downward slopping part of this Laffer curve. Increasing τ^k decreases the tax revenue

[39]Note that falls in τ^u only, imply a ξ_{re} of approximately -1 and -0.2 percent for the capitalists and skilled workers respectively and of 1.8 percent for unskilled workers. The implied ζ's in this case are effectively zero.

collected from capital, while the reverse is true for decreases in τ^k. This is consistent with data on effective capital tax rates, which suggest that capital taxation in the UK is much higher than the rest of Europe and the USA (see e.g. Angelopoulos *et al.* (2008), for a discussion of these data). However, the debt Laffer curve indicates that it is possible to reduce debt by either increasing ($\tau^k = 44.7\%$) or decreasing ($\tau^k = 7.8\%$) τ^k. This is because, total tax revenue can fall by either increases or decreases in the capital tax rate.

[Figure 3a about here]

As can be seen in Figure 3b, increases in τ^k decrease the supply of capital and, via the complementarities in the production function, tend to decrease skilled labour supply as well. However, for the capitalists, who own the biggest share of capital in the economy, the negative income effects from increased capital taxation dominate, so that their skilled labour supply increases. More importantly, the substitutability between capital and unskilled labour implies that, after the increase in the capital tax and the resulting fall in capital supply, firms will substitute capital with unskilled labour in production. Overall, these movements decrease tax revenue, so that the economy is on the downward slopping part of the total revenue Laffer curve as well. Hence, a lower debt-to-GDP target, which, as explained previously requires a lower tax revenue in the long run, is consistent with an increase in τ^k.

[Figure 3b about here]

On the other hand, the decrease required in τ^k to reduce debt-to-GDP to its target is very large, so that the required capital tax rate is outside of the historical experience of the recent decades. The reason is that when τ^k initially falls, tax revenue increases, due to the complementarities between the tax bases, as discussed above. Namely, the increase in capital crowds-in skilled labour and crowds-out unskilled labour, while the positive income effect decreases the labour supply of capitalists (see Figure 3c). Recall, that a lower debt target for a given level of spending requires lower tax revenue due to the fall in interest payments. Hence, the capital tax rate has to decrease enough to take the economy back to the upward slopping part of the Laffer curve.

[Figure 3c about here]

5.5.1 Transition dynamics and welfare effects

We first present the results from increasing the capital income tax, τ^k, to 0.447 to achieve the debt target. The welfare losses for all agents under the different scenarios are reported in Table 5.

Table 5: Welfare costs/gains of reducing B/Y to 0.6

	$\tau^k = 0.447$			$\tau^k = 0.078$		
	ξ_{re}	ξ_{al}	ζ	ξ_{re}	ξ_{al}	ζ
Capitalist	-0.0114	-0.0130	0.0016	0.3362	NL	NL
Skilled	-0.0062	-0.0071	8.8e-04	0.2990	NL	NL
Unskilled	0.0031	0.0036	-4.4e-04	-0.1580	NL	NL
Aggregate	-0.0043	-0.0049	6.1e-04	0.1317	NL	NL

As can be seen in Figure 3b, the post-reform economy implies less consumption for capitalists and skilled workers, although the effects are much smaller compared to the case where the debt reduction was achieved by increases in the labour income taxes The consumption of the unskilled workers increases in the post-reform economy. In addition, work time falls for skilled workers and increases for capitalists and unskilled labour, although the quantitative changes are again small. After the increase in the capital tax, firms will substitute capital with unskilled labour in production. In addition, the capitalists, who own the biggest share of capital in the economy and thus suffer the greatest loss in income due to the increase in τ^k, will increase their labour effort to make up for the loss in income. Skilled workers, on the other hand, will reduce their labour supply, as the substitution effects, from a lower return to labour because of the lower capital base, dominate.

It is worth noting the importance of heterogeneity for the above results. First, skilled labour responds to the capital tax changes differently from unskilled labour, due to their different relationship (complements vs substitutes) with capital. Second, the heterogeneity in capital holdings imply that the income effects dominate for the capitalist, as assets constitute a large share of his resources, whereas substitution effects dominate for the skilled labour.

As a result of the above, the capitalist and the skilled worker are worse off after the capital tax increase, whereas unskilled labour is better off. However, the differences between the two pre- and post-reform economy are smaller, compared to the case where labour taxes fall. This is reflected in the welfare losses under the increase in τ^k, which are smaller for the capitalist and the skilled worker, under all cases considered, while, as noted, the unskilled worker makes welfare gains in this regime. Therefore, all agents in the economy would prefer an increase in the capital tax, relative to the labour tax in skilled labour and/or unskilled labour, to reduce the debt. Unskilled workers gain under the increase in capital taxes, because firms will find it optimal to substitute capital for unskilled labour, which explains, as already said, the rise in the labour hours and consumption for the unskilled workers. In addition, note that the paths under heterogeneous learning deviate and again over- and under-shoot the rational expectations solution. This is particularly so for the capitalist and results in reductions in his welfare, compared to rational expectations by about 0.16%. The additional gains/losses for the other two agents are trivial.

It is useful to emphasise the importance of the finding that the welfare losses for the capitalist and the skilled worker are lower under increased capital taxation, than under increased labour taxation. This implies that it is in the best interest of capitalists and skilled workers for capital and not labour (especially unskilled) to be taxed by more. Therefore, it is beneficial in this case for the capital holders that unskilled labour is closing the welfare gap with the other two socioeconomic groups.

We next consider the effects of the reduction in τ^k to 7.8%. This implies a very large increase in the capital stock, skilled labour and consumption of both the capitalist and the skilled worker, which lead to a very big increase in welfare for these two agents. Note that the results for welfare are comparable to the results obtained for welfare from removing capital taxation in general equilibrium models (see e.g. Lucas (1990) and Garcia-Mila *et al.* (2010)). However, this change also implies a big cost for the unskilled workers. Capital is substituting unskilled labour so that the welfare of the latter is significantly reduced. Unless unskilled labour is compensated by the capitalists and skilled labour for its losses, it will not support cuts in capital taxes. Note that, from the society's point of view, this would be desirable, as aggregate welfare increases after the fall in capital tax, which suggests that the welfare gains for capitalists and skilled workers are more than enough to compensate unskilled labour.

Finally, note that given the very large differences between the two steady-states, adaptive learning (both under the homogeneous and heterogeneous assumptions) does not imply well-defined solutions, because of the big forecast errors and the unrealistic adjustments that they imply. Effectively, when the learners start from the pre-reform economy, the post-reform steady-state is not learnable.[40]

[40] Evans and Honkapohja (2003, 2006) suggest that policymakers should rule out policy reforms that lead to unstable solutions under learning since in such cases economic agents unsuccessfully try to correct their forecast functions over time and hence the economy fails to converge to the desired rational expectations equilibrium.

5.6 Laffer curves in all tax rates

Finally, we consider the case where the government changes all tax rates proportionately to meet the debt target. Figure 4a shows the Laffer curves for the tax revenue and debt with respect to all the tax rates. Note that the horizontal axis of the total tax revenue and debt Laffer curves plots τ^u, since the other tax rates are given by $\tau^h = 1.5 * \tau^u$, $\tau^k = 2.21 * \tau^u$.

