FINANCIALLY CONSTRAINED ARBITRAGE & CROSS-MARKET CONTAGION

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Textbook Arbitrage

- Liquidity demand:
  - Agents value similar assets differently.
  - Gains from trade $\Rightarrow$ Demand for liquidity.

- Arbitrage $\Leftrightarrow$ Intermediation:
  - Unsatisfied liquidity demand $\Rightarrow$ Price wedge between similar assets.
  - Arbitrageurs exploit the price wedge.
  - $\Rightarrow$ Bring asset prices closer.
  - $\Rightarrow$ Provide liquidity to other investors.

- Finance textbook: Costless arbitrage.
  - $\Rightarrow$ Absence of Arbitrage Opportunities.
  - $\Rightarrow$ No unsatisfied demand for liquidity.
  - $\Rightarrow$ No rationale for policy intervention (Welfare Theorems).

Trouble in Paradise

- Limited arbitrage:
  - Prices can diverge from fundamental values (e.g. bubbles, crashes).
  - Contagion.

- Limited intermediation:
  - Liquidity dry-ups.
  - Liquidity linkages.

- Leading approaches:
  1. Behavioral Finance.
  2. **Agency / Financial constraints.**

Research Agenda

- Financially constrained arbitrage.
  - Arbitrageurs are "special".
  - Arbitrageurs face financial constraints.
  - $\Rightarrow$ Arbitrage capital is relevant:

Arbitrage capital $\Rightarrow$ Asset prices and liquidity $\Rightarrow$ Arbitrage capital

- Implications:
  - Investment policy by intermediaries.
  - Asset prices and market liquidity.
  - Welfare.
  - Policy.

Examples

- **Stocks + Market Makers**
  - MM: Higher inventory, low revenues.
  - $\Rightarrow$ Lower daily stock market liquidity + Contagion across different stocks.

- **Currencies + Hedge Funds**
  - Carry Trade: Borrow/invest in low/high interest rate country.
  - Lower AUM + Greater outflows.
  - $\Rightarrow$ Interest rate gap widens + Low interest rate currency appreciates.
  - Fall 2008:
    - Large outflow from hedge funds.
    - $\Rightarrow$ Low interest rate currencies appreciated (e.g. Yen vs. GBP).

This Paper

Framework

- Dynamics + Multiple assets.
- Nests standard asset pricing model.

Riskfree arbitrage

- Dynamics.
- Closed form $\Rightarrow$ Many properties.

Risky arbitrage

- Amplification.
- Contagion.
- Arbitrageurs: Stabilizing vs. destabilizing?

Literature

- Pre-1998 crisis: Tuckman and Vila (1992), Shleifer and Vishny (1997)

- Post-1998 crisis: Basak and Croitoru (2000, 2006), Xiong (2001), Liu and Longstaff (2004), Pavlova and Rigobon (2008), Zigrand (2006), Rahi and Zigrand (2007), Krishnamurthy and He (2009a, 2009b), Kondor (2009), Duffie and Strulovici (2009).

- More to come.

- Survey in Gromb and Vayanos (2010).

- Closest papers:
  - Kyle and Xiong (2001): No constraints.
  - Gromb and Vayanos (2002): Single arbitrage opportunity.
  - Brunnermeier and Pedersen (2009): Static.

Roadmap

- **Model**
- Riskfree arbitrage
- Risky arbitrage

MODEL: RISKFREE ARBITRAGE

- Continuous time, infinite horizon ($t \in \mathbb{R}^+$).

Assets

- Riskless asset with exogenous return r.
- Pairs of risky assets $(i, -i) \in \mathcal{I}^2$
  - Zero net supply.
  - Dividends

$$
\begin{aligned}
dD_{i,t} &= Ddt + \sigma dB_{i,t} \\
dD_{-i,t} &= Ddt + \sigma dB_{i,t}
\end{aligned}
$$

Regular Investors

- **Market segmentation**
  - The i-investors can only hold the riskfree and risky asset i.
  - Competitive, measure 1, wealth $w_{i,t}$.

$$\max \quad \mathsf{E}_t \int_t^{\infty} -\exp\left[-ac_s\right] \cdot \exp\left[-\gamma s\right] ds \qquad \text{with } a, \gamma > 0$$

- **Unrealized gains from trade**
  - Investors hold an endowment in shares:
    - i-investors: u_i shares of asset i $\quad i \in \mathcal{A}$
    - $-i$-investors: $-u_i$ shares of asset $-i$ $\quad i \in \mathcal{I}/\mathcal{A}$

Arbitrageurs

- Infinitely-lived, competitive, measure 1

$$\max \quad \mathsf{E}_t \int_t^\infty \log c_s \cdot \exp\left[-\beta s\right] ds \qquad \text{with } \beta > 0$$

- **"Special":** Can invest in all assets.

