Investment Shocks and the Comovement Problem*

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Abstract

Recent work based on sticky price-wage estimated dynamic stochastic general equilibrium (DSGE) models suggests investment shocks are the most important drivers of post-World War II US business cycles. Consumption, however, typically falls after an investment shock. This finding sits oddly with the observed business cycle comovement where consumption, along with hours-worked and investment, moves with economic activity. We show that this comovement problem is resolved in an estimated DSGE model when (i) the cost of capital utilization is specified in terms of increased depreciation of capital, as originally proposed by Greenwood et al. (1988) in a neoclassical setting, or (ii) there is no wealth effect on labor supply. The data, however, favours the first channel. Traditionally, the cost of utilization is specified in terms of forgone consumption following Christiano et al. (2005), who studied the effects of monetary policy shocks. The alternative specification we consider has two additional implications relative to the traditional one: (i) it has a substantially better fit with the data and (ii) the contribution of investment shocks to the variance of consumption is over three times larger. The contributions to output, investment, and hours, are also relatively higher, suggesting that these shocks may be quantitatively even more important than previous estimates based on the traditional specification.

JEL classification: E2, E3

Key words: Investment shocks, comovement, estimated DSGE models

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1. Introduction

Recent research based on estimated dynamic stochastic general equilibrium (DSGE) models suggests investment shocks are the most important drivers of business cycle fluctuations in the post-World War II US economy. Justiniano et al. (2009a) find, using a model with a variety of real and nominal frictions similar to Christiano et al. (2005) and Smets and Wouters (2007), that over half the fluctuations in output and hours, and over 80% of the fluctuations in investment are driven by investment shocks. These shocks may manifest as shocks either to the marginal efficiency of investment, as in Greenwood et al. (1988), or to the investment-specific technology as in Greenwood et al. (1997), and recent work favours the former interpretation (see Justiniano et al. (2009b)). Previously, using a structural vector autoregression methodology, Fisher (2006) found that investment-specific shocks are the dominant source of business cycles in the US.

Despite their quantitative importance, however, one difficulty remains: consumption typically falls (or does not rise immediately) after a positive investment shock in the model. Thus the model economy does not produce comovement among macroeconomic variables in response to an investment shock, unlike observed business cycles in which consumption, investment, hours, and output all move together. This lack of comovement is clearly problematic in viewing investment shocks as an important source of business cycles. Early work of Barro and King (1984) pointed out this problem in the neoclassical model. Subsequently, Greenwood et al. (1988) showed that incorporating variable capital utilization in that model can introduce a channel which can potentially lead to a rise in current consumption after an investment shock. In DSGE models with real and nominal frictions, the countercyclicality of markups can, in theory, provide yet another channel that can help alleviate the comovement problem (see Justiniano et al. (2009a)). But despite the presence of variable capital utilization and countercyclical markups, it is puzzling that the current generation of estimated DSGE models continue to display the comovement problem in the estimated response of consumption to investment shocks.

In this paper we show that a crucial feature behind this failure is the way the cost of utilization is typically modeled in estimated DSGE models. Following Christiano et al. (2005) (hereafter CEE), the cost of increasing capital utilization enters directly in the household's budget constraint as lost consumption. The reason why the cost of utilization is specified in terms of lost consumption goods is that it allows utilization to rise after an expansionary monetary policy shock (see footnote 20 in Christiano et al. (2001)). This mechanism prevents a sharp rise in marginal cost and limits the extent to which labor productivity falls in response to a positive monetary policy shock, thereby generating persistent output and inflation responses. The motivation behind this modeling choice is, therefore, the need to be consistent with what happens after a monetary policy innovation. Subsequently, the CEE specification has been widely adopted in the estimated DSGE literature (prominent early examples include Smets and Wouters (2003), Altig et al. (2005), and Levin et al. (2005), among others). The downside of specifying the cost of utilization this way, however, is that it shuts down an amplification channel that turns out to be potentially important for the effects of investment shocks. This channel is missing in the CEE specification but it was originally considered by Greenwood et al. (1988) (hereafter GHH) in a neoclassical real business cycle model.

The main objective of this paper is to assess the consequences of the GHH specification and contrast them with those of the traditional CEE specification. To accomplish this task, we estimate an augmented version of the Smets and Wouters (2007) model with the preference structure suggested by Jaimovich and Rebelo (2009), which allows for a varying wealth elasticity of labor supply. This preference structure nests as special cases the standard King et al. (1988) preferences and the one which imply no wealth effect on labor supply (Greenwood et al. (1988)). Jaimovich and Rebelo (2009) show that these preferences help generate comovement in response to anticipated shocks. The advantage of considering these preferences in our context is that it enables us to examine the relative roles of the capital utilization specifications and varying wealth elasticity preferences in generating the comovement result. We use quarterly US data on seven macroeconomic time series over the period 1954:3 - 2004:4 and Bayesian methods to estimate the model and conduct quantitative analysis.

We show that in an estimated DSGE model, when the cost of higher capital utilization is in terms of a higher depreciation rate of capital then comovement occurs. Specifically, after an investment shock, consumption rises along with other macroeconomic variables. The reason behind this finding is that optimal capital utilization under the GHH specification depends on the difference between the rental rate of capital *and* the value of installed capital. A positive investment shock implies that this difference rises and, therefore, boosts utilization. On impact, the fall in the value of installed capital is the primary driver of this difference as the rental rate of capital does not immediately increase. The shock creates strong incentives to build new capital which is more productive relative to current capital stock, therefore, the shadow value of installed capital falls on impact and investment rises. Capital utilization is relatively cheaper which means quicker depreciation of less productive installed capital. It has a direct effect of increasing output on impact. Moreover, capital utilization further amplifies the positive effect of the countercyclical price markup on labor demand, and hence equilibrium hours. The amplification in hump-shaped investment and hours responses leads to an amplification in the output response beyond the first quarter. Consumption rises on impact due to both the larger availability of output on impact and the consumption-smoothing behaviour of the households, thereby ensuring comovement. By contrast, under the CEE specification, utilization depends *only* on the rental rate of capital, and the second amplification channel through the value of installed capital is absent. The GHH specification, therefore, provides useful amplification in an estimated DSGE model and overcomes the comovement problem in response to investment shocks. When the wealth effect on labor supply is absent, the CEE specification generates comovement. The data, however, favours the GHH cost of utilization channel to resolve the comovement problem.

We find that the GHH specification has additional implications. First, the empirical fit of the DSGE model with the GHH specification substantially dominates the one with the CEE specification. Interestingly, the responses of real variables to monetary policy shocks remain broadly similar across the two specifications. Second, the contribution of investment shocks to the unconditional variance of consumption growth is over 3 times larger under GHH relative to CEE. The contributions to the variance of output growth, hours, investment growth, wage growth, nominal interest rate, and inflation are also relatively higher. These findings suggest that investment shocks may be quantitatively even more important than previous estimates based on the CEE specification.

Based on our findings, we conclude that adopting the GHH specification for modeling the cost of utilization in DSGE models may help in better understanding the effects of investment shocks, especially since these shocks appear to be relatively more important than monetary shocks as sources of business cycles.

The rest of the paper is structured as follows. Section 2 describes the model and the two specifications, section 3 presents the estimation methodology, while section 4 presents estimation results and section 5 concludes.

2. The model

We consider a DSGE model that is widely used in the literature following Christiano et al. (2005), Smets and Wouters (2003), and Smets and Wouters (2007). This model has a variety of real and nominal frictions that are helpful in accounting for the conditional responses of macroeconomic variables to unanticipated shocks. The model has households that consume goods and services, supply specialized labor on a monopolistically competitive labor market, rent capital services to firms and make investment decisions. Firms choose the optimal level of labor and capital and supply differentiated products on a monopolistically competitive goods market. Prices and wages are re-optimized at random intervals as in Calvo (1983) and Erceg et al. (2000). When they are not re-optimized, prices and wages are partially indexed to past inflation rates. There are seven types of orthogonal structural shocks: TFP, investment (interpreted either as investment-specific or marginal efficiency of investment), preference, price and wage markups, government spending, and monetary policy.

2.1 Preferences

We introduce the preference structure suggested by Jaimovich and Rebelo (2009) which conveniently nests two special cases which we describe below. The utility function of household $j \in [0,1]$ is

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\varepsilon_{t+s}^b \left(C_{t+s} - \chi L_{t+s}(j)^{1+\sigma_l} X_{t+s}\right)^{1-\sigma_c} - 1}{1-\sigma_c} \tag{1}$$

where

$$X_t = C_t^{\omega} X_{t-1}^{1-\omega}$$

is a geometric average of current and past consumption levels, C_t and X_{t-1}, respectively, and $L_t(j)$ are labor services (hours) supplied to firms in the production sector. The operator E_t denotes expectation conditional on the information available at time t, $0 < \beta < 1$, $\sigma_l > 0$, $\chi > 0$, $\sigma_c > 0$,

$0 \leq \omega \leq 1$ and ε_t^b is the preference shock specified in section 2.4. When $\omega = 1$ the preferences are the same as in King et al. (1988) with the implication that intertemporal substitution effect influences labor effort. When $\omega = 0$ the preferences are the same as in Greenwood et al. (1988), with the implication that intertemporal consumption-saving choice does not affect labor effort.