[Figure 4a about here]

The Laffer curves for total tax revenue and debt are very similar to the ones analysed above in the τ^k case, suggesting that the effects of the changes in τ^k are dominant. In this case, the outcome of the changes in the after tax returns to factor inputs, results in increases in the supply of labour of capitalists and unskilled workers and a decrease in capital supply and the supply of skilled labour (see Figure 4b). After a decrease in the tax rates, the capital stock and the labour supply of the capitalists increase, while the labour input from skilled and unskilled workers decreases (see Figure 4c).

[Figures 4b and 4c about here]

5.6.1 Transition dynamics and welfare effects

The welfare gains/losses are reported in Table 6 and as can be seen they are qualitatively similar to the case of changes in τ^k. However, the welfare losses are higher in this case for the capitalist and the skilled worker, compared to the case where τ^k increases only. At the same time, the welfare gains are lower for the unskilled worker. As this case is a mix of capital and labour tax increases, the results fall between the two cases. Therefore, no agent in the economy would prefer this case to the case where τ^k increases only.

Table 6: Welfare costs/gains of reducing B/Y to 0.6

	$\tau^u = 0.203$, $\tau^h = 0.304$, $\tau^k = 0.448$			$\tau^u = 0.170$, $\tau^h = 0.113$, $\tau^k = 0.249$		
	ξ_{re}	ξ_{al}	ζ	ξ_{re}	ξ_{al}	ζ
Capitalist	-0.0138	-0.0162	0.0024	0.2182	NL	NL
Skilled	-0.0100	-0.0113	0.0013	0.2680	NL	NL
Unskilled	0.0015	0.0021	-6.4e-04	-0.0341	NL	NL
Aggregate	-0.0072	-0.0080	8.9e-04	0.1528	NL	NL

Similarly, the welfare gains for the capitalist and the skilled worker are lower when all tax rates fall, compared to a fall in τ^k only. At the same time, the losses for the unskilled worker are lower too. However, in terms of aggregate welfare, this change in tax rates is better than a decrease in τ^k only. Therefore, at the aggregate level, reducing the capital tax rate (and taxing only labour) is not the preferred choice.

6 Summary and Conclusions

Using a heterogenous agent model under both rational expectations and adaptive learning, we have addressed the issue on whether supply-side reforms consistent with lower public debt-to-GDP in the long-run can lead to a more efficient and equitable economy. To implement these tax reforms, we calibrated the model so that the steady-state represented the current UK economy and then simulated the model, under rational expectations and bounded rationality, after a permanent change in a tax rate, starting from the current, or pre-reform steady state. We evaluated welfare for each agent over the transition period to the new, or post-reform steady state. The change in the tax rate was chosen so that in the post-reform steady state the debt-to-GDP ratio was 60%. As Schmitt-Grohé and Uribe (1997) have shown, when a distorting income tax rate is chosen to close the government budget constraint, as we do for the post-reform steady state, there will be two long-run equilibrium solutions for the tax rate, as long as there is a Laffer curve between tax revenue and the tax rates. We find that both long-run solutions are possible for the model UK economy and thus considered both increases and decreases in the income tax rates. The complementarities and substitutabilities between the capital and labour supplies of the heterogeneous agents are critical in obtaining these multiple equilibria, since they imply that the Laffer curve between total tax revenue and the tax rates is empirically plausible, even though only the upward-slopping part of the curve is obtained for the relationship between tax revenue from individual labour incomes and the tax rates. We then welfare ranked these equilibria, across different tax reforms, for the aggregate economy and for all types of agents.

Our modeling approach and analysis contributes some interesting and useful results to a growing literature. First, we find that even without increases in the labour taxes to accompany capital tax decreases, the latter imply a large increase in inequality. In particular, they result in big welfare gains for the capitalists and skilled workers and big welfare losses for the unskilled workers. Our quantitative results are comparable to those of Garcia-Mila *et. al.* (2010) and lend further support to their conclusions. However, the reason that unskilled labour is hurt in our model is because of the substitutability between capital and unskilled labour in production.

We also obtain the result, as in the literature, that a capital tax decrease is beneficial at the aggregate level. However, the distributional effects referred to above suggest that some compensation from the capitalists and skilled workers to the unskilled workers would be required if the government did not want to disadvantage one particular group in the economy.

Second, and more importantly, we find a tax reform that is Pareto improving. This corresponds to a fall in the taxation for skilled labour. An increase in skilled labour supply, which follows a cut in skilled labour taxes, increases the returns for capital and unskilled labour as well, given the complementarities of skilled labour with the other two sources of production.

Third, we find that if the menu of policies was restricted to considering increases in tax rates, then increasing the tax rate on capital is preferred for every agent in the economy since welfare losses are minimised for capitalists and skilled workers and welfare gains are maximised for unskilled workers. Therefore, the general message coming from our analysis is that if the government wants to increase taxes, it should increase the capital tax and if it wants to decrease taxes, it should decrease the tax on skilled labour.

Fourth, we find that heterogeneity in initial beliefs matters quantitatively for the welfare losses/gains for the agents. In particular, the losses (gains) are increased after rises (falls) in the tax rates, as the interaction of the initial beliefs results in the economy overshooting and actually converging faster to the new steady state, compared to the rational expectations solution. Since the new steady state is worse (better) under tax increases (decreases), learning under this form of heterogeneity in initial beliefs implies bigger losses (gains). This is different from the results under homogeneous learning after tax reforms, where convergence to the new equilibrium is faster under the rational expectations solution (see e.g. Giannitsarou (2006)).

Finally, we find that the UK economy is on the upward slopping part of the Laffer curve for tax revenue from labour income, but on the downward slopping side of the Laffer curve for tax revenue from assets and for total tax revenue. Nevertheless, as discussed above, reducing capital taxes and/or increasing labour taxes is a bad idea, if the goal is to increase welfare for all agents.

7 Appendix

7.1 E-Stability

E-stability determines the stability of the RE solution under a learning rule such as RLS, in which the estimates $\tilde{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\tilde{\boldsymbol{\phi}}_{\mathbf{z}}$ used in the PLM (29) are adjusted slowly in the direction of the implied ALM parameters shown in (31). In fact, if this adjustment process is completed, feeding the latest estimates $\tilde{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\tilde{\boldsymbol{\phi}}_{\mathbf{z}}$ in the two ALM parameters in (31) should yield exactly the same two estimates for $\tilde{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\tilde{\boldsymbol{\phi}}_{\mathbf{z}}$. In such a case, these estimates must be equal to the RE solution parameters $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$, since the model at hand has a unique equilibrium.[41] Evans and Honkapohja (2001) show that such condition can be verified by computing the following two matrices (associated to $\bar{\boldsymbol{\phi}}_{\mathbf{x}}$ and $\bar{\boldsymbol{\phi}}_{\mathbf{z}}$, respectively):

$$\begin{aligned} \mathbf{Q_x} &= \left[(\mathbf{I} - \mathbf{M}_1\bar{\boldsymbol{\phi}}_{\mathbf{x}})^{-1}\mathbf{M2}\right]' \otimes \left[(\mathbf{I} - \mathbf{M}_1\bar{\boldsymbol{\phi}}_{\mathbf{x}})^{-1}\mathbf{M}_1\right], \\ \mathbf{Q_z} &= \boldsymbol{\rho}' \otimes \left[(\mathbf{I} - \mathbf{M}_1\bar{\boldsymbol{\phi}}_{\mathbf{x}})^{-1}\mathbf{M}_1\right] \end{aligned} \tag{41}$$

and then testing if all their corresponding eigenvalues have real parts less than one.