- **Financial constraint**
  - Long/short 1 share of asset i or $-i \Rightarrow$ Haircut $m_i > 0$.
  - $\Rightarrow$ Arbitrageurs' wealth W_t and positions $x_{i,t}$ satisfy

$$\boxed{\sum_{i \in \mathcal{I}} m_i \cdot |x_{i,t}| \leq W_t}$$

Roadmap

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- **Riskfree arbitrage**
- Risky arbitrage

EQUILIBRIUM

Notation

- Arbitrageur positions: $x_{i,t}$
- Regular investors positions: $y_{i,t}$
- Asset prices and risk premia: $p_{i,t} = \frac{D}{r} - \phi_{i,t}$

Definition: Symmetric Equilibrium

- *Arbitrageurs (optimally) enter spread trades:* $x_{i,t} = -x_{-i,t}$.
- *All risky asset markets clear:* $x_{i,t} = -y_{i,t}$.
- *Risk premia are opposites*: $\phi_{i,t} = -\phi_{i,t}$.

- **Note:** Risk premium = price wedge

$$\phi_{i,t} = \frac{p_{-i,t} - p_{i,t}}{2}$$

Arbitrageurs

- Dynamic budget constraint

$$dW_t = \underbrace{-\beta W_t dt}_{\substack{\text{consumption} \\ \text{(log utility)}}} + \underbrace{r W_t dt}_{\substack{\text{riskfree} \\ \text{return}}}$$

$$+ \sum_{i \in \mathcal{I}} x_{i,t} \underbrace{(D dt + dp_{i,t} - r p_{i,t} dt)}_{\substack{\equiv \Phi_{i,t} dt \\ \text{expected excess return per leg}}} + \underbrace{\sum_{i \in \mathcal{A}} (x_{i,t} + x_{-i,t}) \, \sigma dB_{i,t}}_{\substack{= 0 \\ \text{no dividend risk}}}$$

- By symmetry:

$$\boxed{dW_t = \left[-(\beta - r) W_t + 2 \sum_{i \in \mathcal{A}} x_{i,t} \Phi_{i,t} \right] dt}$$

Proposition 1: *Each arb maxes out his constraint with trades* $(i, -i)$ *s.t.*

$$i \in \arg\max_{j \in \mathcal{A}} \; \frac{\Phi_{j,t}}{m_j}$$

Intuition:

- Arbitrageurs face riskfree opportunities.
- $\Rightarrow$ Seek the highest "excess return on collateral".

i-investors

- Guess and verify value function:

$$V(w_{i,t}) = \exp\left[-r \cdot a \cdot w_{i,t} - b_{i,t}\right]$$

- FOC:

$$\underbrace{\Phi_{i,t}}_{\text{expected excess return}} = \underbrace{r \cdot a \cdot \sigma^2 \cdot (u_i + y_{i,t})}_{\text{marginal cost of risk}}$$

- Market clearing ($y_{i,t} = -x_{i,t}$) $\Rightarrow$

$$\boxed{\Phi_{i,t} = r \cdot a \cdot \sigma^2 \cdot (u_i - x_{i,t})}$$

Corollary 1: *All opportunities in which arbitrageurs invest yield the same return on collateral*

$$\exists \Phi_t \geq 0, \quad \forall i, \qquad \frac{\Phi_{i,t}}{m_i} = \Phi_t$$

Intuition:

- Arbitrageurs seek the highest return (on collateral).
- $\Rightarrow$ Equalization in equilibrium.

- **Preview:** Source of contagion.