2.2 Budget constraint, capital accumulation, and the aggregate resource constraint

Under the CEE specification, utilization of capital is costly for the households in terms of consumption units. Utilization of capital, however, does not influence the depreciation rate. Under the GHH specification, the cost of capital utilization is in terms of the depreciation of existing capital. These differences in the two specifications affect three equations in the model: the household's budget constraint, the capital accumulation equation, and the aggregate resource constraint. We present these equations below. The rest of the model is exactly identical under both specifications.

2.2.1 The CEE specification

The budget constraint and the capital accumulation equation are given as

$$C_t + I_t + \frac{B_t}{R_t P_t} - T_t \leq \frac{B_{t-1}}{P_t} + \frac{W_t(j)L_t(j)}{P_t} + \frac{R_t^k Z_t K_{t-1}}{P_t} - a(Z_t)K_{t-1} + \frac{Div_t}{P_t} \tag{2}$$

and

$$K_t = (1-\delta)K_{t-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{3}$$

respectively, where I_t is investment, B_t are nominal government bonds, R_t is the gross nominal interest rate, P_t is the price level, T_t is lump-sum taxes, $W_t(j)$ is the nominal wage, R_t^k is the rental rate on capital, Z_t is the utilization rate of capital, $a(Z_t)$ is an increasing and convex function of the utilization rate, and Div_t the dividends distributed to the households from labor unions. The left hand side of (2) represents real expenditures at time t net of taxes on consumption, investment, and bonds. The right hand side of (2) indicates real receipts from wage income, earnings from supplying capital services net of cost, and dividends. In (3), $S(\frac{I_t}{I_{t-1}})$ is a convex investment adjustment cost function. In the steady state it is assumed that, $S = S' = 0$ and $S'' > 0$.

The aggregate resource constraint is

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t \tag{4}$$

where the term $a(Z_t)K_{t-1}$ indicates the cost of variable utilization in terms of consumption. The definition of the gross domestic product (GDP) is

$$C_t + I_t + G_t = Y_t - a(Z_t)K_{t-1} \equiv X_t \tag{5}$$

2.2.2 The GHH specification

The budget constraint and the capital accumulation equation are given as

$$C_t + I_t + \frac{B_t}{R_t P_t} - T_t \leq \frac{B_{t-1}}{P_t} + \frac{W_t(j)L_t(j)}{P_t} + \frac{R_t^k Z_t K_{t-1}}{P_t} + \frac{Div_t}{P_t} \tag{6}$$

and

$$K_t = (1 - \delta(Z_t))K_{t-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{7}$$

where $0 \leq \delta(Z_t) \leq 1$ shows that the depreciation rate of existing capital depends on its utilization. As in Greenwood et al. (1988), this is a convex function that satisfies the following properties. $\delta^{'} > 0$ indicating that a higher utilization of capital is costly in terms of the depreciation of capital stock, and $\delta^{''} > 0$ indicating that the marginal depreciation rate increases as utilization rises. In the steady state (denoted by a $_*$) we assume, $Z_* = 1$. Using the restriction, $\delta'(Z_*) = r_*^k = R_*^k/P_*$, the second order derivative, δ'' governs the dynamics of utilization in the model around the steady state.[1] Its this additional parameter we estimate in the GHH version.

The aggregate resource constraint is

$$C_t + I_t + G_t = Y_t \tag{8}$$

Note that the cost of capital utilization does not appear explicitly in the aggregate resource constraint and hence $X_t = Y_t$.

[1] The restriction is, $\delta'(Z_*)Q_* = r_*^k$. The value of installed capital in the steady state Q_*=1.

2.3 Optimal utilization of capital

Since the only modification we consider is the cost of utilization, the first-order conditions for optimal utilization of capital under the two specifications are different, and are given by the following equations. Under the CEE specification

$$\frac{R_t^k}{P_t} = a'(Z_t) \tag{9}$$

and under the GHH specification

$$\frac{R_t^k}{P_t} = Q_t \delta'(Z_t) \tag{10}$$

where $Q_t \equiv \frac{\upsilon_t}{\lambda_t}$ is the shadow value of installed capital in consumption units, given by the ratio of the marginal value of installed capital, υ_t, and the marginal value of consumption, λ_t. These variables, λ_t and υ_t, are the Lagrange multipliers associated with (6) and (7) in the household's optimization problem, respectively. As evident from (10), optimal utilization under the GHH depends on Q_t, and this property has important consequences for generating comovement after an investment shock as shown in section 4.

2.4 The log-linearized model

Using the same notation as in Smets and Wouters (2007), we present the log-linearized equations of the model here where lower case letters denote log deviations from steady state values.

From (4), the log-linearized aggregate resource constraint under the CEE specification is

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \tag{11}$$

Output, y_t, is the sum of consumption, c_t, investment, i_t, capital utilization costs, $z_y z_t$, and an government spending disturbance, ε_t^g. The coefficient $c_y = 1 - g_y - i_y$ is the steady state share of consumption in output. The coefficients g_y and i_y are the steady state shares of government spending and investment in output, respectively. The coefficient $z_y = r_*^k k_y$, where k_y is the steady state capital output ratio. These steady state shares are linked to other model parameters, which are shown in Table 1. The government spending disturbance is assumed to follow a first-order autoregressive (AR (1)) process with a mean zero IID normal error term, $\eta^g \sim N(0, \sigma_g)$, where σ_g

is the standard deviation, given as

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g \tag{12}$$

Similarly, from (8) the log-linearized aggregate resource constraint under the GHH specification is

$$y_t = c_y c_t + i_y i_t + \varepsilon_t^g \tag{13}$$

where y_t corresponds to the log-linearized output. The log-linearized equation for GDP under the CEE specification, denoted by χ_t, is given as

$$\chi_t = y_t - z_y z_t = c_y c_t + i_y i_t + \varepsilon_t^g \tag{14}$$

Capital services used in production are a function of capital installed in the previous period, k_{t-1}, and capital utilization, z_t, and given as

$$k_t^s = k_{t-1} + z_t \tag{15}$$

where capital utilization under the CEE specification is a function of the rental rate of capital (log-linearizing (9)),

$$r_t^k = \psi z_t \tag{16}$$

and $\psi = \frac{a''(1)}{a'(1)}$ is a parameter governing the elasticity of capital utilization. Log-linearizing (10) we get the optimal capital utilization under the GHH specification as

$$z_t = \frac{r_*^k}{\delta'' Z_*}(r_t^k - q_t) \tag{17}$$

Log-linearizing (3) we obtain the capital accumulation equation under the CEE specification as

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + \left(1 - \frac{(1-\delta)}{\gamma}\right) i_t + \left(1 - \frac{(1-\delta)}{\gamma}\right) \varepsilon_t^i \tag{18}$$

and log-linearizing (7) we obtain the capital accumulation equation under the GHH specification as

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + \left(1 - \frac{(1-\delta)}{\gamma}\right) i_t + \left(1 - \frac{(1-\delta)}{\gamma}\right) \varepsilon_t^i - \frac{\delta' Z_*}{\gamma} z_t \tag{19}$$

The remaining equations of the model are identical for the two cases. The log-linearized first-order condition for consumption is

$$c_t = E_t c_{t+1} + c_1(r_t - E_t \pi_{t+1}) + c_2 E_t(l_{t+1} - l_t) + c_3 E_t(x_{t+1} - x_t) + c_1(E_t \varepsilon_{t+1}^b - \varepsilon_t^b) \tag{20}$$

where the coefficients c_1 and c_2 depend on the underlying model parameters and the steady state level of hours worked, and $c_3 = c_2(1+\sigma_l)^{-1}$. The expressions for c_1 and c_2 are given in the Appendix. In the equation above, current consumption depends on future expected and past consumption (through the x_t variable), expected hours growth, the real interest rate and the preference shock. The preference shock is assumed to follow a first-order autoregressive (AR (1)) process with a mean zero IID normal error term, $\eta^b \sim N(0, \sigma_b)$, where σ_b is the standard deviation, given as

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b \tag{21}$$

Investment is described by the Euler equation

$$i_t = \frac{1}{1+\beta\gamma^{1-\sigma_c}} \left(i_{t-1} + \beta\gamma^{1-\sigma_c} E_t i_{t+1} + \frac{1}{\gamma^2 \varphi}(q_t + \varepsilon_t^i) \right) \tag{22}$$

where in the above equation, φ is the second derivative of the investment adjustment cost function, $S\left(\frac{I_t}{I_{t-1}}\right)$, evaluated at the steady state. The parameter γ is the common, deterministic, growth rate of output, consumption, investment and real wages. The shock to investment-specific technology, ε_t^i, is assumed to follow a first-order autoregressive (AR(1)) process with a mean zero IID normal error term, $\eta^i \sim N(0, \sigma_i)$, where σ_i is the standard deviation, given as

$$\varepsilon_t^i = \rho_I \varepsilon_{t-1}^i + \eta_t^i \tag{23}$$

The dynamics of the value of capital, q_t, are described by

$$q_t = -(r_t - E_t \pi_{t+1}) + \frac{r_*^k}{(r_*^k + (1-\delta))} E_t r_{t+1}^k + \frac{(1-\delta)}{(r_*^k + (1-\delta))} E_t q_{t+1} \tag{24}$$

where r_t^k denotes the rental rate on capital and δ is the depreciation rate.