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[41] See Propositions 10:3 and 10:4 in Evans and Honkapohja (2001) for more details regarding the derivation of the E-stability condition and the local convergence result discussed in the text.

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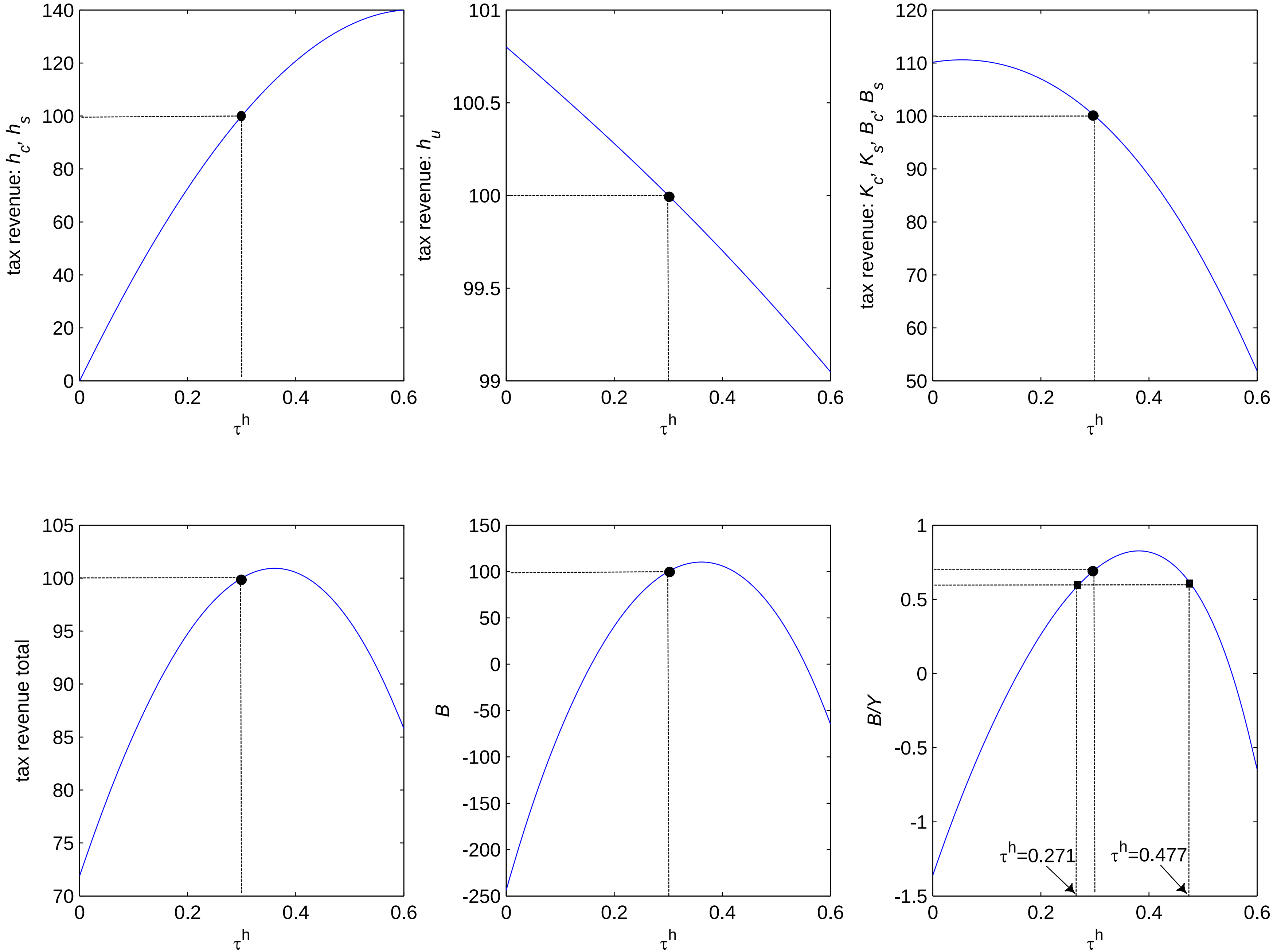
Figure 1a: Laffer curves for changes in τ^h