Equilibrium

- Financial constraint:

$$\sum_{i \in \mathcal{I}} m_i |x_{i,t}| \leq W_t$$

- FOC:

$$m_i \Phi_t = r \cdot a \cdot \sigma^2 \cdot (u_i - x_{i,t})$$

- Dynamic budget constraint:

$$dW_t = \left[- (\beta - r) W_t + 2\Phi_t \sum_{i \in \mathcal{A}} m_i x_{i,t} \right] dt$$

Dynamics

Lemma 2:

- *If* $W_t \geq W_\infty$*:*
  - *Arbitrage capital* W_t *decreases towards* W_∞*.*
  - *Excess returns* Φ_t *increase towards* Φ_∞*.*
  - *Risk premia* $\phi_{i,t}$ *increase towards* $\phi_{i,\infty}$*.*

- *If* $W_t \leq W_\infty$*:*
  - *Arbitrage capital* W_t *increases towards* W_∞*.*
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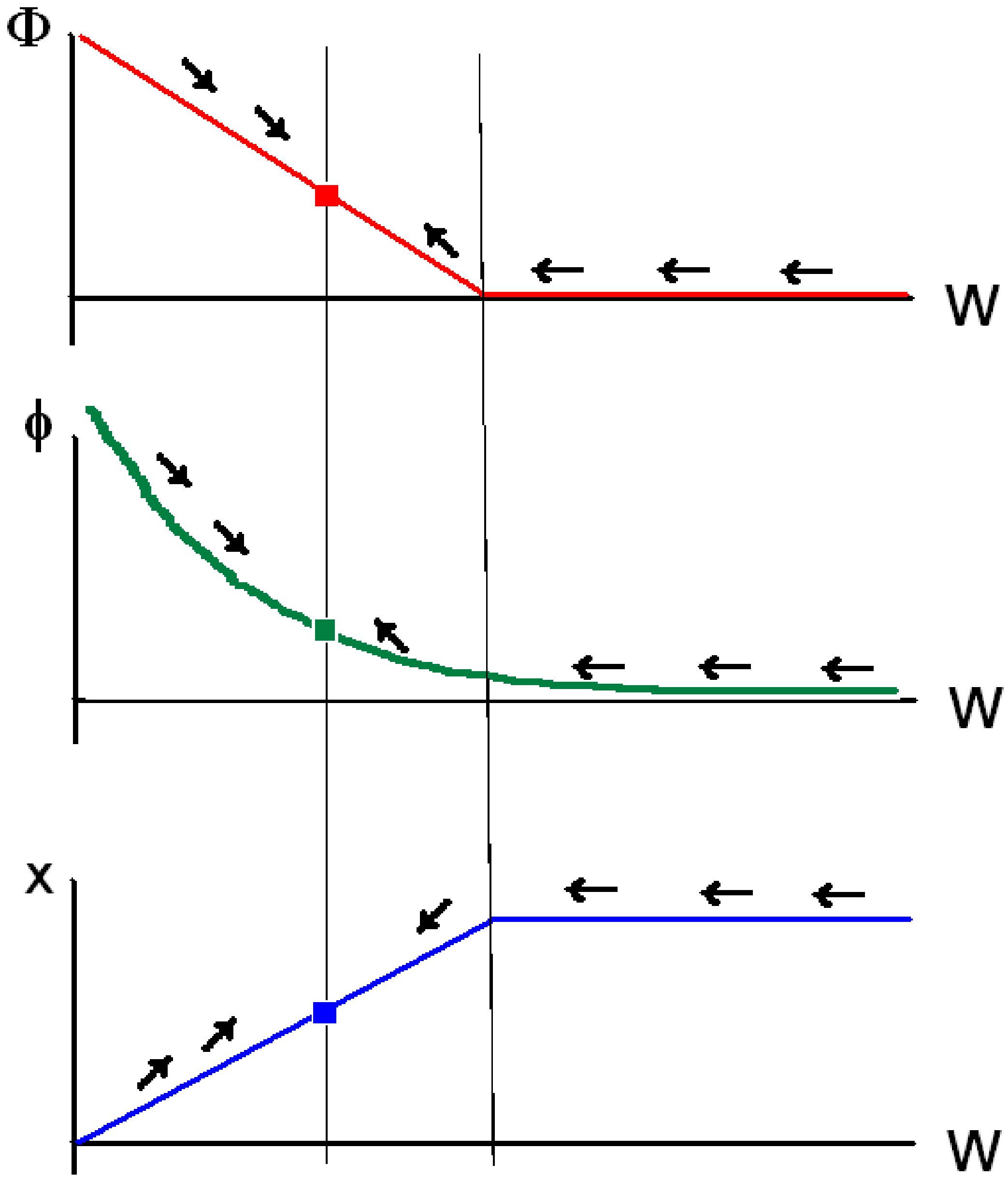
  - *Risk premia* $\phi_{i,t}$ *decrease towards* $\phi_{i,\infty}$*.*

- **Preview:** Source of predictability, mean reversion, etc.