The aggregate production function is given by

$$y_t = \phi_p(\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a) \tag{25}$$

That is, output is produced using capital (k_t^s) and labor services (l_t). The parameter ϕ_p is one plus the share of fixed costs in production. The variable ε_t^a is the total factor productivity shock and is assumed to follow a first-order autoregressive (AR (1)) process with a mean zero IID normal error term, $\eta^a \sim N(0, \sigma_a)$, where σ_a is the standard deviation, given as

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a \tag{26}$$

In the goods market, we can define the price markup as,

$$\mu_t^p = \text{mpl}_t - w_t = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t \tag{27}$$

where mpl_t is the marginal product of labor, and w_t is the real wage.

Inflation dynamics are described by the New-Keynesian Phillips curve

$$\pi_t = \pi_1 \pi_{t-1} + +\pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \tag{28}$$

where $\pi_1 = \iota_p/(1+\beta\gamma^{1-\sigma_c}\iota_p)$, $\pi_2 = \beta\gamma^{1-\sigma_c}/(1+\beta\gamma^{1-\sigma_c}\iota_p)$, $\pi_3 = 1/(1+\beta\gamma^{1-\sigma_c}\iota_p)[(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)/\xi_p((\phi_p-1)\varepsilon_p+1)]$. In the notation above $1-\xi_p$ denotes the probability that a given firm will be able to reset its price and ι_p denotes the degree of indexation to past inflation by firms who do not optimally adjust prices. Finally, ε_p is a parameter that governs the curvature of the Kimball goods market aggregator, and $(\phi_p - 1)$ denotes the share of fixed costs in production.[2] The price markup disturbance follows an ARMA(1,1) process given as

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \tag{29}$$

with a mean zero IID normal error term $\eta_t^p \sim N(0, \sigma_p)$ and σ_p is the standard deviation.

Cost minimization by firms implies that the capital-labor ratio is inversely related to the rental rate of capital and positively related to the wage rate.

$$r_t^k = -(k_t^s - l_t) + w_t \tag{30}$$

Similar to the goods market, in the labor market the wage markup is given by

$$\begin{aligned}
\mu_t^w &= w_t - \text{mrs}_t \\
&= w_t - (1-\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega})^{-1}\left((1-\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega}\sigma_l + \chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega})l_t\right) \\
&+ (1-\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega})^{-1}\left((1-\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega} + \chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega})x_t\right) \\
&- (1-\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega})^{-1}\left(\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega}c_t\right)
\end{aligned} \tag{31}$$

Note that the mrs_t expression is implied by the preferences in (1).

[2] The Kimball goods (and labor) market aggregator implies that the demand elasticity of differentiated goods under monopolistic competition depends on their relative price (see Kimball (1995)). This helps obtain plausible duration of price and wage contracts.

The wage inflation dynamics are described by

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \tag{32}$$

where $w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}$ $w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}$, $w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}$, and $w_4 = \frac{(1-\xi_w)(1-\beta\gamma^{1-\sigma_c}\xi_w)}{((1+\beta\gamma^{1-\sigma_c})\xi_w)(1/((\phi_w-1)\varepsilon_w+1))}$ The parameters $(1 - \xi_w)$ and ι_w denote the probability of resetting wages and the degree of indexation to past wages, respectively. σ_l is the elasticity of labor supply with respect to the real wage, and $(\phi_w - 1)$ denotes the steady state labor market markup. Similar to the goods market formulation ε_w denotes the curvature parameter for the Kimball labor market aggregator. The wage mark up disturbance is assumed to follow an ARMA(1,1) process

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{33}$$

where η^w is a mean zero IID normal error term $\eta_t^w \sim N(0, \sigma_w)$ and σ_w is the standard deviation.

The monetary authority follows a generalized Taylor rule

$$r_t = \rho r_{t-1} + (1 - \rho)[r_\pi \pi_t + r_y(y_t - y_t^f)] + r_{\Delta y}[(y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)] + \varepsilon_t^r \tag{34}$$

under the GHH specification and

$$r_t = \rho r_{t-1} + (1 - \rho)[r_\pi \pi_t + r_y(\chi_t - \chi_t^f)] + r_{\Delta y}[(\chi_t - \chi_t^f) - (\chi_{t-1} - \chi_{t-1}^f)] + \varepsilon_t^r \tag{35}$$

under the CEE specification. Note that the terms involving y_t under the GHH specification are replaced by χ_t under the CEE specification to ensure that monetary policy responds to the same concept of output under both cases.

The policy instrument is the nominal interest rate, r_t, which is adjusted gradually in response to inflation and the GDP gap, defined as the difference between GDP and potential GDP, where the latter is the level of GDP that would prevail in equilibrium with flexible prices and in the absence of the two markup shocks. In addition, policy responds to the growth of the GDP gap. The parameter ρ captures the degree of interest rate smoothing. The disturbance ε_t^r is the monetary policy shock and is assumed to follow an AR(1) process with a mean zero IID normal error term, $\eta^r \sim N(0, \sigma_r)$, where σ_r is the standard deviation,

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r \tag{36}$$

3. Estimation methodology and data

In this section we describe the data and the Bayesian estimation methodology used in the empirical analysis.

3.1 Data

We estimate the model using quarterly US data (1954:3 - 2004:4) on output, consumption, inflation, investment, hours worked, wages and the nominal interest rate. All nominal series are expressed in real terms by dividing with the GDP deflator. Moreover, output, consumption, investment and hours worked are expressed in per capita terms by dividing with civilian non-institutional population between 16 and 65. We define nominal consumption as the sum of personal consumption expenditures on nondurable goods and services. As in Justiniano et al. (2009a), we define nominal gross investment as the sum of personal consumption expenditures on durable goods and gross private domestic investment. Real wages are defined as compensation per hour in the non-farm business sector divided by the GDP deflator. Hours worked is the log of hours of all persons in the non-farm business sector, divided by the population. Inflation is measured as the quarterly log difference in the GDP deflator. The nominal interest rate series is the effective Federal Funds rate. All data except the interest rate are in logs and seasonally adjusted. Notice that we do not demean or de-trend the data but estimate a common (deterministic) trend of the trending variables along the balanced growth path of the model.

3.2 Bayesian methodology

We use the Bayesian methodology to estimate a subset of model parameters. This methodology is now extensively used in estimating DSGE models and recent overviews are presented in An and Schorfheide (2007) and Fernández-Villaverde (2009). The key steps in this methodology are as follows. The model presented in the previous sections is solved using standard numerical techniques and the solution is expressed in state-space form as follows:

$$v_t = Av_{t-1} + B\varepsilon_t$$

$$\mathbf{Y}_t = \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix} + \begin{bmatrix} \gamma \\ \gamma \\ \gamma \\ \gamma \\ L_* \\ \overline{\pi} \\ \overline{r} \end{bmatrix}$$

where A and B denote matrices of reduced form coefficients that are non-linear functions of the structural parameters. v_t denotes the vector of model variables, ε_t the vector of exogenous disturbances and

$$\mathbf{Y}_t = [\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta \log \frac{W_t}{P_t}, \log L_t, \pi_t, R_t]$$

the vector of observable variables at time t to be used in the estimation below, where Δ denotes the first-difference operator. Note that when estimating the CEE specification we replace $\Delta \log Y_t$ with $\Delta \log X_t$ in the vector of observable variables, i.e. we use the model's concept of GDP. Let Θ denote the vector that contains all the structural parameters of the model. The non-sample information is summarized with a prior distribution with density $p(\Theta)$.[3] The sample information (conditional on version $\mathcal{M}_i$ of the DSGE model) is contained in the likelihood function, $p(\mathbf{Y}_T|\Theta, \mathcal{M}_i)$, where $\mathbf{Y}_T = [Y_1, ..., Y_T]'$ contains the data. The likelihood function allows one to update the prior distribution of Θ, $p(\Theta)$. Then, using Bayes' theorem, we can express the posterior distribution of the parameters as

$$p(\Theta|\mathbf{Y}_T, \mathcal{M}_i) = \frac{p(\mathbf{Y}_T|\Theta, \mathcal{M}_i)p(\Theta)}{p(\mathbf{Y}_T|\mathcal{M}_i)} \tag{37}$$

where the denominator, $p(\mathbf{Y}_T|\mathcal{M}_i) = \int p(\Theta, \mathbf{Y}_T|\mathcal{M}_i)d\Theta$, in (37) is the marginal data density conditional on model $\mathcal{M}_i$. In Bayesian analysis the marginal data density constitutes a measure of model fit with two dimensions: goodness of in-sample fit and a penalty for model complexity. The posterior distribution of parameters is evaluated numerically using the random walk Metropolis-Hastings algorithm. We simulate the posterior using a sample of 2,000,000 draws and use this (after dropping the first 20% of the draws) to (i) report the mean, and the 10 and 90 percentiles of the