● denotes pre-reform steady-state

Figure 1b: τ^h=0.477, B/Y=0.60

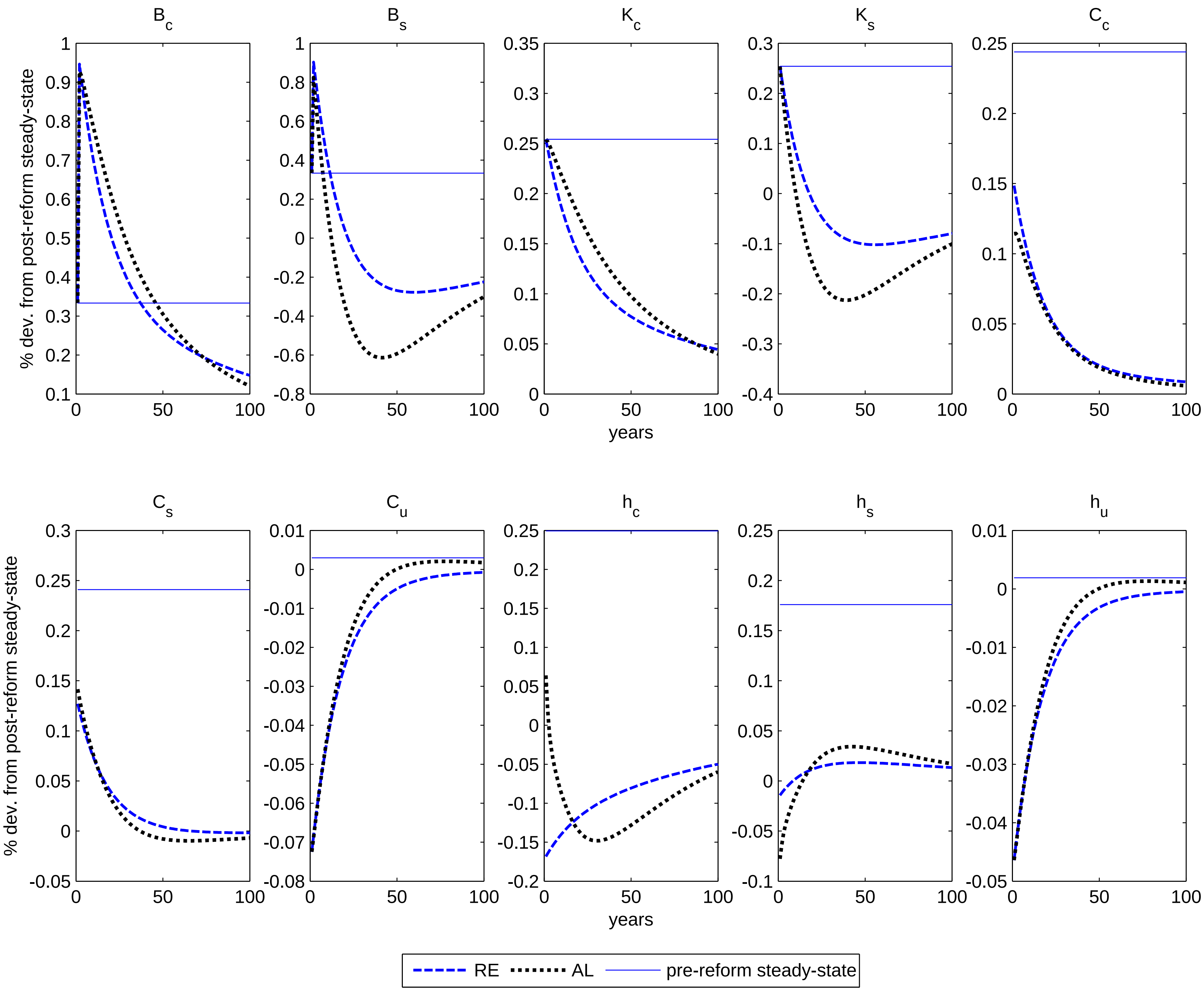


Figure 1c: τ^h=0.271, B/Y=0.60