Φ
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Roadmap

- Model
- Riskfree arbitrage
- **Risky arbitrage**

RISKY ARBITRAGE

- *Fundamental risk*: Assets i and $-i$ pay different dividends:

$$dD_{i,t} = D_i dt + \sigma_i dB_{i,t} + \sigma_i^f dB_{i,t}^f$$

$$dD_{-i,t} = D_i dt + \sigma_i dB_{i,t} - \sigma_i^f dB_{i,t}^f$$

- *Supply risk*: Endowments $u_{i,t}$ are stochastic:

$$du_{i,t} = \kappa_i^u (u_i - u_{i,t}) dt + \sigma_i^u f(u_{i,t}) dB_{i,t}^u$$

Amplification and Contagion

Lemma 3 (Contagion): *Shocks in one market affect all asset prices.*

Intuition:

- Shocks affect arbitrage capital.
- Arbitrage capital W_t affects all asset prices.

Lemma (Amplification): *"Small shocks" can have large effects.*

Intuition:

- Shocks affect arbitrage capital.
- Arbitrage capital W_t affects asset prices.
- Asset prices affect arbitrage capital.
- Etc.

Arbitrageurs: (De)Stabilizing?
Volatilities and Correlations

Proposition 7: *Premia volatility is $\cap$-shaped in arbitrage capital.*

Intuition:

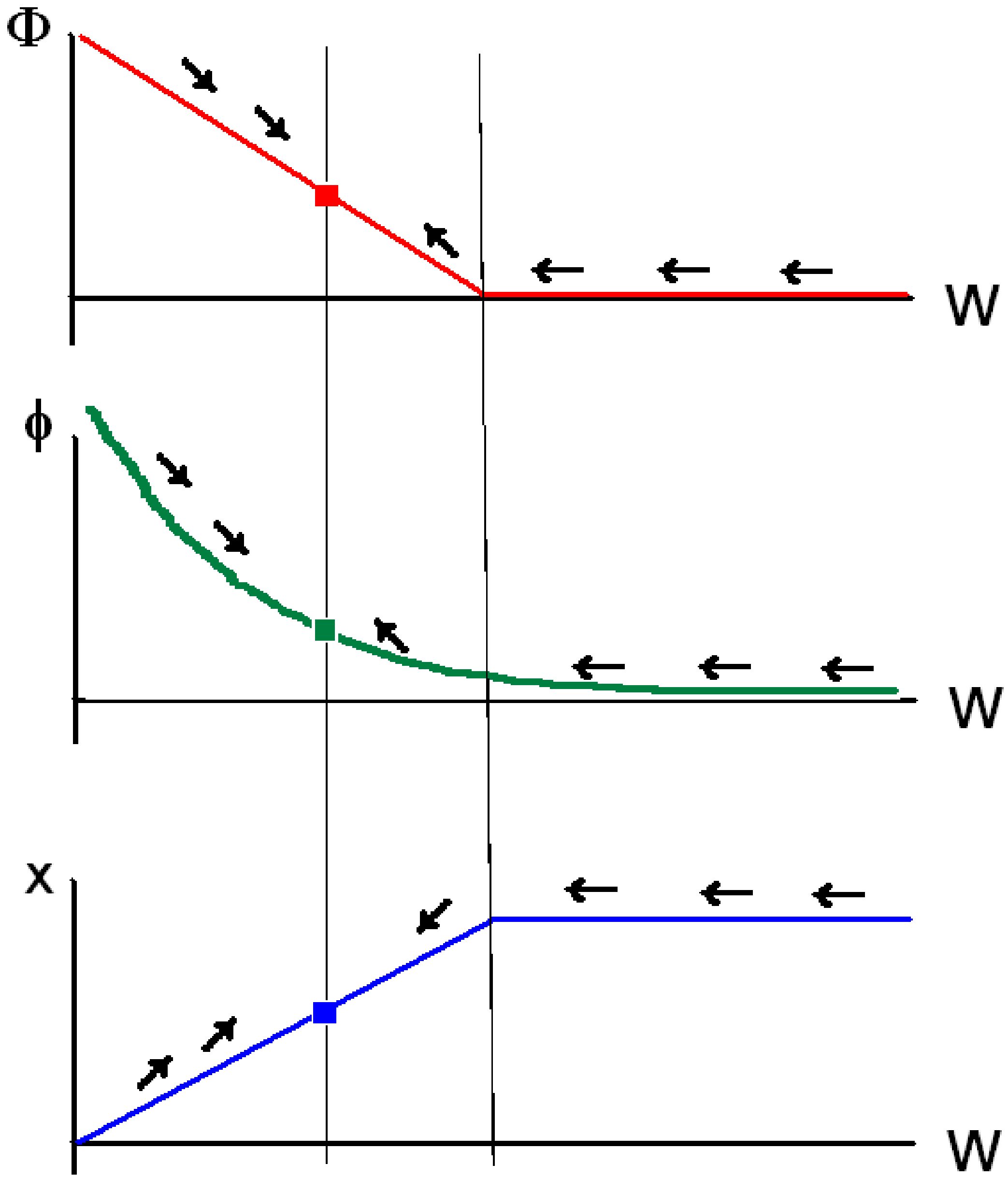
- Arbitrage capital affects premia $\Rightarrow$ Its volatility affect premia's volatility.
- Arbitrage capital's volatility is $\cap$-shaped in arbitrage capital.