[3] We assume that parameters are *a priori* independent from each other. This is a widely used assumption in the applied DSGE literature and implies the joint prior distribution equals the product of marginal priors.

posterior distribution of the estimated parameters and (ii) evaluate the marginal likelihood of the model.[4] All estimations are done using DYNARE.[5]

3.3 Prior distribution

Tables 1 and 2 lists the choice of priors for the GHH and CEE specifications, respectively. We use prior distributions that conform to the assumptions used in Smets and Wouters (2007) and Justiniano et al. (2009a). A number of parameters are held fixed prior to estimation. The curvature parameters for the Kimball goods and labor market aggregators, ε_p, and ε_w, are both set equal to 10 and the steady state labor market markup, ϕ_w, is set at 1.5 as in Smets and Wouters (2007). We set the capital share parameter in production, α, equal to 0.3, and the steady state government spending to output ratio equal to 0.22, the average value in the sample. Finally, we normalize χ in the utility function equal to one. In the CEE specification, we set the depreciation rate for capital, δ, equal to 0.025 a value conventional at the quarterly frequency.

The first five columns in Tables 1 and 2 list the parameters and the assumptions on the prior distributions. All parameters of the model are the same under the CEE and the GHH specifications except for one: the capital utilization elasticity.

Under the GHH specification, capital depreciation is endogenous, and δ'' determines the capital utilization elasticity, which we estimate. We set the mean of the prior distribution for this parameter at 0.15 with a standard deviation of 0.10. Under the CEE specification, parameter ψ governs this elasticity. The prior mean we assume for δ'' implies a relatively low elasticity of capital utilization with respect to the rental rate, equal to 0.22. Similarly under the CEE specification, the prior mean of ψ is 5 which implies an elasticity of 0.20.

4. Results

In this section we present the parameter estimates, variance decompositions, and the impulse responses to investment and monetary policy shocks as well as two experiments designed to clarify the role of JR preferences versus GHH utilization for the comovement result.

[4]We also calculate convergence diagnostics in order to check and ensure the stability of the posterior distributions of parameters as described in Brooks and Gelman (1998).

[5]http://www.cepremap.cnrs.fr/dynare/. The replication files are available upon request.

4.1 Parameter estimates

It is evident from comparing Tables 1 (column 6) and 2 (column 6) that most parameters estimates are similar across the two specifications. There are, however, some differences which we highlight here. First, the parameter that governs capital utilization elasticity under the GHH specification, δ'' is 0.08. This implies an elasticity of capacity utilization equal to 0.47. By contrast, under the CEE specification, $\psi = 4.2$ which implies an elasticity of 0.24, approximately half of the GHH specification. Second, the estimated wealth elasticity parameter is smaller under the GHH (0.53) relative to the CEE (0.81) specification, but still higher than the prior mean of 0.5. For both specifications, therefore, the data seem to support an intermediate preference structure with consumption-saving choice having an influence on labor supply. Third, the Calvo price and wage stickiness parameters imply a slightly longer contract durations under the GHH specification relative to the CEE specification. Finally, estimates of the investment adjustment cost parameter, ϕ are slightly larger under the GHH specification.

4.2 Variance decompositions

Table 3 presents the contribution of each shock to the unconditional variance of the seven observed variables, along with the 10-90 percentile intervals. There are four notable findings. First, the contribution of investment shocks to *all* the seven data series is higher under the GHH specification (Panel A) relative to the CEE specification (Panel B). In particular, the variance shares in output growth (72.44%) and hours (38.80%) are substantially higher under the GHH specification relative to the CEE specification, which are equal to 59.10% and 26.65%, respectively. Second, the contribution to the variance of consumption growth under the GHH specification is over 3 times that under the CEE specification. It is useful to note that, in the literature, the estimates of the variance share of consumption accounted for by investment shocks are typically below 4% (see, for example, Justiniano et al. (2009a)). The contributions to the variance of investment growth, wage growth, nominal interest rate, and inflation are also relatively higher. Third, the contributions of TFP shocks to the variance of five of the seven data series are smaller under the GHH specification. In particular, the contribution to output growth falls from 15.5% under CEE specification to below 9% under the GHH specification. TFP shocks, therefore, turn out to be even less important than previously estimated in DSGE models. Fourth, the quantitative significance of monetary shocks, however, is small and similar across both specifications. Their contribution to the variance of investment growth, hours, wage growth, nominal interest rate and inflation is below 5%, and to the variance in consumption and output growth is ranked behind TFP shocks. The key implication of these findings is that investment shocks may play an even bigger role as drivers of business cycles than suggested by previous estimates (see, for example, Justiniano et al. (2009a)).[6]

4.3 Impulse responses

Figure 1 displays the impulse responses to an investment shock. First note that consumption rises upon impact under the GHH specification (solid line) and displays comovement with output, investment, and hours. By contrast, under the CEE specification consumption is initially close to zero (in fact it is slightly below zero).[7] A similar negative impact response of consumption is obtained, for example, when investment shock is interpreted exclusively as a shock to the marginal efficiency of investment (see, for example, Figure 2 in Justiniano et al. (2009b)).

A positive investment shock implies a lower cost of depreciation in consumption units and thus a larger response of utilization to an investment shock. To see the intuition, rewriting (22) as

$$q_t = -\varepsilon_t^i + \varphi\gamma^2 \left[(1+\beta\gamma^{1-\sigma_c})i_t - i_{t-1} - \beta\gamma^{1-\sigma_c} E_t i_{t+1}\right] \tag{38}$$

The shadow value of installed capital depends inversely on the investment shock, and combining (38) with (17) to get

$$z_t = \frac{r_*^k}{\delta'' Z_*}(r_t^k - q_t) = \frac{r_*^k}{\delta'' Z_*}\left(r_t^k + \varepsilon_t^i - \varphi\gamma^2 \left[(1+\beta\gamma^{1-\sigma_c})i_t - i_{t-1} - \beta\gamma^{1-\sigma_c} E_t i_{t+1}\right]\right) \tag{39}$$

From (39) we see that ε_t^i has a contemporaneous *direct* effect on capital utilization, in addition to the usual *indirect* effect through r_t^k. Under the CEE specification (16), however, only the latter effect is present. The difference between the rental rate of capital and the shadow value of capital

[6]Note that, as Justiniano et al. (2009b) show, when the relative price of investment is included in the estimation as an additional observable series to identify investment-specific shocks, the shocks to marginal efficiency of investment dominate the shocks to investment-specific technology in accounting for the variance shares of output, investment, and hours. In the present set up, we use this broad interpretation of the investment shock and do not distinguish between these two types of shocks.

[7]Output under the CEE specification corresponds to GDP in model.

rises after an investment shock as the value of installed capital declines, as shown in Figure 1. This feature allows utilization to respond more strongly to an investment shock under the GHH specification compared to that under the CEE specification and is key to generating comovement. The positive (and persistent) investment shock creates strong incentives to build new capital that is more productive relative to current capital stock, therefore, the shadow value of installed capital q_t falls on impact. Since installed capital is less valuable, it is cheaper to increase the rate of utilization and depreciate the current capital stock. The rental rate on capital rises slowly, contributing to increased utilization of capital.[8]

The positive investment shock, in the presence of investment adjustment costs, translates into a hump-shaped response for investment. The amplification under the GHH specification relative to CEE, as reflected in the gap between the two impulse responses, occurs because higher capital utilization quickly depreciates the less productive capital stock and raises the marginal productivity of future capital, inducing higher investment.