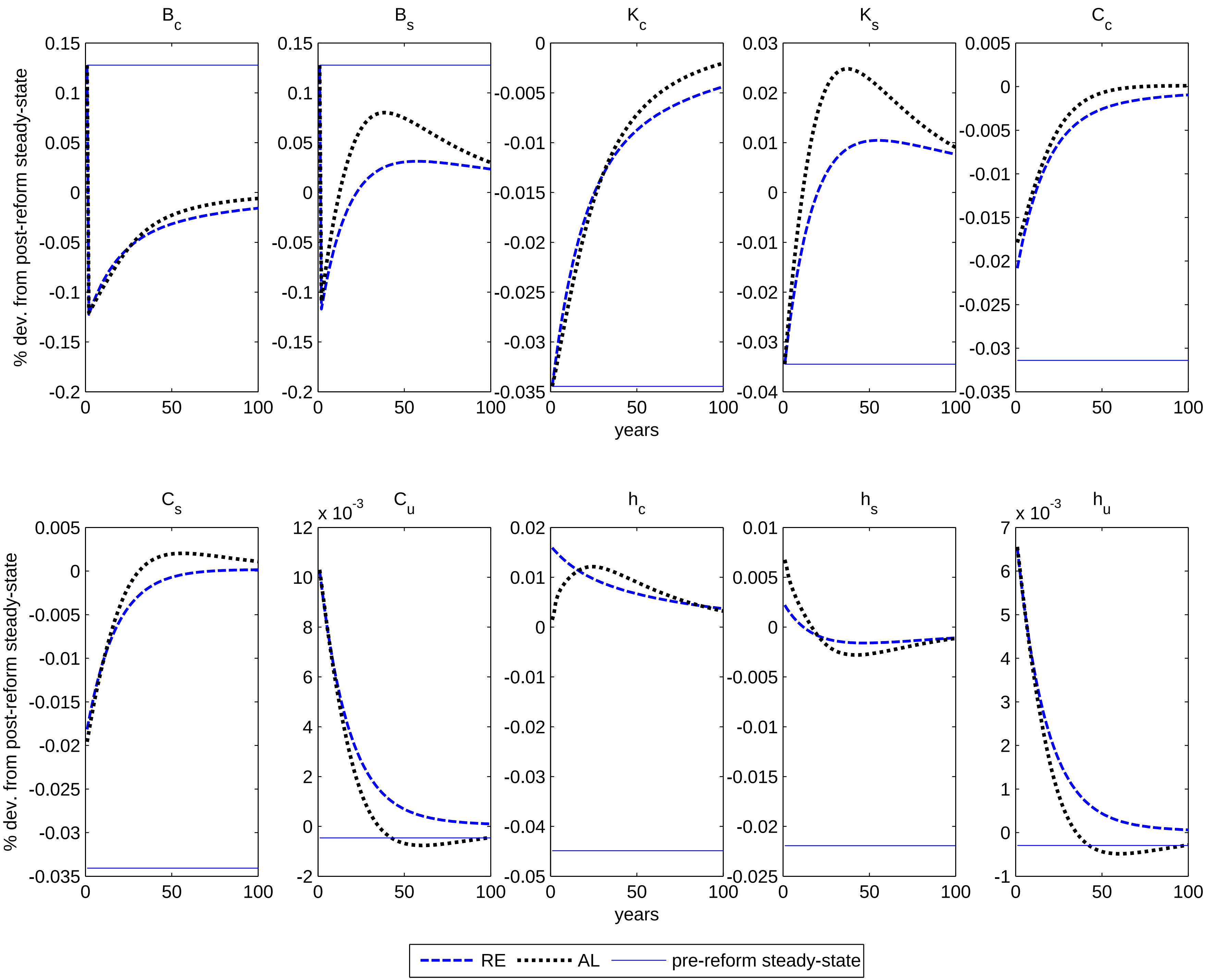
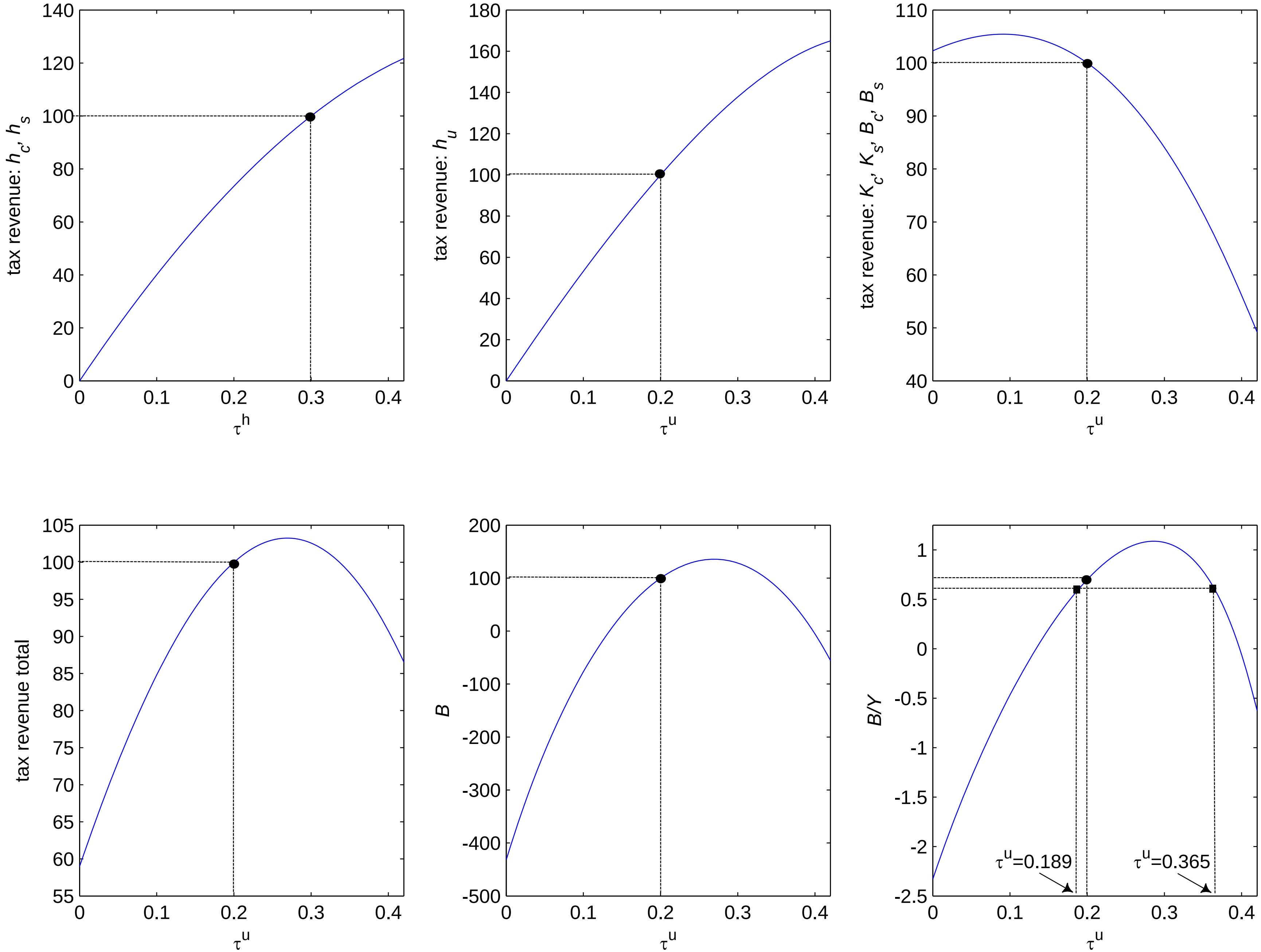
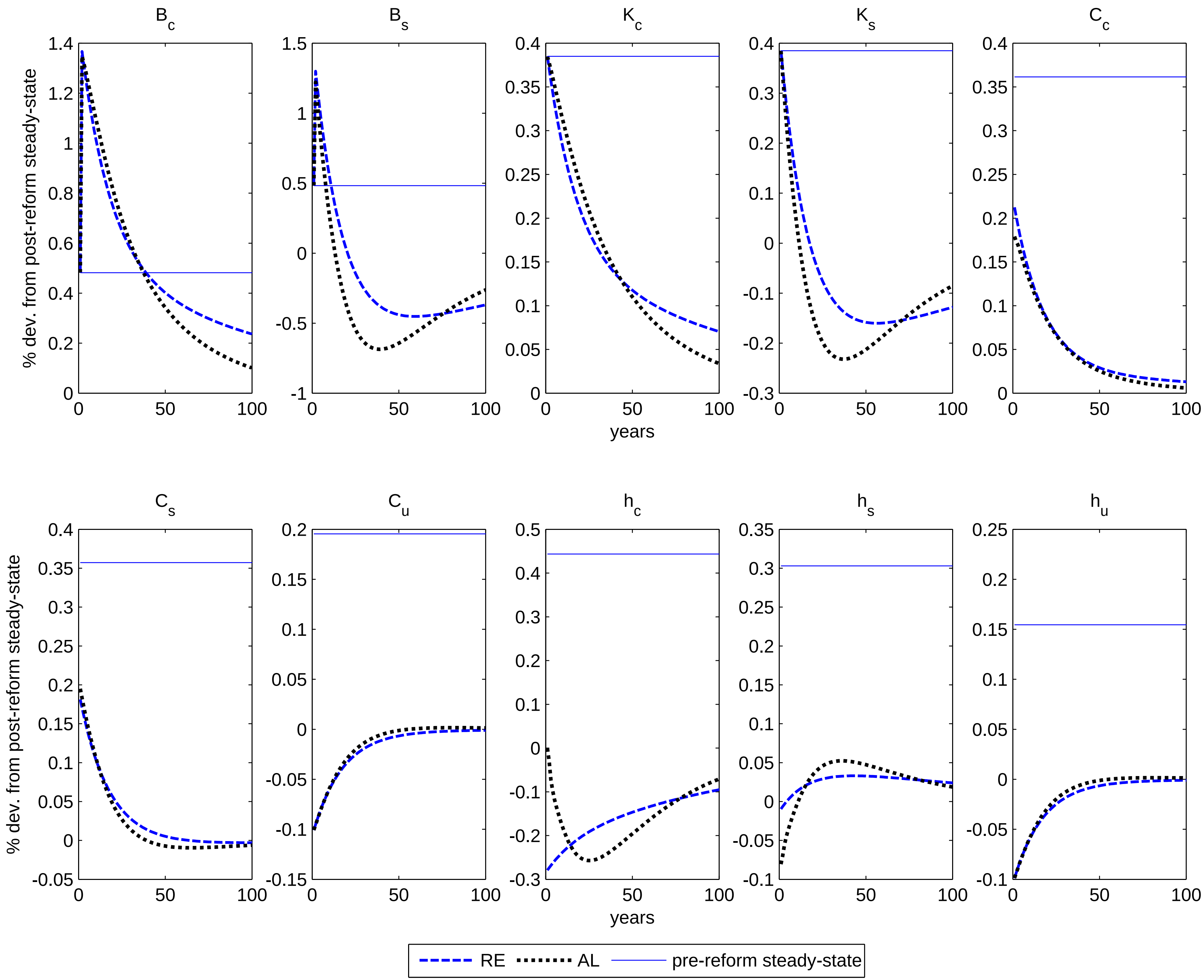