Proposition 8: *Consider $i \in \mathcal{A}$:*

- *Correlation with $j \in \mathcal{A}$ is $\cap$-shaped in arbitrage capital.*
- *Correlation with $j \notin \mathcal{A}$ is U-shaped in arbitrage capital.*

Intuition:

- Arbitrage capital has positive correlation with $\mathcal{A}$ and negative with $\mathcal{I}/\mathcal{A}$.



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RESEARCH AGENDA

- Applications + Extensions:
  - Relation to standard models with incomplete markets, transaction costs, etc.
  - Diversification vs. Contagion.
  - Mobility of arbitrage capital.

- Endogenous constraints:
  - Information asymmetry? Moral hazard?
  - Technical: Optimal contract in a dynamic principal-agent model... in GE.

- Welfare:
  - Equilibrium is not constrained efficient (Gromb-Vayanos 2002).
  - $\Rightarrow$ Policy.

WELFARE

- Welfare question:
  - Arbitrageurs benefit all investors by increasing liquidity.
  - Liquidity provision depends on arbitrage capital.
  - Do arbitrageurs put their capital at risk in a socially optimal way?
  - At the heart of financial regulation.

Proposition: *The equilibrium may fail to be constrained efficient.*

- Risk management:
  - Recall: After losses, investment opportunities are more attractive.
  - $\Rightarrow$ Arbitrageurs save capital for bad times.

- Arbitrageurs' equilibrium risk taking is socially suboptimal.
  - Fail to internalize their price impact.
  - Pecuniary externality: Matters under incomplete markets (financial constraints).
  - Opens the door for discussion of policy.

Intuition:

- Suppose that arbitrageurs take less risk ex-ante.

- More capital in bad state ex-post.

  $\Rightarrow$ Smaller price discrepancy.

  $\Rightarrow$ Transfer from regular investors to arbitrageurs if the latter sell.

  $\Rightarrow$ Arbitrageurs can repay through greater liquidity provision in the future.

- Transfer to arbitrageurs can be Pareto improving!

- Fire-sale externality.

POLICY

Result 1: *Tightening arbitrageurs' constraints can increase welfare.*

- Intuition:
  - Suppose arbitrageurs are overinvested ex-ante.
  - Constraining them reduces their positions.
  - In essence, force them to do better risk management.
- Note: Not about default risk.

Result 2: *Softening competition between arbitrageurs can increase welfare.*

- Intuition: Market power $\Rightarrow$ Arbitrageurs (partly) internalize their price impact.