The response of hours on impact under both specifications is similar. The reason is as follows. The wage markup and the price markup (inverse of marginal cost) are relatively more countercyclical on impact under the CEE specification, as shown in Figure 1. These markups induce rightward shifts in implicit labor demand and labor supply schedules, respectively, and should cause equilibrium hours to rise relatively more under the CEE specification. The utilization of capital, which amplifies the rightward shift in the labor demand schedule, is, however, relatively stronger under the GHH, making up for the relatively weaker impact response of the two markups. The equilibrium response of hours on impact is, therefore, similar. Beyond two quarters, the effects of price and wage markups, and capital utilization imply a relatively more amplified response of hours under the GHH specification.

The response of output on impact under GHH is slightly larger than CEE (despite the similar hours response) due to the direct effect of higher utilization of installed capital. In the subsequent quarters the output response is further amplified due to the relatively larger response of hours,

[8] As seen in Figure 1, the rental rate on capital initially falls on impact under the GHH specification. This occurs because, in equilibrium, the strong increase in current utilization raises the capital services-hours ratio, $(k_t^s - l_t)$ relatively more than the wage. From (30), it is clear that this can reduce r_t^k. The fall in q_t, however, ensures that $(r_t^k - q_t)$ remains positive on impact. Under the CEE specification, the response of utilization is not strong enough to raise $(k_t^s - l_t)$, therefore, the rental rate on capital rises on impact.

capital utilization, and capital accumulation (not shown). The output and hours responses are consistent with the relatively higher labor productivity response under the GHH specification. Consumption rises on impact due to both the larger availability of output on impact and the consumption-smoothing behaviour of the households, thereby ensuring comovement.

One way to avoid the comovement problem is to consider capital depreciation shocks as in Liu et al. (2009). Although this prevents consumption from moving in the opposite direction from output, hours, and investment, the response of consumption at impact still remains quite close to zero (see, for example, Figure 2 in Liu et al. (2009)). By contrast, the GHH specification we examine generates a relatively strong response of consumption upon impact, and has additional quantitative implications as discussed in section 4.2.

Figure 2 displays the impulse responses to a contractionary monetary policy shock. The responses of output, hours, and wages are very similar under both the GHH and the CEE specifications. Investment also displays a hump-shaped response under both specifications due to the presence of investment adjustment costs, the response under the CEE specification is, however, more amplified. Capital utilization initially rises under the GHH specification. The reason is that the shadow value of capital falls relatively more than the rental rate on capital, pushing utilization up initially. Under the CEE specification, the cost of utilization falls with the fall in the rental rate on capital. This difference in the response of utilization implies that the initial response of marginal cost is slightly muted under the CEE specification. Since marginal cost falls under both, the implied reduction in the inflation rate, however, is similar across the two specifications.

4.3.1 Wealth effects and comovement

We examine how the strength of the wealth effect on labor supply influences the response of consumption under the GHH and CEE specifications. We first consider the case where the preferences imply the maximum possible wealth effect (setting $\omega = 1$). Figure 3 shows that the GHH specification produces comovement whereas the CEE specification does not. Under the latter, the stronger wealth effect implies slightly lower equilibrium hours and output which in turn leads to a further decline in consumption on impact, relative to the baseline case (Figure 1).

Next, we consider the case where the preferences imply no wealth effect on labor (setting

$\omega = 0$). As shown in Figure 4, consumption is no longer negative on impact under the CEE specification. Both price and wage markups are relatively less countercyclical which offset the positive impact on hours due to the absence of the wealth effect, relative to the baseline case. The output response is the same compared to the baseline case. The source of the positive impact on consumption, therefore, comes not from higher equilibrium hours (hence output) but from a slightly lower investment response. This is driven by the lower value of capital relative to the baseline case (see Figure 1).

These experiments reveal that the GHH specification for capital utilization produces comovement independent of the wealth effect on labor supply. On the other hand, the success of the CEE specification in generating comovement depends on the assumed strength of the wealth effect. Since the data indicates positive wealth effects (estimated $\omega > 0.5$) the GHH specification is preferable for generating comovement.

4.4 Model fit

We can compare the fit of the CEE and GHH specifications using the log marginal densities, $\ln(p(\mathbf{Y}_T|\mathcal{M}_i))$, i = CEE, GHH. We find $\ln(p(\mathbf{Y}_T|\mathcal{M}_{CEE}))$ = -1306.02 and $\ln(p(\mathbf{Y}_T|\mathcal{M}_{GHH}))$ = -1290.25. These values imply a large Bayes factor in favour of the GHH specification indicating its superior fit with the data relative to the CEE specification.[9]

5. Conclusion

Recent literature based on estimated DSGE models suggests investment shocks are the most important drivers of business cycles. But it is puzzling that despite the presence of capital utilization and countercyclical markups the comovement problem persists: consumption typically falls (or does not rise) after the investment shock, unlike observed business cycles where consumption, hours, investment all move with economic activity. We show that a source of this shortcoming is the way in which the cost of capital utilization is typically modeled. Traditionally, in the estimated DSGE literature, the cost of utilization has been specified in terms of foregone consumption following

[9]The log marginal data densities are computed using the modified harmonic mean estimator suggested by Geweke (1999).

Christiano et al. (2005), who studied the effects of monetary shocks. We find that when the cost of capital utilization manifests as higher depreciation of capital, as originally proposed by Greenwood et al. (1988) in a neoclassical setting, investment shocks produce comovement. When preferences are restricted to have no wealth effect on labor supply, the traditional specification displays comovement but that version is not supported by the data. The alternative specification has two additional implications relative to the traditional one. First, the fit of the estimated DSGE model is superior. Second, the contributions of investment shocks to the variance of output, consumption, hours, and investment are higher. In particular, the contribution to the variance of consumption is over 3 times that under the traditional specification. The alternative specification, therefore, reveals that investment shocks may be quantitatively even more important drivers of business cycles than previously estimated assuming the traditional specification. Based on our findings, we conclude that adopting the alternative specification for modeling the cost of utilization in DSGE models may be helpful in better understanding the effects of investment shocks, especially since these shocks appear to be relatively more important than monetary policy shocks as sources of business cycles.

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A. Appendix

We present the first-order conditions of the household's problem in the model of section 2. Household j maximizes the following objective function

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\varepsilon_t^b \left(C_t - \chi L_t^{1+\sigma_l} X_t\right)^{1-\sigma_c} - 1}{1-\sigma_c}$$

with $X_t = C_t^{\omega} X_{t-1}^{1-\omega}$, subject to the budget constraint, and the capital accumulation equation,
Under CEE

$$C_t + I_t + \frac{B_t}{R_t P_t} - T_t \leq \frac{B_{t-1}}{P_t} + \frac{W_t(j) L_t(j)}{P_t} + \frac{R_t^k Z_t K_{t-1}}{P_t} - a(Z_t) K_{t-1} + \frac{Div_t}{P_t} \tag{A.1}$$

$$K_t = (1-\delta) K_{t-1} + \varepsilon_t^i \left[1 - S(\frac{I_t}{I_{t-1}})\right] I_t \tag{A.2}$$

Under GHH

$$C_t + I_t + \frac{B_t}{R_t P_t} - T_t \leq \frac{B_{t-1}}{P_t} + \frac{W_t^h(j) L_t(j)}{P_t} + \frac{R_t^k Z_t K_{t-1}}{P_t} + \frac{Div_t}{P_t} \tag{A.3}$$

$$K_t = (1-\delta(Z_t)) K_{t-1} + \varepsilon_t^i \left[1 - S(\frac{I_t}{I_{t-1}})\right] I_t \tag{A.4}$$

where C_t is consumption, I_t is investment, B_t are nominal government bonds, $W_t(j)$ is the nominal wage, R_t is the gross nominal interest rate, T_t is lump-sum taxes, R_t^k is the rental rate on capital, Z_t is the utilization rate of capital, $a(Z_t)$ is a convex function of the utilization rate and Div_t the dividends distributed to the households from labor unions. $S(\frac{I_t}{I_{t-1}})$ is a convex investment adjustment cost function. In the steady state it is assumed that, $S = S' = 0$ and $S'' > 0$. Let λ_t, υ_t denote the lagrange multipliers associated with (A.1) and (A.2) respectively. Using the fact that in equilibrium all households make the same decisions for the variables, the FOCs for this problem (dropping the j index) are given as