Figure 2a: Laffer curves for proportional changes in τ^h & τ^u



● denotes pre-reform steady-state

Figure 2b: τ^h=0.548, τ^u=0.365, B/Y=0.60



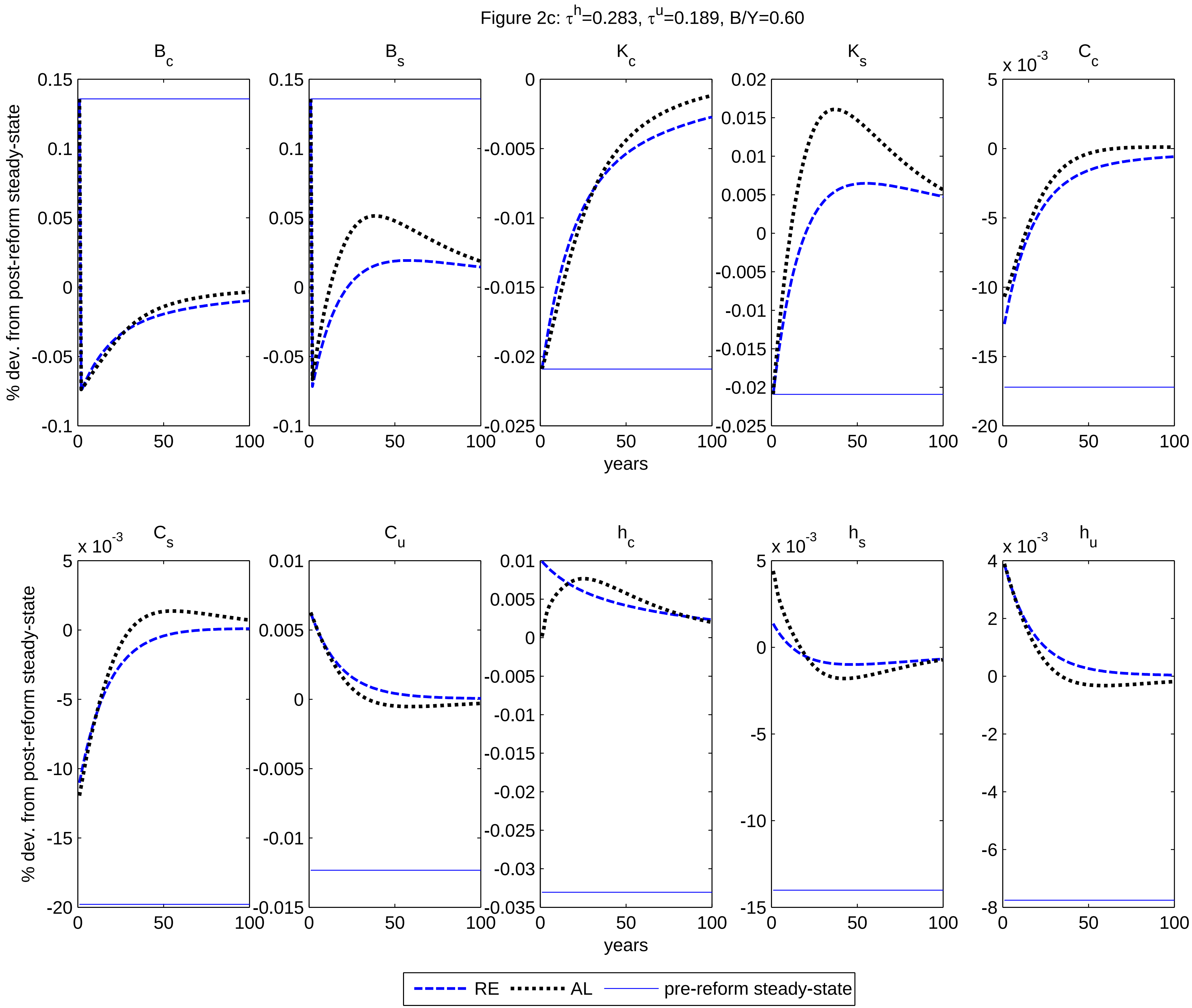
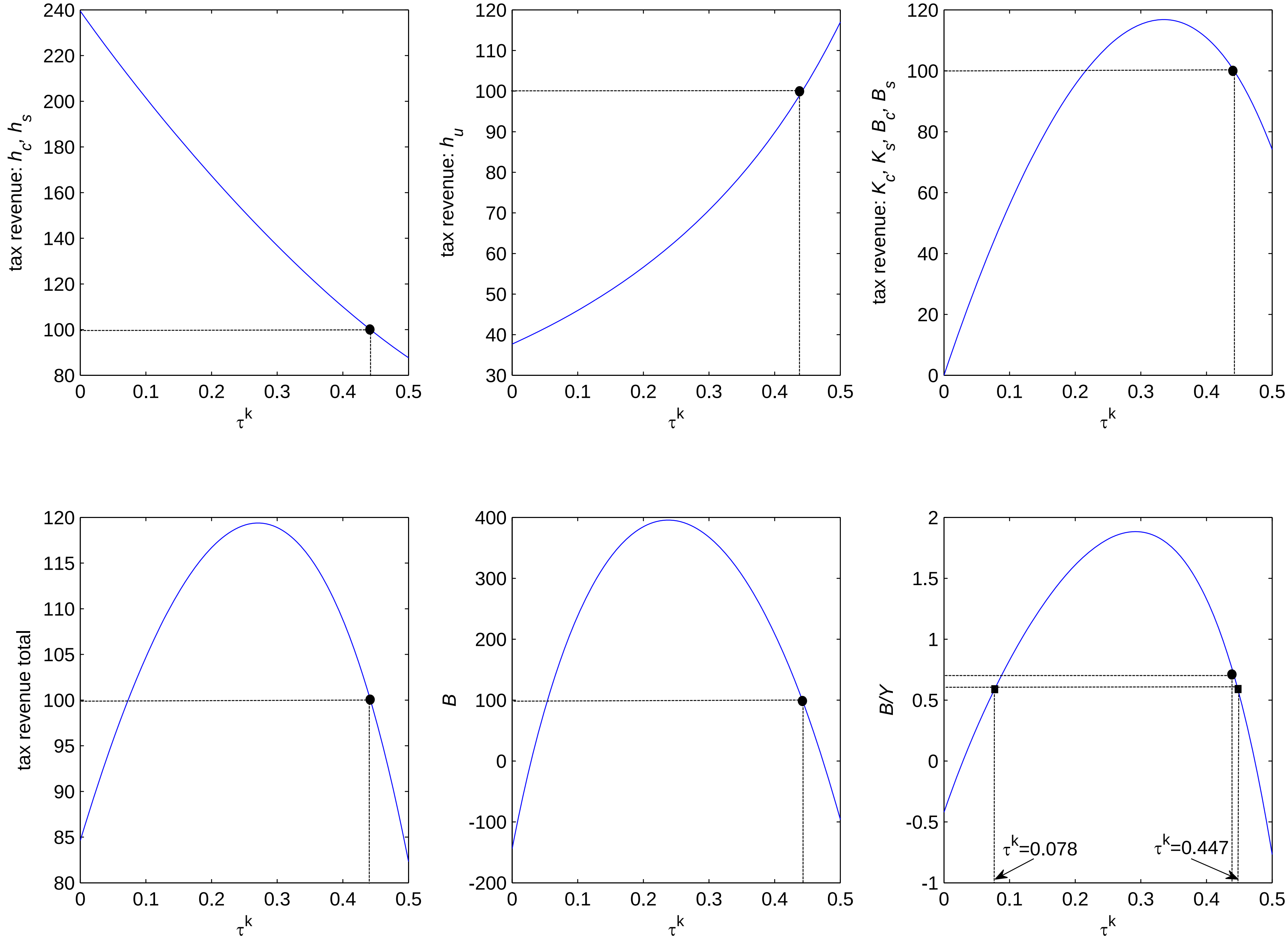


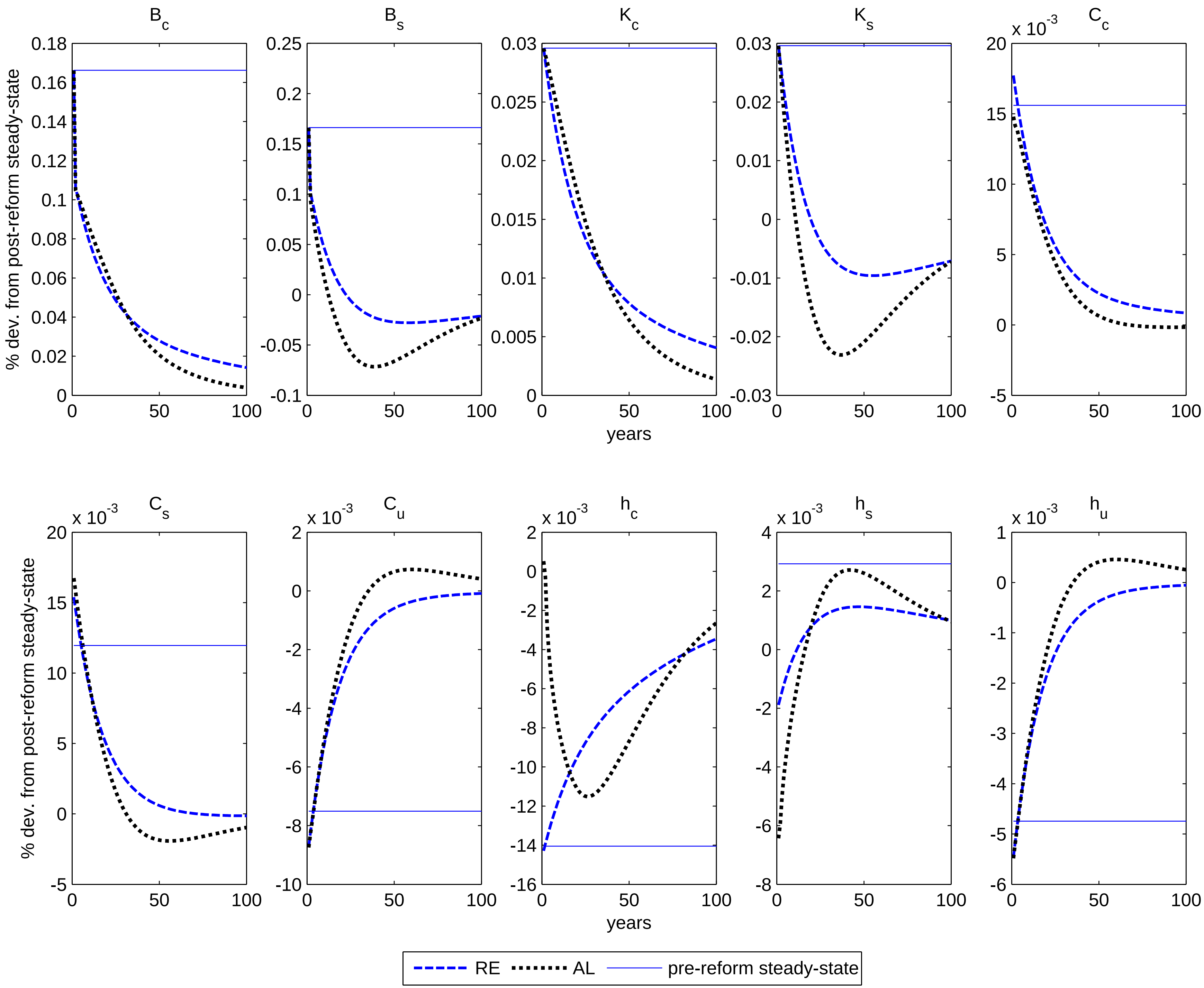
Figure 2c: τ^h=0.283, τ^u=0.189, B/Y=0.60
B_c
B_s
K_c
K_s
C_c
C_s
C_u
h_c
h_s
h_u
% dev. from post-reform steady-state
years
x 10^{-3}
RE
AL
pre-reform steady-state

Figure 3a: Laffer curves for changes in τ^k



● denotes pre-reform steady-state

Figure 3b: τ^k=0.447, B/Y=0.60



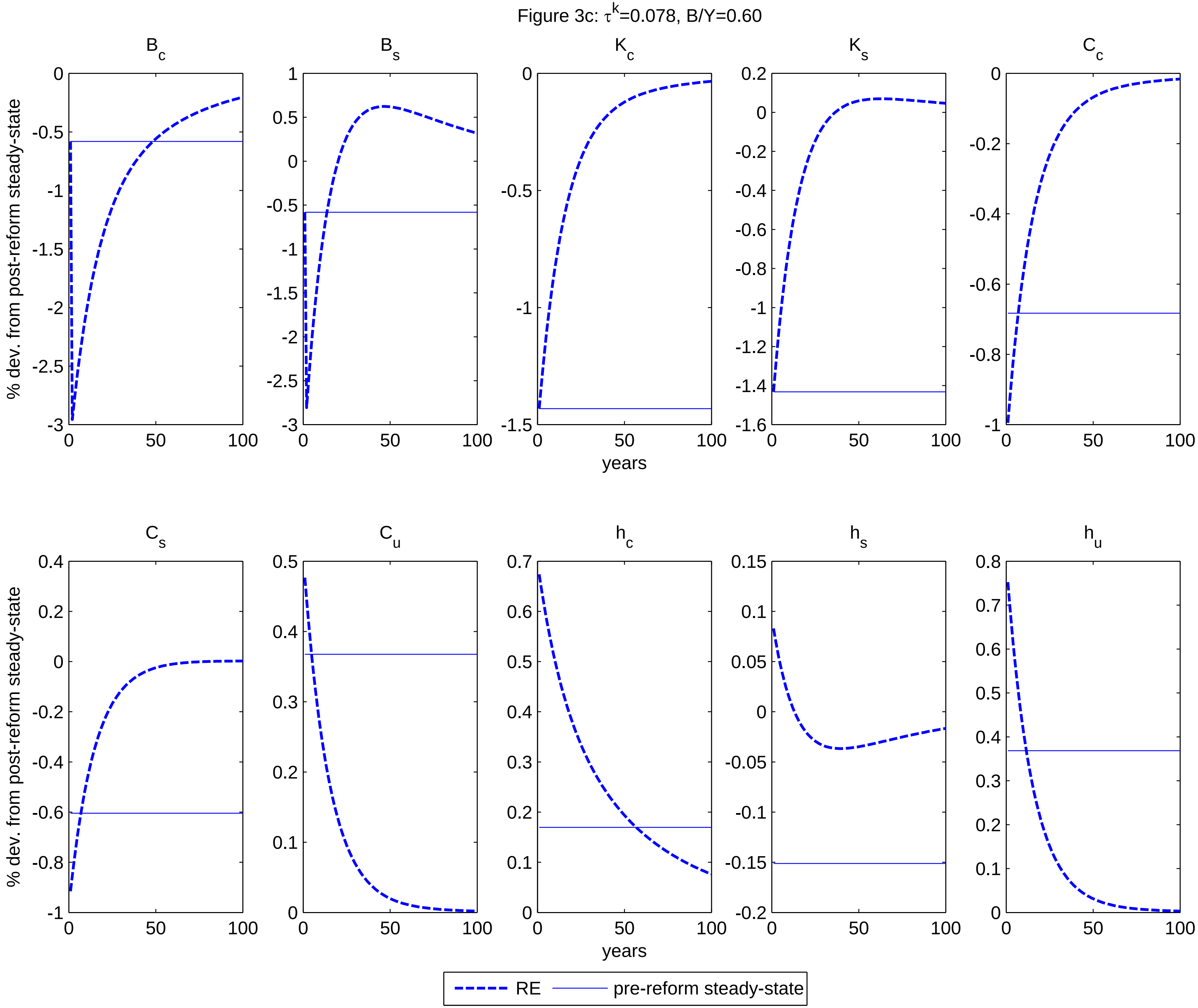


Figure 3c: τ^k=0.078, B/Y=0.60
B_c
B_s
K_c
K_s
C_c
C_s
C_u
h_c
h_s
h_u
% dev. from post-reform steady-state
years
RE
pre-reform steady-state