CEE specification,

$$\text{Consumption} : \lambda_t = \left(C_t - \chi L_t^{1+\sigma_l} X_t\right)^{-\sigma_c} \left(1 - \chi \omega L_t^{1+\sigma_l} C_t^{\omega-1} X_{t-1}^{1-\omega}\right) \varepsilon_t^b \tag{A.5}$$

$$\text{Hours}: \lambda_t \frac{W_t}{P_t} = \left(C_t - \chi L_t^{1+\sigma_l} X_t\right)^{-\sigma_c} \chi(1+\sigma_l) L_t^{\sigma_l} X_t \varepsilon_t^b \tag{A.6}$$

$$\text{Bonds}: \lambda_t = \beta R_t E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{A.7}$$

$$\text{Investment}: \lambda_t = \upsilon_t \varepsilon_t^i \left(1 - S(\frac{I_t}{I_{t-1}}) - S'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}\right) + \beta E_t \left(\upsilon_{t+1} \varepsilon_{t+1}^i S'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2\right) \tag{A.8}$$

$$\text{Capital}: \upsilon_t = \beta E_t \left(\lambda_{t+1}\left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a(Z_{t+1})\right) + \upsilon_{t+1}(1-\delta)\right) \tag{A.9}$$

$$\text{Capital Utilization}: \frac{R_t^k}{P_t} = a'(Z_t) \tag{A.10}$$

GHH specification

$$\text{Consumption}: \lambda_t = \left(C_t - \chi L_t^{1+\sigma_l} X_t\right)^{-\sigma_c} \left(1 - \chi\omega L_t^{1+\sigma_l} C_t^{\omega-1} X_{t-1}^{1-\omega}\right) \varepsilon_t^b \tag{A.11}$$

$$\text{Hours}: \lambda_t \frac{W_t}{P_t} = \left(C_t - \chi L_t^{1+\sigma_l} X_t\right)^{-\sigma_c} \chi(1+\sigma_l) L_t^{\sigma_l} X_t \varepsilon_t^b \tag{A.12}$$

$$\text{Bonds}: \lambda_t = \beta R_t E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{A.13}$$

$$\text{Investment}: \lambda_t = \upsilon_t \varepsilon_t^i \left(1 - S(\frac{I_t}{I_{t-1}}) - S'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}\right) + \beta E_t \left(\upsilon_{t+1} \varepsilon_{t+1}^i S'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2\right) \tag{A.14}$$

$$\text{Capital}: \upsilon_t = \beta E_t \left(\lambda_{t+1}\left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1}\right) + \upsilon_{t+1}(1-\delta(Z_{t+1}))\right) \tag{A.15}$$

$$\text{Capital Utilization}: \frac{R_t^k}{P_t} = \frac{\upsilon_t}{\lambda_t}\delta'(Z_t) \tag{A.16}$$

Using (A.11) in (A.13), and log-linearizing around the steady state, we obtain

$$c_t = E_t c_{t+1} + c_1(r_t - E_t \pi_{t+1}) + c_2 E_t(l_{t+1} - l_t) + c_3 E_t(x_{t+1} - x_t) + c_1(E_t \varepsilon_{t+1}^b - \varepsilon_t^b) \tag{A.17}$$

where the expressions for coefficients c_1 and c_2 in (A.17) are given as

$$c_1 = \frac{1-\chi\omega L_*^{(1+\sigma_l)}\gamma^{(\omega-1)/\omega}}{-\sigma_c(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}+\chi\omega L_*^{1+\sigma_l}\sigma_c\gamma^{(\omega-1)/\omega}(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}+\chi\omega L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega}} \text{ and}$$

$$c_2 = \frac{\chi L_*^{1+\sigma_l}\sigma_c(1+\sigma_l)\gamma^{(\omega-1)/\omega}(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}-\chi\omega(1+\sigma_l)L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega}}{-\sigma_c(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}+\chi\omega L_*^{1+\sigma_l}\sigma_c\gamma^{(\omega-1)/\omega}(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}+\chi\omega L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega}} -$$
$$- \frac{\chi^2\omega L_*^{2(1+\sigma_l)}\sigma_c(1+\sigma_l)\gamma^{2(\omega-1)/\omega}(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}}{-\sigma_c(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}+\chi\omega L_*^{1+\sigma_l}\sigma_c\gamma^{(\omega-1)/\omega}(1-\chi L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega})^{-1}+\chi\omega L_*^{1+\sigma_l}\gamma^{(\omega-1)/\omega}},$$

and

$$c_3 = c_2(1+\sigma_l)^{-1}.$$

Table 1: Prior and Posterior distributions: GHH specification

	Prior distribution				Posterior distribution		
	Description	Distr.	Mean	Std.dev.	Mean	10%	90%
σ_c	Inverse intertemporal elasticity	Normal	1.0	0.37	2.45	2.18	2.72
ω	Wealth elasticity	Beta	0.5	0.20	0.53	0.32	0.74
ξ_w	Calvo wages	Beta	0.66	0.10	0.82	0.73	0.93
σ_l	Inverse labor elasticity	Gamma	2.00	0.75	0.65	0.18	1.12
ξ_p	Calvo prices	Beta	0.66	0.10	0.77	0.71	0.84
ι_w	Wage indexation	Beta	0.50	0.15	0.52	0.31	0.72
ι_p	Price indexation	Beta	0.50	0.15	0.21	0.10	0.32
δ''	Capital utilization elasticity	Gamma	0.15	0.10	0.08	0.04	0.11
Φ	Fixed cost share	Normal	1.25	0.12	1.32	1.19	1.45
r_π	Taylor rule inflation	Normal	1.70	0.30	2.11	1.80	2.42
ρ	Taylor rule smoothing	Beta	0.60	0.20	0.83	0.79	0.87
r_y	Taylor rule GDP gap	Normal	0.12	0.05	0.13	0.09	0.16
$r_{\Delta y}$	Taylor rule GDP gap growth	Normal	0.12	0.05	0.30	0.24	0.35
φ	Investment adjustment cost	Gamma	4.00	1.0	2.63	1.76	3.45
$\overline{\pi}$	SS Quarterly inflation	Normal	0.5	0.10	0.64	0.47	0.81
γ	Deterministic technology growth	Normal	0.5	0.03	0.47	0.44	0.49
$100(\beta^{-1}-1)$	Discount factor	Gamma	0.25	0.10	0.12	0.05	0.19
L_*	Steady state hours	Normal	0.00	0.50	0.45	0.40	0.52
ρ_a	Neutral technology	Beta	0.60	0.20	0.96	0.95	0.98
ρ_b	Preference	Beta	0.60	0.20	0.92	0.87	0.96
ρ_g	Government spending	Beta	0.60	0.20	0.99	0.98	0.99
ρ_I	Investment	Beta	0.60	0.20	0.72	0.65	0.79
ρ_r	Monetary policy	Beta	0.40	0.20	0.05	0.01	0.09
ρ_p	Price markup	Beta	0.60	0.20	0.98	0.96	0.99
ρ_w	Wage markup	Beta	0.60	0.20	0.92	0.81	0.99
μ_p	Price markup MA	Beta	0.50	0.20	0.89	0.83	0.96
μ_w	Wage markup MA	Beta	0.50	0.20	0.88	0.75	0.98
σ_a	Neutral technology	InvGamma	0.5	2.0	0.54	0.48	0.59
σ_g	Government spending	InvGamma	0.5	2.0	0.34	0.31	0.37
σ_b	Preference	InvGamma	0.1	2.0	2.87	2.01	3.71
σ_I	Investment	InvGamma	0.5	2.0	6.50	4.89	8.04

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Table 1 – continued from previous page

	Prior distribution				Posterior distribution		
σ_r	Monetary policy	InvGamma	0.1	2.0	0.24	0.22	0.27
σ_p	Price markup	InvGamma	0.1	2.0	0.12	0.10	0.15
σ_w	Wage markup	InvGamma	0.1	2.0	0.25	0.22	0.29

Notes. Posterior distributions are obtained via the Metropolis-Hastings algorithm using 2,000,000 draws. The first 400,000 draws are discarded.