Figure 4a: Laffer curves for proportional changes in all tax rates

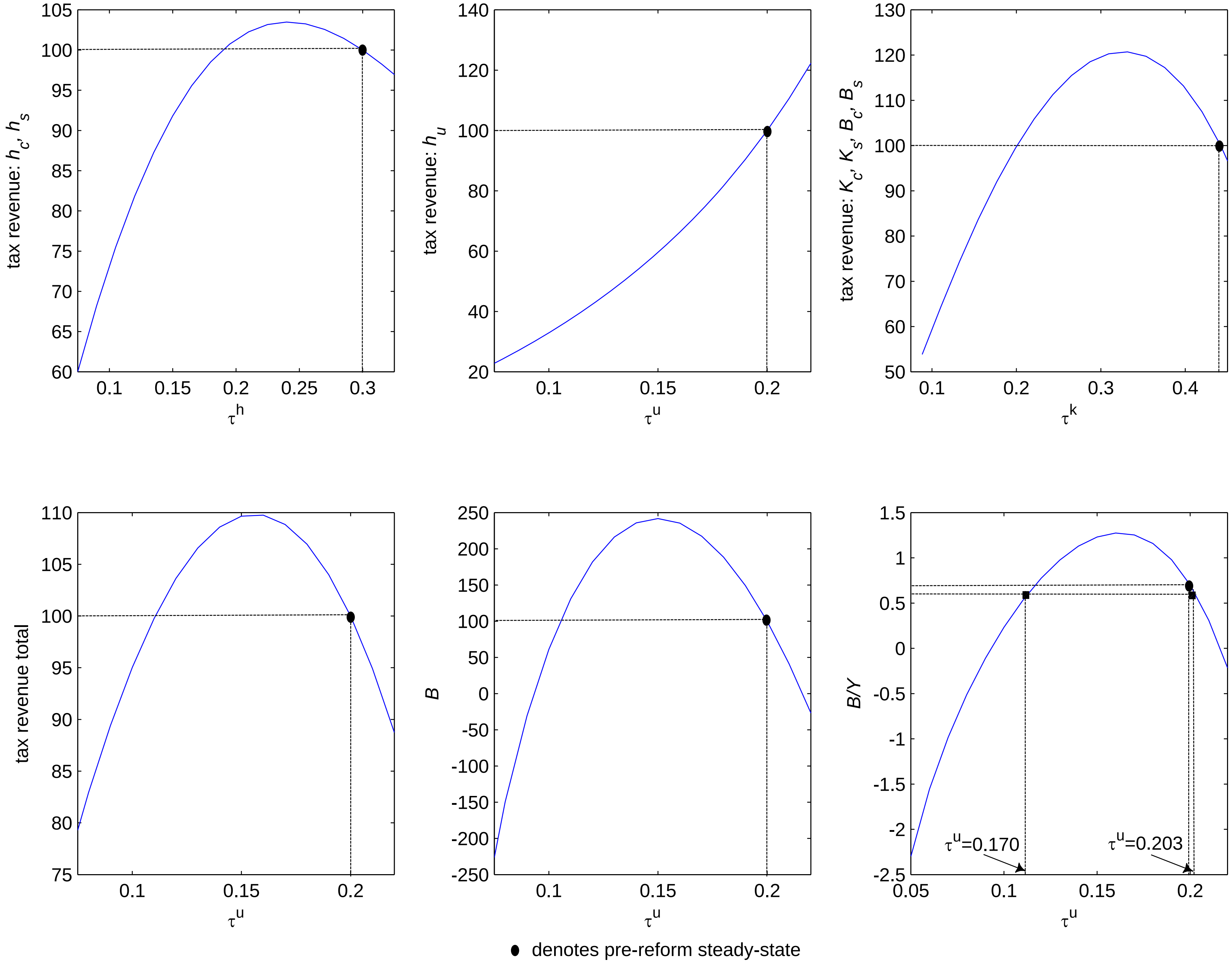


Figure 4b: τ^h=0.304, τ^u=0.203, τ^k=0.448, B/Y=0.60

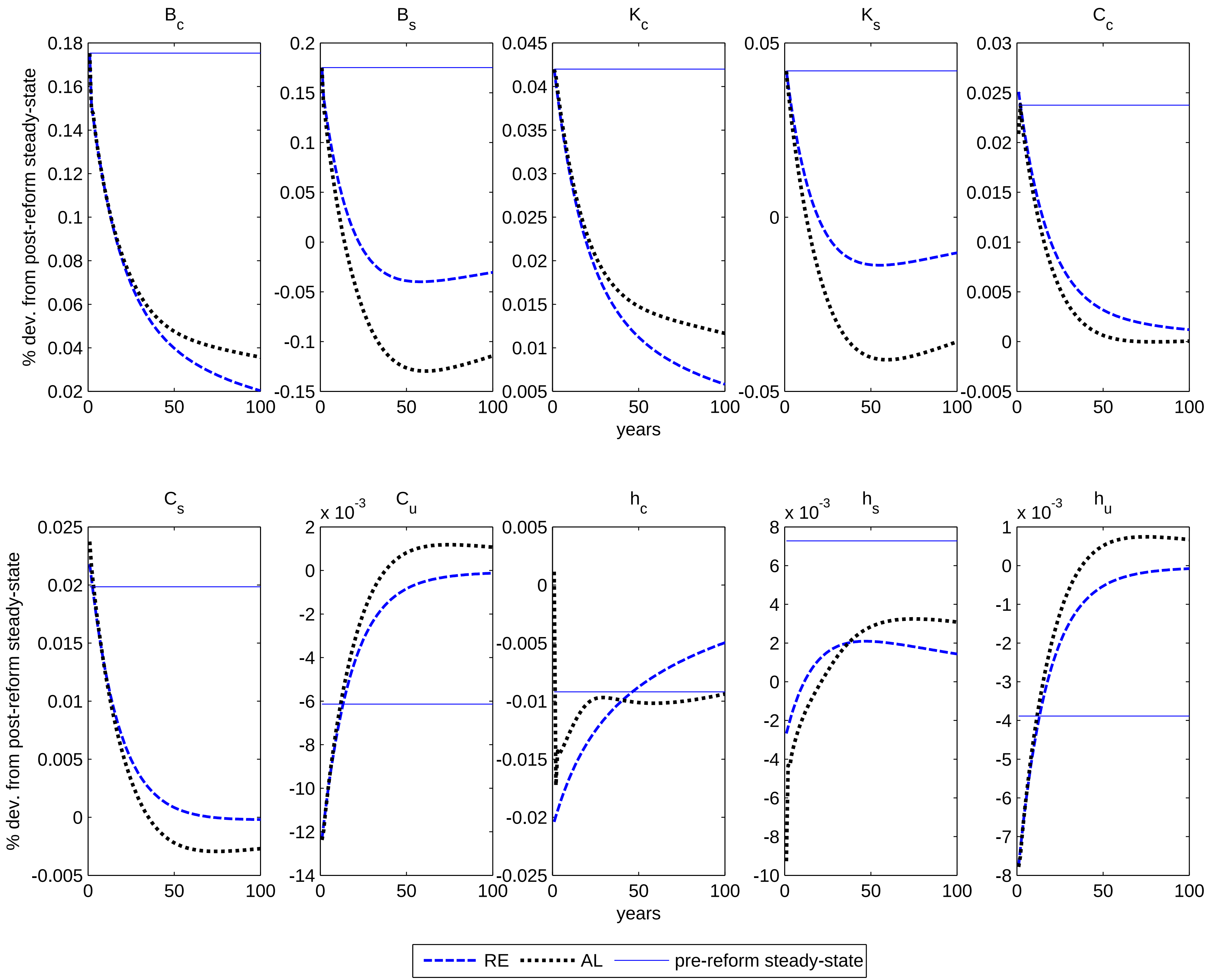


Figure 4c: τ^h=0.170, τ^u=0.113, τ^k=0.249, B/Y=0.60