Table 2: Prior and Posterior distributions: CEE specification

	Prior distribution				**Posterior distribution**		
	Description	Distr.	Mean	Std.dev.	Mean	10%	90%
σ_c	Inverse intertemporal elasticity	Normal	1.0	0.37	2.23	1.96	2.15
ω	Wealth elasticity	Beta	0.5	0.20	0.81	0.69	0.95
ξ_w	Calvo wages	Beta	0.66	0.10	0.74	0.67	0.81
σ_l	Inverse labor elasticity	Gamma	2.00	0.75	0.83	0.28	1.36
ξ_p	Calvo prices	Beta	0.66	0.10	0.66	0.58	0.73
ι_w	Wage indexation	Beta	0.50	0.15	0.53	0.33	0.73
ι_p	Price indexation	Beta	0.50	0.15	0.20	0.09	0.32
ψ	Capital utilization elasticity	Gamma	5.00	1.00	4.20	2.58	5.75
Φ	Fixed cost share	Normal	1.25	0.12	1.45	1.33	1.57
r_π	Taylor rule inflation	Normal	1.70	0.30	2.17	1.86	2.46
ρ	Taylor rule smoothing	Beta	0.60	0.20	0.86	0.83	0.89
r_y	Taylor rule GDP gap	Normal	0.12	0.05	0.16	0.11	0.20
$r_{\Delta y}$	Taylor rule GDP gap growth	Normal	0.12	0.05	0.29	0.25	0.34
φ	Investment adjustment cost	Gamma	4.00	1.0	1.78	1.22	2.33
$\overline{\pi}$	SS Quarterly inflation	Normal	0.5	0.10	0.56	0.40	0.73
γ	Deterministic technology growth	Normal	0.5	0.03	0.47	0.45	0.50
$100(\beta^{-1}-1)$	Discount factor	Gamma	0.25	0.10	0.16	0.07	0.25
L_*	Steady state hours	Normal	0.00	0.50	0.36	0.27	0.45
ρ_a	Neutral technology	Beta	0.60	0.20	0.95	0.94	0.97
ρ_b	Preference	Beta	0.60	0.20	0.94	0.90	0.98
ρ_g	Government spending	Beta	0.60	0.20	0.98	0.97	0.99
ρ_I	Investment	Beta	0.60	0.20	0.70	0.62	0.77
ρ_r	Monetary policy	Beta	0.40	0.20	0.06	0.01	0.11

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Table 2 – continued from previous page

	Prior distribution				**Posterior distribution**		
ρ_p	Price markup	Beta	0.60	0.20	0.98	0.97	0.99
ρ_w	Wage markup	Beta	0.60	0.20	0.98	0.96	0.99
μ_p	Price markup MA	Beta	0.50	0.20	0.84	0.75	0.93
μ_w	Wage markup MA	Beta	0.50	0.20	0.95	0.93	0.98
σ_a	Neutral technology	InvGamma	0.5	2.0	0.53	0.48	0.58
σ_g	Government spending	InvGamma	0.5	2.0	0.34	0.31	0.37
σ_b	Preference	InvGamma	0.1	2.0	3.07	1.86	4.34
σ_I	Investment	InvGamma	0.5	2.0	4.87	3.77	5.94
σ_r	Monetary policy	InvGamma	0.1	2.0	0.24	0.22	0.27
σ_p	Price markup	InvGamma	0.1	2.0	0.12	0.10	0.15
σ_w	Wage markup	InvGamma	0.1	2.0	0.28	0.25	0.31

Notes. Posterior distributions are obtained via the Metropolis-Hastings algorithm using 2,000,000 draws. The first 400,000 draws are discarded.

Table 3: Contribution of each shock to the unconditional variance of observable variables (in %): median and 10-90% percentiles (in square brackets)

Variable	ε^a	ε^i	ε^b	ε^g	ε^r	ε^p	ε^w
A. GHH specification							
Output growth	8.95	72.44	4.78	3.25	7.00	2.00	1.20
	[6.78, 11.65]	[64.85, 78.10]	[2.60, 7.90]	[2.30, 4.45]	[5.65,8.73]	[1.00, 3.90]	[0.63, 2.21]
Consumption growth	19.30	12.31	39.30	3.30	17.00	2.35	4.62
	[14.56, 24.45]	[6.10, 22.50]	[25.90,49.60]	[1.80, 5.75]	[13.90, 20.80]	[0.94, 5.35]	[2.80, 7.23]
Investment growth	3.00	90.80	2.50	0.00	1.90	1.10	0.60
	[2.20, 4.05]	[88.30, 92.40]	[1.85, 3.20]	[0.00, 0.03]	[1.55, 2.35]	[0.50, 2.17]	[0.40, 1.10]
Hours	3.65	38.80	1.30	11.83	4.35	17.55	16.00
	[2.30, 5.95]	[25.20, 52.63]	[0.70, 2.30]	[6.15, 23.50]	[2.60, 6.80]	[7.90, 35.40]	[9.00, 27.05]
Wage growth	5.27	5.40	0.30	0.00	0.35	19.40	68.35
	[3.27, 7.52]	[2.00, 9.82]	[0.00, 0.60]	[0.00, 0.00]	[0.05, 0.90]	[13.70, 28.00]	[57.60, 78.00]
Nominal interest rate	9.03	42.20	5.20	0.85	3.25	24.20	10.90
	[5.25, 13.70]	[22.75, 57.85]	[2.90, 8.10]	[0.45, 1.45]	[1.85, 5.25]	[7.12, 55.00]	[5.60, 18.50]
Inflation	4.45	2.63	1.00	0.30	0.90	62.50	26.50
	[1.81, 8.15]	[0.90, 5.86]	[0.42, 2.10]	[0.12, 0.58]	[0.23, 2.45]	[40.70, 83.35]	[11.27, 44.75]
B. CEE specification							
Output growth	15.50	59.10	7.60	3.65	10.10	2.10	1.50
	[12.50, 19.50]	[51.20, 66.30]	[3.85, 12.10]	[2.70, 4.80]	[8.40,11.95]	[1.20, 3.45]	[0.90, 2.40]
Consumption growth	25.00	3.60	43.65	4.05	13.30	2.40	7.35
	[19.15, 33.23]	[2.15, 5.80]	[25.40,56.60]	[2.45, 6.62]	[9.65, 17.75]	[1.10, 4.70]	[4.50, 11.87]
Investment growth	4.25	82.70	7.60	0.00	3.45	1.00	0.90
	[3.30, 5.70]	[79.10, 86.10]	[4.30, 10.23]	[0.00, 0.00]	[2.80, 4.25]	[0.50, 1.74]	[0.55, 1.45]
Hours	3.70	26.65	4.30	10.10	4.05	15.45	28.68
	[2.23, 5.60]	[16.10, 38.62]	[2.55, 6.25]	[5.10, 19.80]	[2.44, 6.50]	[7.85, 29.30]	[17.60, 47.40]
Wage growth	7.95	4.30	1.00	0.00	0.60	20.32	65.10
	[5.83, 10.25]	[2.72, 6.40]	[0.70, 1.56]	[0.00, 0.00]	[0.30, 1.10]	[14.90, 27.95]	[56.90, 71.30]
Nominal interest rate	5.27	25.00	9.60	0.40	1.85	27.90	23.42
	[2.70, 8.00]	[10.87, 43.30]	[4.90,14.90]	[0.10, 0.60]	[0.90, 3.00]	[9.75, 54.95]	[13.53, 39.10]
Inflation	2.65	2.22	1.77	0.15	1.15	51.05	38.60
	[1.13, 5.05]	[0.80, 5.24]	[0.80, 3.70]	[0.00, 0.25]	[0.45, 2.55]	[30.75, 73.12]	[20.84, 54.50]

Notes. ε^a = Total factor productivity shock, ε^i = Investment shock, ε^b = preference shock, ε^g = government spending shock, ε^r = monetary policy shock, ε^p = price markup shock, ε^w = wage markup shock. Entries decompose the forecast error variance in each variable into percentages due to each shock.

Figure 1: Impulse responses (median) to an investment shock (solid line is GHH specification and dotted line is CEE specification)

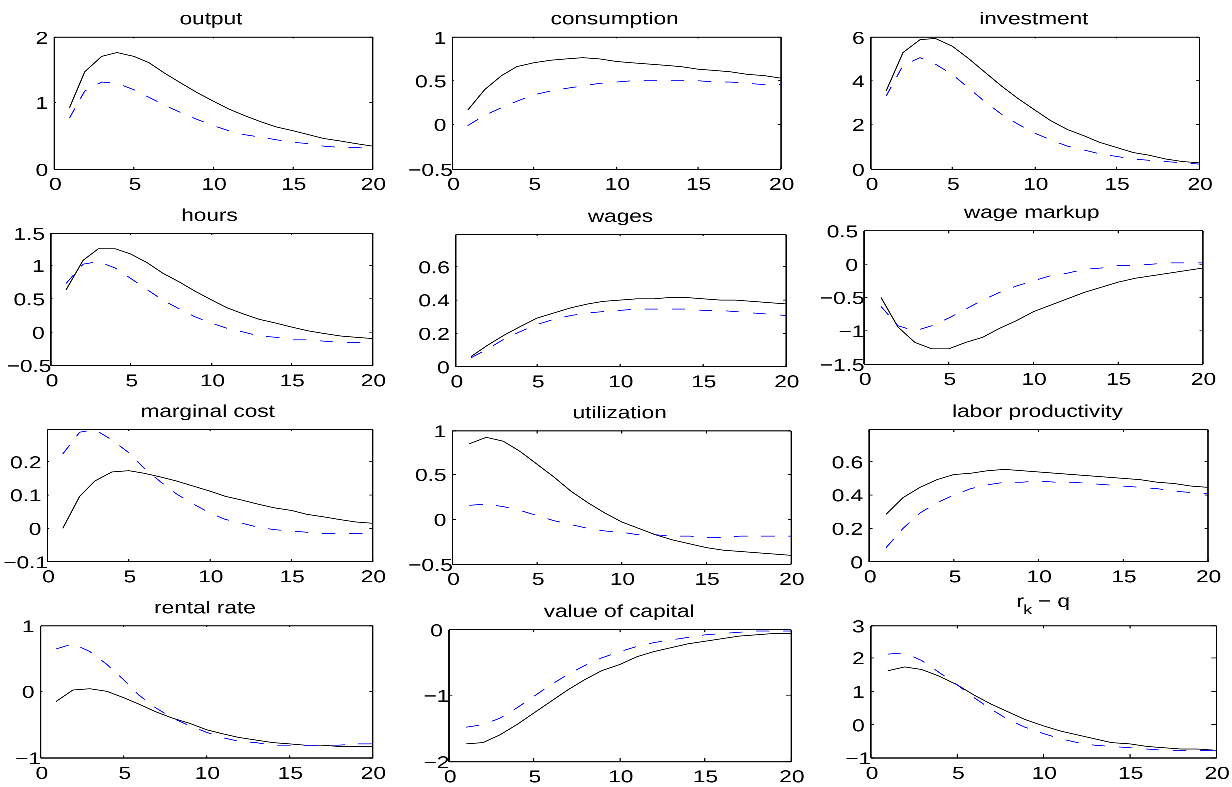

Figure 2: Impulse responses (median) to a contractionary monetary policy shock (solid line is GHH specification and dotted line is CEE specification),

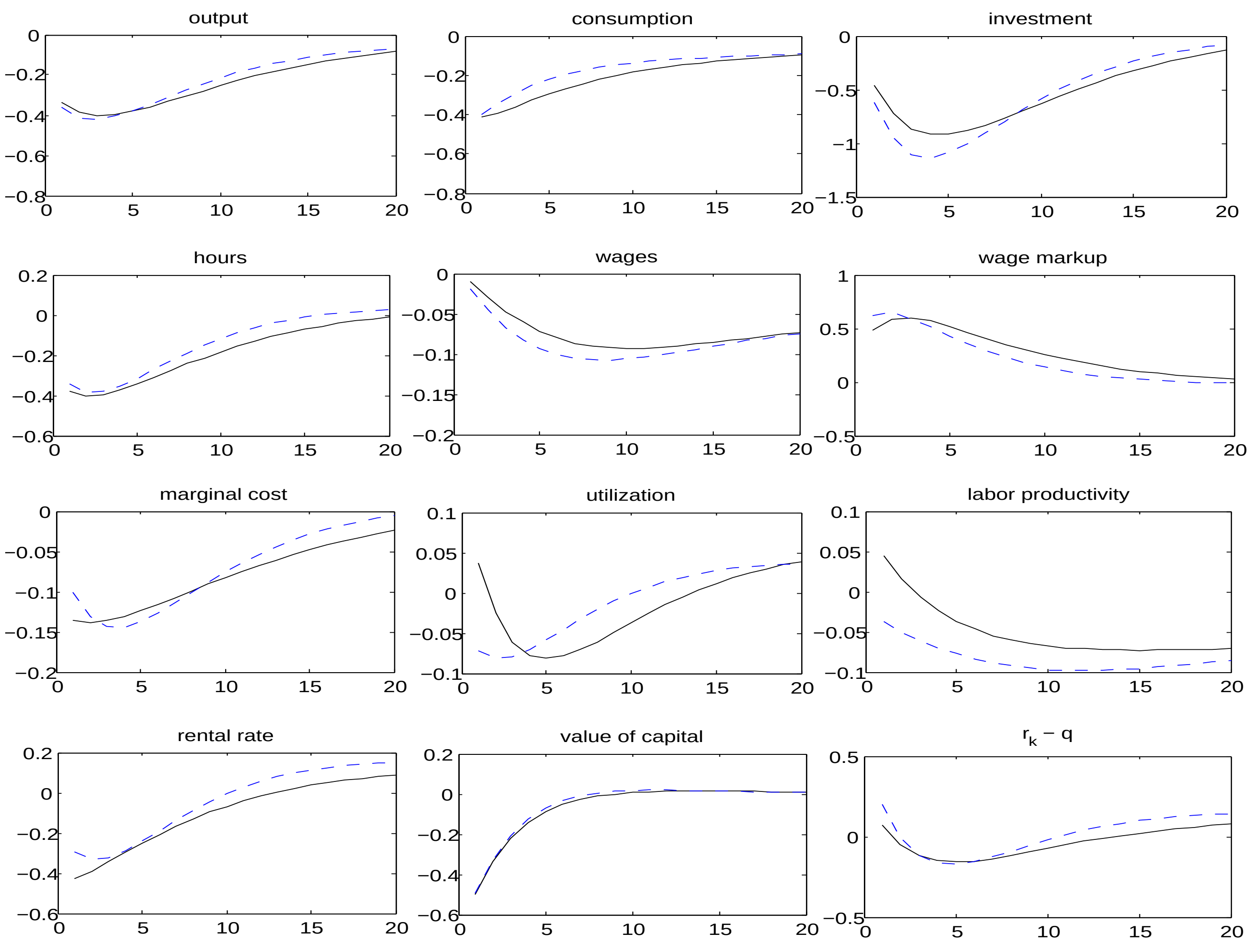

Figure 3: Impulse responses (median) to a positive investment shock (solid line is GHH specification and dotted line is CEE specification $\omega = 1$, and all other parameters at estimated values)

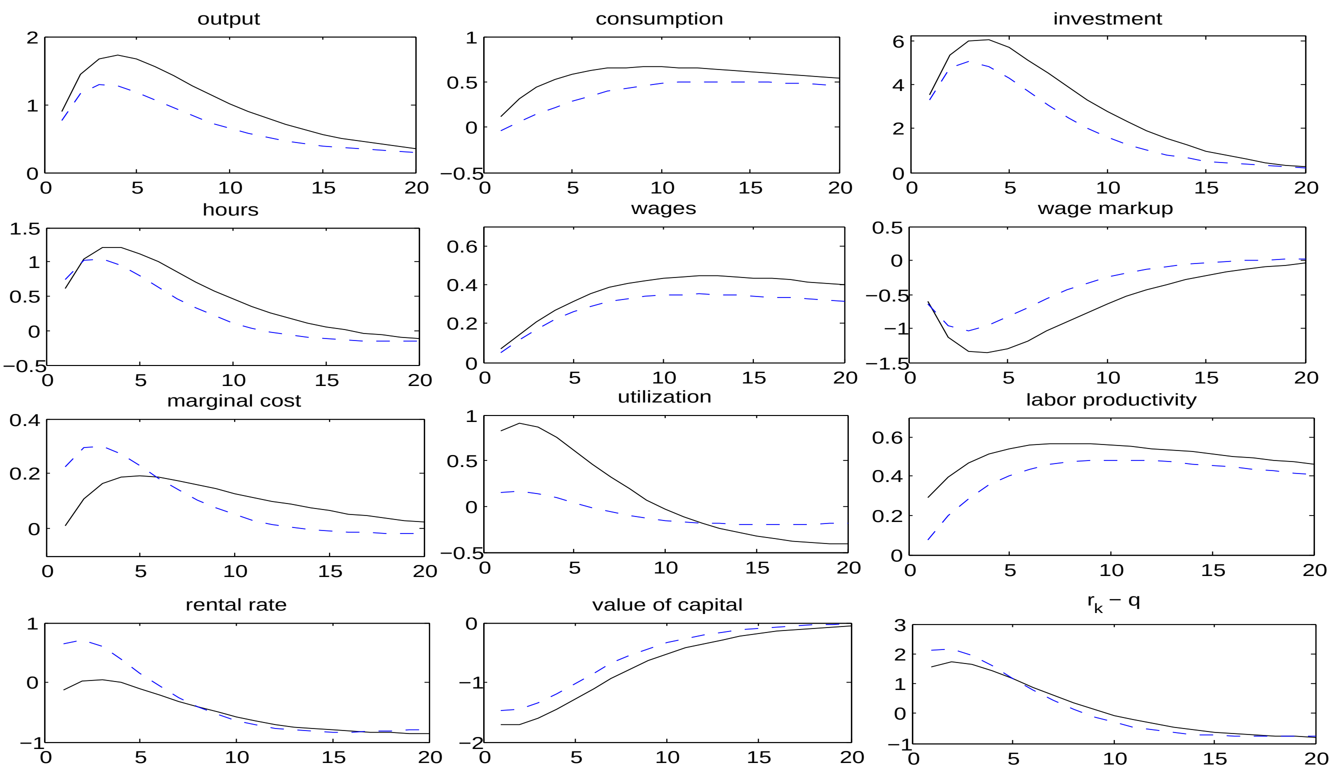

Figure 4: Impulse responses (median) to an investment shock (solid line is GHH specification and dotted line is CEE specification, $\omega = 0$ and all other parameters at estimated values)